LETTER

Design of Linear Phase IIR Digital Filters Based on Eigenvalue Problem

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SUMMARY It is known that an anticausal IIR filter can be realized in real time by using the time reversed section technique. When combined with a causal IIR filter, the overall transfer function can yield exact linear phase characteristic in theory. This paper presents a new method for designing complex IIR digital filters with exact linear phase. The design problem of IIR filters with exact linear phase can be reduced to magnitude-only filter design. The proposed procedure is based on the formulation of an eigenvalue problem by using Remez exchange algorithm. By solving the eigenvalue problem to compute the real maximum eigenvalue, the solution of the rational interpolation problem can be achieved. Therefore, the optimal filter coefficients are easily obtained through a few iterations. The proposed design algorithm not only retains the speed inherent in Remez exchange algorithm, but also simplifies the interpolation step because it has been reduced to the computation of the real maximum eigenvalue. Several examples are presented to demonstrate the effectiveness of the proposed method.

key words: *IIR digital filter, linear phase filter, eigenvalue problem, approximation theory*

1. Introduction

In many applications of digital signal processing, digital filters are required to possess linear phase responses. It is well known [1], [2] that FIR filters with symmetric or antisymmetric impulse responses yield exact linear phase. Unfortunately, FIR filters generally require higher order than IIR filters to meet the same magnitude specifications. IIR filters can be classified into two categories, causal and noncausal. Design of causal IIR filters have been studied in [3]–[8], but its phase response is only approximately linear in passband since poles are restricted inside the unit circle. Noncausal IIR filters can yield exact linear phase when both poles and zeros exist in mirror-image pairs. To realize noncausal IIR filters, it must be divided into causal and anticausal parts. Anticausal filters can be realized by using time reversal for finite length inputs, or using the time reversed section technique for infinite length inputs [9]. It has been shown in [9] that noncausal IIR filters with exact linear phase have better performance than FIR filters and causal IIR filters. Design of noncausal IIR filters with exact linear phase only needs to optimize the magnitude response. The procedures proposed in [9] and [10] are based on the conventional analog filter theory, so the resulting digital filters are restricted to have equal order numerator and denominator. For magnitude optimization of IIR filters, some methods have also been proposed by using Remez exchange algorithm in [11] and [12]. Since all zeros are restricted on the unit circle, the algorithm of [11] used two approximation intervals and worked separately with numerator and denominator polynomials, but the extent of the designable filters is restricted. In [12], the rational interpolation problem was solved by using Newton method, and then a large amount of computational cost is required with increasing order. It is also shown in [11] and [12] that IIR filters with different order numerator and denominator are more effective than one of equal order numerator and denominator in narrow-band and wide-band applications.

Recently, complex digital signal processing have found many applications in communication, radar, sonar and so on [14]. Design of complex digital filters has attracted considerable attention. Complex filters have greater freedom than real filters. Real filters can be viewed as a special case of complex filters if the frequency response satisfies complex conjugate relation between positive and negative frequency. In this paper, we consider design of complex IIR filters with exact linear phase, and different order numerator and denominator.

The purpose of this paper is to develop a new method based on eigenvalue problem for designing complex IIR filters with exact linear phase. First of all, we derive a necessary and sufficient condition of filter coefficients for obtaining exact linear phase characteristics. According to the necessary and sufficient condition, the design problem of linear phase IIR filters can be reduced to magnitude-only filter design. We formulate the design problem in the form of an eigenvalue problem by using Remez exchange algorithm. After solving the eigenvalue problem, we can get more than one eigenvalues in general. Then, we must search for one eigenvalue that corresponds to the solution of the rational interpolation problem. In this paper, we introduce a new and very simple selection rule where the rational interpolation is performed if and only if the real maximum eigenvalue is chosen. Therefore, we can obtain the solution of the rational interpolation prob-

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lem by computing only one eigenvector corresponding to the real maximum eigenvalue. To arrive at equiripple magnitude response, we make use of an iteration procedure so that the optimal filter coefficients can be easily obtained. The new design algorithm not only retains the speed inherent in Remez exchange algorithm, but also simplifies the interpolation step because it has been reduced to the computation of the real maximum eigenvalue. In general, the design algorithm converges rapidly with a few iterations and computes efficiently without any initial guess of the solution. Several examples are designed to demonstrate the effectiveness of the proposed method.

2. IIR Filters with Exact Linear Phase

Let $H(z)$ be the transfer function of a complex IIR filter with numerator order N and denominator order M

$$H(z) = \frac{\sum_{n=0}^{N} a_n z^{-n}}{\sum_{m=0}^{M} b_m z^{-m}} = H_1(z^{-1})H_2(z), \tag{1}$$

where $a_n = a_{nr} + ja_{ni}$ and $b_m = b_{mr} + jb_{mi}$ are complex coefficients. $H(z)$ is an arbitrary transfer function whose poles lie inside and outside the unit circle, and can be allocated to two transfer functions $H_1(z^{-1})$ and $H_2(z)$. Both $H_1(z)$ and $H_2(z)$ are causal IIR filters. It has been shown in [9] that the anticausal filter $H_1(z^{-1})$ can be realized by using time reversal for finite length inputs, or using the time reversed section technique for infinite length inputs. Therefore, $H(z)$ can be realized in real time.

To obtain exact linear phase response, the zeros of $H(z)$ have to lie on the unit circle or occur in mirror-image pairs, and the poles have to occur in mirror-image pairs off the unit circle. Therefore, the filter coefficients satisfy $a_n = a^*_{N-n}$ and $b_m = b^*_{M-m}$, where x^* indicates complex conjugate of x. The magnitude response of $H(z)$ can be obtained by

$$|H(e^{j\omega})| = \frac{N(\omega)}{D(\omega)} = \frac{\sum_{n=0}^{N} c_n TC_n(\omega)}{\sum_{m=0}^{M} d_m TD_m(\omega)}, \tag{2}$$

where when N is even,

$$c_n = \begin{cases} a_{(n+N/2)r} & 0 \leqq n \leqq \frac{N}{2} \\ a_{ni} & \frac{N}{2} < n \leqq N \end{cases}, \tag{3}$$

$$TC_n(\omega) = \begin{cases} 0.5 & n = 0 \\ \cos n\omega & 1 \leqq n \leqq \frac{N}{2} \\ \sin(n - \frac{N}{2})\omega & \frac{N}{2} < n \leqq N \end{cases}, \tag{4}$$

and when N is odd,

$$c_n = \begin{cases} a_{(n+(N+1)/2)r} & 0 \leqq n \leqq \frac{N-1}{2} \\ a_{ni} & \frac{N+1}{2} \leqq n \leqq N \end{cases}, \tag{5}$$

$$TC_n(\omega) = \begin{cases} \cos(n + \frac{1}{2})\omega & 0 \leqq n \leqq \frac{N-1}{2} \\ \sin(n - \frac{N}{2})\omega & \frac{N+1}{2} \leqq n \leqq N \end{cases}. \tag{6}$$

M must be even since the poles are not allowed to locate on the unit circle,

$$d_m = \begin{cases} b_{(m+M/2)r} & 0 \leqq m \leqq \frac{M}{2} \\ b_{mi} & \frac{M}{2} < m \leqq M \end{cases}, \tag{7}$$

$$TD_m(\omega) = \begin{cases} 0.5 & m = 0 \\ \cos m\omega & 1 \leqq m \leqq \frac{M}{2} \\ \sin(m - \frac{M}{2})\omega & \frac{M}{2} < m \leqq M \end{cases}. \tag{8}$$

Therefore, the design problem can be reduced to magnitude-only filter design.

3. Design of Linear Phase IIR Filters

In this section, we describe design of linear phase IIR filters with optimum magnitude response in the Chebyshev sense based on eigenvalue problem. When N and M are given, and the desired magnitude response $|H_d(e^{j\omega})|$ is specified in the interest bands $R \in [-\pi, \pi]$ (e.g., passband and stopband), the aim is to find a set of filter coefficients a_n and b_m to minimize the maximum error between the magnitude response and the desired magnitude response. Therefore, we want to find c_n and d_m in Eq. (2) in such a way that the magnitude function with a positive denominator satisfies

$$W(\omega)|E(\omega)| = W(\omega)||H(e^{j\omega})| - |H_d(e^{j\omega})|| \leq \delta \qquad (\omega \in R), \tag{9}$$

where $E(\omega)$ is an error function, $W(\omega)$ is a weighting function, and δ (> 0) is the maximum error to be minimized.

To solve the magnitude Chebyshev approximation problem, we utilize Remez exchange algorithm and formulate the condition for $|H(e^{j\omega})|$ of Eq. (2) in the form of an eigenvalue problem. By selecting extremal frequencies ω_i ($i = 0, 1, \cdots, N + M + 1$) in the bands R , we formulate $|H(e^{j\omega})|$ as

$$W(\omega_i)E(\omega_i) = (-1)^{(i+l)}\delta, \tag{10}$$

where $l = 0$ or 1 to guarantee $\delta > 0$, and the denominator polynomial $D(\omega)$ must satisfy

$$D(\omega) \neq 0 \qquad \text{(for all } \omega). \tag{11}$$

Substituting Eq. (2) into Eq. (10), we can rewrite Eq. (10) in the matrix form as

$$\boldsymbol{P}\,\boldsymbol{A} = \delta \boldsymbol{Q}\,\boldsymbol{A}\,, \tag{12}$$

where $\boldsymbol{A} = [c_0, c_1, \cdots, c_N, d_0, d_1, \cdots, d_M]^T$, and the elements of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ are given by

$$P_{ij} = \begin{cases} TC_j(\omega_i) & j = 0, 1, \cdots, N \\ -|H_d(e^{j\omega_i})| TD_{j-N-1}(\omega_i) & , \\ & j - N - 1 = 0, 1, \cdots, M \end{cases} \tag{13}$$

$$Q_{ij} = \begin{cases} 0 & j = 0, 1, \cdots, N \\ \dfrac{(-1)^{(i+l)}}{W(\omega_i)} TD_{j-N-1}(\omega_i) & . \\ & j - N - 1 = 0, 1, \cdots, M \end{cases} \tag{14}$$

Once the desired magnitude response $|H_d(e^{j\omega})|$ and the weighting function $W(\omega)$ are given, it is seen from Eqs. (13) and (14) that the elements of the matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are known. Therefore, it should be noted that Eq. (12) corresponds to a generalized eigenvalue problem, i.e., δ is an eigenvalue and $\boldsymbol{A}$ is a corresponding eigenvector. It is well known that there is a nontrivial solution $\boldsymbol{A}$ in Eq. (12) if and only if the determinant satisfies

$$|\boldsymbol{P} - \delta \boldsymbol{Q}| = 0\,. \tag{15}$$

Since $\boldsymbol{P}$ and $\boldsymbol{Q}$ are $(N+M+2)$ by $(N+M+2)$ matrices, Eq. (15) has more than one solutions of δ in general. Therefore, we can obtain at least two solutions by solving the eigenvalue problem of Eq. (12). To minimize the maximum magnitude error, the filter coefficients must satisfy the condition of Eq. (11). However, it is not guaranteed that the solutions obtained from Eq. (12) have satisfied Eq. (11). Therefore, we must search for which solution satisfies Eq. (11) in the obtained solutions. When $\boldsymbol{P}$ is a singular matrix, we can get $\delta = 0$ from Eq. (15), hence a solution can be obtained by solving the linear equations $\boldsymbol{P}\,\boldsymbol{A} = \mathbf{o}$. If the solution satisfies Eq. (11), then we have obtained the desired magnitude response. However, it is generally impossible to obtain the desired magnitude response in the practical design problem. Therefore, $\boldsymbol{P}$ is a nonsingular matrix in general. For example, it can be easily proven that the matrix $\boldsymbol{P}$ is nonsingular in the cases of lowpass filters, bandpass filters and so on. Equation (12) can be rewritten into the standard eigenvalue problem;

$$\boldsymbol{T}\,\boldsymbol{A} = \lambda \boldsymbol{A}\,, \tag{16}$$

where $\boldsymbol{T} = \boldsymbol{P}^{-1}\boldsymbol{Q}$, and $\lambda = 1/\delta$. Here, will we ask whether Eq. (16) has a solution that satisfies Eq. (11)? If exists, does which eigenvalue correspond to the solution? We see from Eq. (10) that the sign change of $E(\omega)$ is caused by the sign change of whose numerator or denominator polynomial. When the numerator polynomial changes its sign, $E(\omega)$ crosses 0 to change its sign. When the denominator polynomial changes its sign, $E(\omega)$ crosses ∞. Therefore, there exist more than one solutions depending on the sign change of $E(\omega)$ through 0 or ∞. To satisfy Eq. (11), $E(\omega)$ must change its sign through 0. When the optimum Chebyshev approximation to the desired response exists, there are $(N+M+2)$ extremal frequencies of $E(\omega)$ [15], [16]. Hence Eq. (16) has at least one solution that satisfies Eq. (11) if the extremal frequencies are appropriately selected. By the uniqueness of the optimal solution, the solution is unique. Now, we answer the second question. In Eq. (10), we can choose $l = 0$ or 1 to guarantee the solution that satisfies Eq. (11) having a positive error δ. Therefore, we seek through only the positive and real eigenvalues.

Theorem 1: The real maximum eigenvalue corresponds to the solution that satisfies Eq. (11) when the optimum Chebyshev approximation exists.

Proof: Let $|H_o(e^{j\omega})|$ be the solution with $\delta_o(> 0)$ that satisfies Eq. (11), $|H_n(e^{j\omega})|$ another solution with $\delta_n(> 0)$ that doesn't satisfy Eq. (11), and $\hat{H}(\omega) = |H_o(e^{j\omega})| - |H_n(e^{j\omega})| = E_o(\omega) - E_n(\omega)$.

A) Assume that $\delta_o = \delta_n$, we have $\hat{H}(\omega_i) = 0$ from Eq. (10), then $\hat{H}(\omega)$ has $(N + M + 2)$ zeros in $(-\pi, \pi]$. However, $\hat{H}(\omega)$ has at most $(N + M)$ zeros in $(-\pi, \pi]$. Therefore, we can conclude that $\delta_o \neq \delta_n$.

B) Assume that $\delta_o > \delta_n$. It is seen in Fig. 1 that $\hat{H}(\omega)$ has one zero in the interval $[\omega_i, \omega_{i+1}]$ when $E_n(\omega)$ crosses 0 to change its sign, and two zeros when $E_n(\omega)$ crosses ∞. There are $(N + M + 2)$ interpolated intervals in $(-\pi, \pi]$ (including transition band). We suppose that there are I intervals where $E_n(\omega)$ changes its sign through ∞, hence $\hat{H}(\omega)$ has $(N + M + I + 2)$ zeros in $(-\pi, \pi]$. However, $\hat{H}(\omega)$ has at most $(N + M)$ zeros in $(-\pi, \pi]$. Therefore, we can conclude that $\delta_o < \delta_n$. $\lambda_o = 1/\delta_o$ is the real maximum eigenvalue. The theorem is proven.

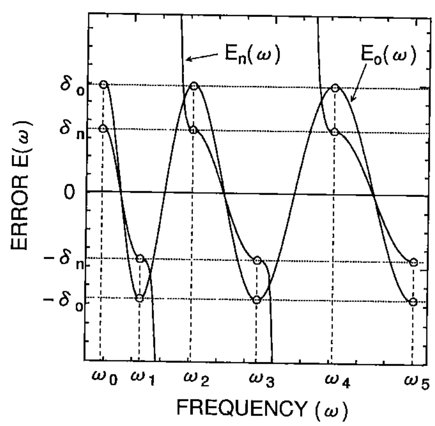


Fig. 1 Interpolation of $E(\omega)$.

We have proven that the real maximum eigenvalue corresponds to the solution that satisfies Eq. (11). Therefore, we can obtain the solution of the rational interpolation problem by finding only one eigenvector corresponding to the real maximum eigenvalue. Since we are interested in only the real maximum eigenvalue, this computation can be done efficiently by using the iterative power method without invoking general methods such as the QR technique [17], [18]. In order to obtain an equiripple magnitude response, we make use of an iteration procedure to get the optimal filter coefficients. Since we have obtained the solution that satisfies Eq. (11), we assume that the denominator polynomial is positive without any loss in generality, and can consider it as a weighting function in the FIR applications. Therefore, the algorithm converges in general with a few iterations as same as the design of FIR linear phase filters. The design algorithm is shown as follows.

Procedure {Design Algorithm of Linear Phase IIR Filters}

Begin

1. Read numerator order N and denominator order M, the desired magnitude response $|H_d(e^{j\omega})|$, and weighting function $W(\omega)$.
2. Select initial extremal frequencies Ω_i $(i = 0, 1, \cdots, N+M+1)$ in the interest bands R.

Repeat

3. Set $\omega_i = \Omega_i$ $(i = 0, 1, \cdots, N+M+1)$.
4. Compute $\boldsymbol{P}, \boldsymbol{Q}$ by using Eqs. (13) and (14), then find the real maximum eigenvalue to obtain the filter coefficients c_n and d_m that satisfies Eq. (11).
5. Search the peak frequencies $\hat{\omega}_i$ $(i = 0, 1, \cdots, J)$ of the error function $E(\omega)$ within R.
6. Reject the $(J - N - M - 1)$ superfluous peak frequencies and store the remaining frequencies into the corresponding Ω_i.

Until Satisfy the following condition for the prescribed small constant ϵ :

$$\{\ |\Omega_i - \omega_i| \leqq \epsilon \quad (\text{for } i = 0, 1, \cdots, N+M+1)\ \}$$

End .

4. Relations of Real-Valued Filters

In this section, we discuss design of linear phase IIR filters with real coefficients. Design of a real-valued filter with real coefficients a_n and b_m is included in the complex filter design problem as a special case if

$$\begin{cases} H_d(e^{-j\omega}) = H_d^*(e^{j\omega}) \\ H(e^{-j\omega}) = H^*(e^{j\omega}) \end{cases} . \tag{17}$$

Therefore, the above design algorithm can be used to obtain the real filter coefficients by the condition of Eq. (17). Unlike the case of complex filters, the approximation can be done only in the frequency range $[0, \pi]$ in the case of real filters. Hence, the computation can be greatly decreased. For obtaining a linear phase IIR filter with real coefficients, the poles of $H(z)$ have to occur in complex conjugate and mirror-image pairs in quadruplets or in reciprocal pairs on the real axis all off the unit circle, while the zeros occur in complex conjugate and mirror-image pairs, or in reciprocal pairs on the real axis, or in complex conjugate pairs on the unit circle, or at $z = \pm 1$. Therefore M must be even, $b_m = b_{M-m}$ and $a_n = \pm a_{N-n}$. By the symmetric or antisymmetric a_n and odd or even N, there are the following four types of filters. We assume that Type I is the filter with even N and $a_n = a_{N-n}$, Type II odd N and $a_n = a_{N-n}$, Type III even N and $a_n = -a_{N-n}$, and Type IV odd N and $a_n = -a_{N-n}$. Similar to linear phase FIR filters, Type I filters can be used to design all kinds of filters, but Type II, III and IV filters have some restrictions, e.g., Type II filters cannot be used to design highpass filters because one zero locates at $z = -1$. The magnitude response of $H(z)$ can be expressed as

$$|H(e^{j\omega})| = F(\omega)\,\frac{c_0 + 2\sum_{n=1}^{L} c_n \cos n\omega}{d_0 + 2\sum_{m=1}^{M/2} d_m \cos m\omega}, \tag{18}$$

where

$$L = \begin{cases} N/2 & \text{(Type I)} \\ (N-1)/2 & \text{(Type II)} \\ N/2 - 1 & \text{(Type III)} \\ (N-1)/2 & \text{(Type IV)} \end{cases}, \tag{19}$$

$$c_n = \begin{cases} a_{L+n} & 0 \leqq n \leqq L \\ & \text{(Type I)} \\ \begin{cases} a_{L+n+1} - c_{n+1} & 0 \leqq n \leqq L-1 \\ a_N & n = L \end{cases} \\ & \text{(Type II)} \\ \begin{cases} c_{n+2} - a_{L+n+2} & 0 \leqq n \leqq L-2 \\ -a_{L+n+2} & L-1 \leqq n \leqq L \end{cases} \\ & \text{(Type III)} \\ \begin{cases} c_{n+1} - a_{L+n+1} & 0 \leqq n \leqq L-1 \\ -a_N & n = L \end{cases} \\ & \text{(Type IV)} \end{cases} \tag{20}$$

$$d_m = b_{M/2+m} \qquad 0 \leqq m \leqq M/2. \tag{21}$$

and

$$F(\omega) = \begin{cases} 1 & \text{(Type I)} \\ 2\cos\dfrac{\omega}{2} & \text{(Type II)} \\ 2\sin\omega & \text{(Type III)} \\ 2\sin\dfrac{\omega}{2} & \text{(Type IV)} \end{cases} \quad (22)$$

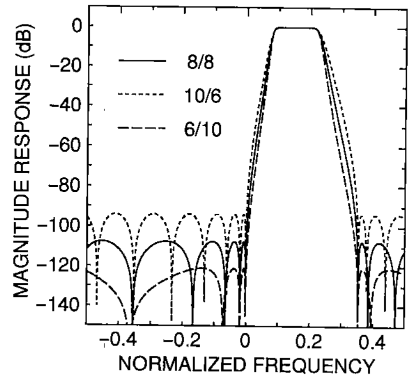
Therefore, we can similarly formulate $|H(e^{j\omega})|$ of Eq. (18) in the form of an eigenvalue problem. The matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ in Eq. (12) will become $(L+M/2+2)$ by $(L+M/2+2)$ matrices, hence the computation can be greatly decreased. FIR filters are included in IIR digital filters as a special case if $M=0$. When $M=0$, linear phase IIR filters will become linear phase FIR filters, and Eq. (12) degenerates into a set of linear equations which is the well-known McClellan–Parks algorithm [1], [2]. In the McClellan-Parks algorithm, the linear equations are not required to solve since the Lagrange interpolation formula is employed. However, the eigenvalue problem is required to solve in our method.

5. Design Examples

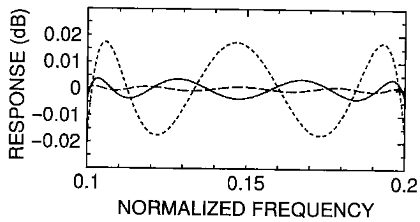
In this section, we present several design examples to demonstrate the effectiveness of the proposed method, and compare the performance of the filters with different order numerator and denominator.

Example 1: We consider design of linear phase IIR filters with $N+M=16$, passband $[0.2\pi, 0.4\pi]$ and stopband $[-\pi, 0], [0.7\pi, \pi]$. The weighting function is set to $(1, 100)$ in the passband and stopband. First, we designed one filter with $N=M=8$. The resulting magnitude response is shown in solid line in Fig. 2, and the stopband attenuation 107.7 dB is obtained. We have also designed two filters with $N=10$ and $M=6$ or $N=6$ and $M=10$. The magnitude responses are shown in Fig. 2 also, and the stopband attenuations are 94 dB and 121.1 dB respectively. It is seen in Fig. 2 that the magnitude errors decrease when N decreases and M increases. Therefore, the filters with more poles and less zeros are effective in narrow passband applications. Next, we compare the number of multipliers per sample of the filters with different N and M. It is known in [9] that the anticausal filter $H_1(z)$ is required to implement twice in the time reversed section scheme. Hence we decompose $H(z)$ into the all-pole filter $H_1(z^{-1})$ and $H_2(z)$ that has all zeros of $H(z)$. $H_1(z)$ and $H_2(z)$ are implemented in direct-form structures. $H_1(z)$ requires $M/2$ complex multipliers, and $H_2(z)$ is $M/2+N/2+1$. Therefore, a total of $3M/2+N/2+1$ complex multipliers are required. The filter of $N=M=8$ requires 17 complex multipliers, and the filters of $N=10, M=6$ and $N=6, M=10$ are 15 and 19 respectively. The filters with $N<M$ require most multipliers.

Example 2: We consider design of linear phase IIR filters with $N+M=12$, passband $[0.2\pi, \pi]$ and stopband $[-0.8\pi, -0.2\pi]$. The weighting function is set to $(1, 10000)$ in the passband and stopband. Three filters with $N=M=6$ or $N=8$ and $M=4$ or $N=4$ and $M=8$ are designed, and the magnitude responses are shown in Fig. 3. The stopband attenuations of the

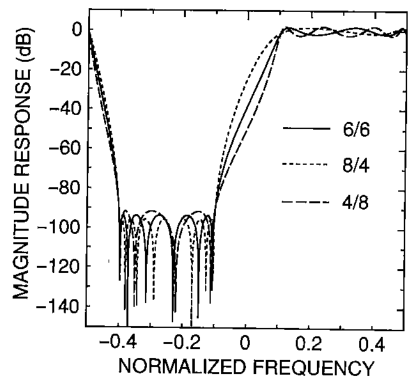


(a) Log magnitude in *dB*.

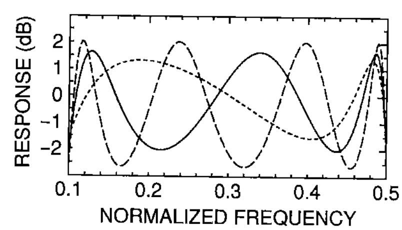


(b) Passband detail.

Fig. 2 Magnitude responses of Example 1.



(a) Log magnitude in *dB*.

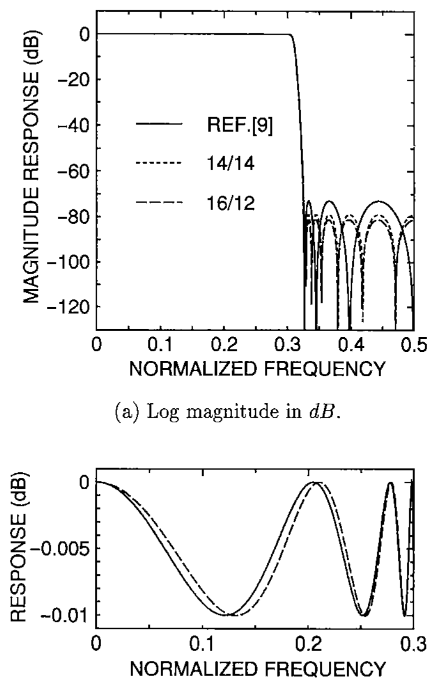


(b) Passband detail.

Fig. 3 Magnitude responses of Example 2.

filters are 93.64 dB, 95.66 dB and 91.64 dB respectively, then the filter of $N = 8$ and $M = 4$ has best magnitude response. We also compare the number of multipliers per sample of the filters. The filter of $N = 8$ and $M = 4$ requires 11 complex multipliers, while the filters of $N = M = 6$ and $N = 4, M = 8$ are 13 and 15 respectively. Therefore, the filters with less poles and more zeros are effective in wide passband applications.

Example 3: We consider design of real IIR filters of [9] with passband $[0, 0.6\pi]$, stopband $[0.65\pi, \pi]$, and passband attenuation 0.01 dB for comparison purposes. In [9], the filter of $N = M = 14$ is designed and the stopband attenuation 73 dB is obtained. The magnitude response of the filter is shown in solid line in Fig. 4, and isn't optimal in the Chebyshev sense because all zeros of $H(z)$ are double zeros on the unit circle. We have designed the filter of $N = M = 14$ with a weighting function $(1, 10.26)$ in the passband and stopband, and the magnitude response is shown in dotted line in Fig. 4. The passband and stopband attenuations are 0.01 dB and 79 dB respectively, and is optimal in the Chebyshev sense which is the same as the result of [10]. There is approximately 6 dB improvement in stopband attenuation. We also designed one filter of $N = 16$ and $M = 12$ with a weighting function $(1, 13.3)$ in the passband and stopband. The magnitude response is shown in breaked line in Fig. 4 also. The stopband attenuation is 81.3 dB and biggest in the above filters, while the passband attenuation is 0.01 dB. To satisfy the above specification, the order of FIR filters is required to exceed 140. The filters with different N and M cannot be designed by using procedures of [9] and [10].



(a) Log magnitude in dB.

(b) Passband detail.

Fig. 4 Magnitude responses of Example 3.

6. Conclusions

In this paper, we have proposed a new method for designing linear phase IIR filters with exact linear phase and optimum magnitude response in the Chebyshev sense. The design procedure is based on the formulation of an eigenvalue problem using Remez exchange algorithm. We have introduced a new and very simple selection rule where the rational interpolation is performed if and only if the real maximum eigenvalue is chosen. Therefore, the solution of the rational interpolation problem can be achieved by computing only one eigenvector corresponding to the real maximum eigenvalue, and the optimal filter coefficients can be easily obtained through a few iterations. The new design algorithm not only retains the speed inherent in Remez exchange algorithm, but also simplifies the interpolation step because it has been reduced to the computation of the real maximum eigenvalue. The proposed procedure can be extended to design of IIR digital filters with arbitrary frequency responses.

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