PAPER

# A New Class of Complex Compact-Supported Orthonormal Symmlets

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**SUMMARY** This paper presents a new class of complex-valued compact-supported orthonormal symmlets. Firstly, some properties of complex-valued compact-supported orthonormal symmlets are investigated, and then it is shown that complex-valued symmlets can be generated by real-valued half-band filters. Therefore, the construction of complex-valued symmlets can be reduced to the design of real-valued half-band filters. Next, a design method of real-valued half-band FIR filters with some flatness requirements is proposed. For the maximally flat half-band filters, a closed-form solution is given. For the filter design with a given degree of flatness, the design problem is formulated in the form of linear system by using the Remez exchange algorithm and considering the given flatness condition. Therefore, a set of filter coefficients can be easily computed by solving a set of linear equations, and the optimal solution is obtained through a few iterations. Finally, some design examples are presented to demonstrate the effectiveness of the proposed method.
***key words:*** *symmlets, orthonormal wavelets, compact-supported wavelets, complex-valued wavelets*

## 1. Introduction

The discrete wavelet transform (DWT), which is implemented by a two-band perfect reconstruction filter bank (PRFB), has been extensively used in many digital signal and image processing applications [1]–[5]. Among the numerous existing wavelets, real-valued compact-supported orthonormal wavelets have been most widely used. The associated PRFB's have real-valued and finite impulse responses (FIR). In many applications such as image coding, one desirable property for wavelets is symmetry. It is well-known that there does not exist any nontrivial real-valued compact-supported orthonormal symmetric wavelets (called symmlets), except for the Haar wavelet [7]. To get symmlets, at least one of the above properties has to be given up. One possible solution to this dilemma is to construct complex-valued symmlets [8], [10], [12]. In case of real-valued symmlets, it is known that the associated PRFB's are exactly linear phase. However, for complex-valued symmlets, symmlets may be symmetric or conjugate-symmetric. If it is conjugate-symmetric, then the associated PRFB's are required to be exactly linear phase, which had been proved to be impossible also [12]. Hence, the only possibility is to construct symmetric (not conjugate-symmetric) symmlets, where the associated PRFB's are not exactly linear phase. Notice that there is a wrong explanation in [10]. A class of complex-valued compact-supported orthonormal symmlets has been presented in [8] and [12], which are constructed from the Daubechies's real-valued wavelets, and has been also applied to image subband coding in [8] and [10]. It is known [5], [8] that symmlets can be efficiently implemented since its symmetry, and the symmetric extension method can be employed when it is applied to image subband coding.

In this paper, we present a new class of complex-valued compact-supported orthonormal symmlets. Firstly, we investigate some properties of complex-valued compact-supported orthonormal symmlets and relation between symmlets and half-band filters. We show that complex-valued symmlets can be generated by a real-valued half-band filter, then the construction of complex-valued symmlets can be reduced to the design of real-valued half-band filters. Next, we propose a design method of real-value half-band FIR filters with some flatness requirements. For the maximally flat half-band filters, we give a closed-form solution. For the filter design with a given degree of flatness, we formulate the design problem in the form of linear system by using the Remez exchange algorithm and considering the given flatness condition. Therefore, a set of filter coefficients can be easily computed by solving a set of linear equations, and the optimal solution is obtained through a few iterations. Finally, we present some design examples to demonstrate the effectiveness of the proposed method.

## 2. Complex-Valued Orthonormal Symmlets

Assume that $\psi(t)$ is a basic wavelet function, the wavelet transform (WT) to $f(t)$ ($f \in \mathbf{L}^2(\mathbf{R})$) is defined as

$$F_W(a,b) = \frac{1}{a^{\frac{1}{2}}} \int_{-\infty}^{\infty} f(t)\bar{\psi}\left(\frac{t-b}{a}\right) dt, \tag{1}$$

where $\bar{x}$ denotes complex conjugate of $x$, and the dilation/contraction and translation parameters are $a \in \mathbf{R}^+$, $b \in \mathbf{R}$. When discretized, $a = 2^{-k}$ and $b = 2^{-k}m$ ($k, m$ : integer), in general.

It is well-known that the dyadic wavelet bases can

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be generated by a two-band PRFB $\{H(z), G(z)\}$, where $H(z)$ is a lowpass filter and $G(z)$ highpass. The scaling function $\phi(t)$ and wavelet function $\psi(t)$ are related with $\{H(z), G(z)\}$ in frequency domain as follows;

$$\begin{cases} \hat{\phi}(\omega) = H(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) = \prod_{k=1}^{\infty} H(e^{j2^{-k}\omega}) \\ \hat{\psi}(\omega) = G(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) \end{cases}, \quad (2)$$

where $\hat{\phi}(\omega), \hat{\psi}(\omega)$ are the Fourier transforms of $\phi(t)$ and $\psi(t)$, respectively. If both $H(z)$ and $G(z)$ are FIR filters, then the generated wavelet bases are compact-supported. The orthonormality condition that $H(z)$ and $G(z)$ have to satisfy is

$$\begin{cases} H(z)\bar{H}(z^{-1}) + H(-z)\bar{H}(-z^{-1}) = 1 \\ G(z)\bar{G}(z^{-1}) + G(-z)\bar{G}(-z^{-1}) = 1 \\ H(z)\bar{G}(z^{-1}) + H(-z)\bar{G}(-z^{-1}) = 0 \end{cases}, \quad (3)$$

where $\bar{H}(z)$ has a set of coefficients that are complex conjugate with ones of $H(z)$. Assume that $H(z)$ is a FIR filter of order $N$ with complex-valued coefficients $h_n$;

$$H(z) = \sum_{n=0}^{N} h_n z^{-n}, \quad (4)$$

where $N$ is odd. To be symmlets, its impulse response has to be symmetric or antisymmeric, i.e.,

$$h_n = \pm h_{N-n}. \quad (5)$$

However, it should be noted that if $h_n$ is antisymmetric, then the magnitude of $H(z)$ becomes zero at $\omega = 0$, i.e., $H(1) = 0$, so that it is not lowpass filter. Hence, the impulse response must be symmetric. The frequency response of $H(z)$ is given by

$$\begin{aligned} H(e^{j\omega}) &= e^{-j\frac{N}{2}\omega} \sum_{n=0}^{(N-1)/2} h_n \cos\left(\frac{N}{2} - n\right)\omega \\ &= e^{-j\frac{N}{2}\omega}\hat{H}(e^{j\omega}). \end{aligned} \quad (6)$$

It is clear that the phase response is not linear since $h_n$ is complex-valued. Only if $h_n$ is real-valued, $H(z)$ will be exactly linear phase. Unfortunately, it is well-known that the only solution for real-valued compact-supported orthonormal symmlets is the Haar wavelet. For complex-valued filters, in order to have exactly linear phase, the coefficients should be conjugate-symmetric, i.e., $h_n = \pm\bar{h}_{N-n}$. However, it has been proved in [12] that there does not exist any nontrivial complex-valued solution also. Notice that the symmetry condition of Eq. (5) is different from the linear phase condition. Hence, there exist some solutions for complex-valued compact-supported orthonormal symmlets. It can be seen in Eq. (6) that $\hat{H}(e^{j\omega})$ is a polynomial of $\cos\omega$ and then an even function. From the orthonormality condition of Eq. (3), the highpass filter $G(z)$ can be constructed from $H(z)$ as

$$G(z) = z^{-N}\bar{H}(-z^{-1}), \quad (7)$$

whose impulse response $g_n$ is antisymmetric, i.e.,

$$g_n = -g_{N-n}. \quad (8)$$

As a result, the generated scaling function is symmetric, while the wavelet function is antisymmetric.

## 3. Design of Symmlet Filters

In this section, we describe how to design the filter $H(z)$ associated with complex-valued compact-supported orthonormal symmlets.

### 3.1 Property

Since $H(z)$ is complex filter, it is difficult to approximate simultaneously its magnitude and phase responses. Before designing $H(z)$, we define a product filter $P(z)$ as

$$P(z) = H(z)\bar{H}(z^{-1}). \quad (9)$$

Since $H(z)$ has a set of symmetric coefficients, the coefficients of $P(z)$ are also symmetric. The coefficients of $H(z)$ and $\bar{H}(z^{-1})$ are complex conjugate each other, then the coefficients of $P(z)$ are real. In order to satisfy the orthonormality condition of Eq. (3), $P(z)$ can be expressed in the form

$$P(z) = \frac{1}{2} + \sum_{m=0}^{M} p_m[z^{-2m-1} + z^{2m+1}], \quad (10)$$

where $p_m$ is real, and $N = 2M + 1$. This is why the order $N$ of $H(z)$ must be odd. In the following, we examine relation of zeros between $H(z)$ and $P(z)$. The symmetric relation in Eq. (5) implies that if $z_i$ is a zero of $H(z)$, then $z_i^{-1}$ will be also its zero. Therefore, all possible zeros of $H(z)$ are a zero located at $z = -1$, or a pair of complex-conjugate zeros $(e^{j\theta_i}, e^{-j\theta_i})$ on the unit circle, or a pair of real reciprocal zeros $(r_i, r_i^{-1})$, or a pair of complex reciprocal zeros $(z_i, z_i^{-1})$, where $\theta_i$ is real ($\neq 0$ or $\pi$), $r_i$ is real ($\neq \pm 1$), and $z_i$ is complex ($|z_i| \neq 1$). Note that $H(z)$ must have odd number of zeros located at $z = -1$ because the order $N$ is odd and the coefficients are symmetric. Then $\bar{H}(z^{-1})$ has a zero located at $z = -1$, and/or a pair of complex-conjugate zeros $(e^{j\theta_i}, e^{-j\theta_i})$, and/or a pair of real reciprocal zeros $(r_i, r_i^{-1})$, and/or a pair of complex reciprocal zeros $(\bar{z}_i, \bar{z}_i^{-1})$. Therefore, $P(z)$ will have sets of double zeros at $z = -1$, and/or double complex-conjugate pairs $(e^{j\theta_i}, e^{-j\theta_i})$, and/or double real reciprocal pairs $(r_i, r_i^{-1})$, and/or quadruple configurations $(z_i, \bar{z}_i, z_i^{-1}, \bar{z}_i^{-1})$. If $P(z)$ with such zeros can be designed, then we can factorize $P(z)$ to get $H(z)$. When $P(z)$ has more than one sets of quadruple configurations $(z_i, \bar{z}_i, z_i^{-1}, \bar{z}_i^{-1})$, there are several possible choices. If all the zeros within the upper semi-unit circle are selected for $H(z)$, then the normal symmetric complex (NSC) filter is obtained [8]. If the zeros within the unit circle are selected alternatively for $H(z)$ and $\bar{H}(z^{-1})$, then the approximately linear phase symmetric complex (ALPSC) filter is obtained [12].

From Eq. (10), the magnitude response of $P(z)$ is

$$P(e^{j\omega}) = \frac{1}{2} + 2\sum_{m=0}^{M} p_m \cos(2m+1)\omega, \tag{11}$$

which satisfies

$$P(e^{j\omega}) + P(e^{j(\pi-\omega)}) = 1. \tag{12}$$

This means that the magnitude response of $P(z)$ is antisymmetric about $(\frac{\pi}{2}, \frac{1}{2})$. That is, $P(z)$ is a half-band filter. If the magnitude is 0 in the stopband $[\omega_s, \pi]$, then the magnitude response of $P(z)$ becomes 1 in the passband $[0, \omega_p]$, where $\omega_p$ and $\omega_s$ are the passband and stopband cutoff frequencies, respectively, and $\omega_p + \omega_s = \pi$. Therefore the approximation needs to be done only in the stopband. It is seen that $P(z)$ in Eq. (10) has $(M+1)$ unknown coefficients $p_m$, thus there are $2(M+1)$ independent zeros because of the symmetry of the filter coefficients. $P(z)$ has a total of $2(2M+1)$ zeros, in which $2M$ dependent zeros have been used for satisfying the orthonormality condition in Eq. (3) so that the passband is naturally formed. Therefore, we just need to optimize the stopband response by locating the $2(M+1)$ independent zeros. To obtain a good stopband response, all the independent zeros are required to locate on the unit circle. For example, in the Daubechies's wavelets, all the independent zeros are located at $z = -1$ to get the maximally flat filters. Since $P(z)$ has a set of real symmetric coefficients, its zeros occur in quadruple configurations $(z_i, \bar{z}_i, z_i^{-1}, \bar{z}_i^{-1})$, and/or in complex-conjugate pairs $(e^{j\theta_i}, e^{-j\theta_i})$ on the unit circle, and/or in real reciprocal pairs $(r_i, r_i^{-1})$, and/or at $z = -1$. Note that the zeros located at $z = -1$ must be double because the order $2N$ of $P(z)$ is even. To construct $H(z)$ from $P(z)$, we should force the complex-conjugate zeros $(e^{j\theta_i}, e^{-j\theta_i})$ and the real reciprocal zeros $(r_i, r_i^{-1})$ to be double. To obtain double complex-conjugate zeros $(e^{j\theta_i}, e^{-j\theta_i})$, we just need to make the magnitude response $P(e^{j\omega}) \geq 0$. Unfortunately, there is not any method to ensure double real zeros. Somtimes $P(z)$ has no real zero, then we can factorize $P(z)$. Recall that $H(z)$ must have odd zeros located at $z = -1$, then the number of the zeros of $P(z)$ at $z = -1$ is an odd multiple of two. Therefore, the number $2(M+1)$ of the independent zeros, which should locate on the unit circle, will be an odd multiple of two also, i.e., $M$ is even. The number $2M$ of the dependent zeros, which do not locate on the unit circle, will be a multiple of four. In this case, we have found through numerical examples that the dependent zeros frequently occur in quadruple configurations $(z_i, \bar{z}_i, z_i^{-1}, \bar{z}_i^{-1})$, and then there is no real zero. Therefore, we can factorize $P(z)$ to get $H(z)$ when $M$ is even. However, there is no solution for odd $M$ since there always exist some real zeros.

### 3.2 Maximally Flat Filters

To get regular symmlets and to have some vanishing moments, $H(z)$ is required to have at least one zero at $z = -1$. We assume that $H(z)$ has $2K+1$ zeros at $z = -1$, then $P(z)$ is

$$P(z) = (1+z)^{2K+1}(1+z^{-1})^{2K+1}Q(z), \tag{13}$$

where $0 \leq K \leq M/2$ and $Q(z)$ is a linear phase FIR filter of order $4(M-K)$ with real-valued coefficients. From Eq. (13), we have

$$\left.\frac{\partial^i P(e^{j\omega})}{\partial \omega^i}\right|_{\omega=\pi} = 0 \qquad (i = 0, 1, \cdots, 4K+1), \tag{14}$$

which corresponds to the flatness condition. By substituting the magnitude response of Eq. (11) into Eq. (14), we get

$$\begin{cases} \displaystyle\sum_{m=0}^{M} p_m = \frac{1}{4} & (i = 0) \\ \displaystyle\sum_{m=0}^{M} (2m+1)^{2i} p_m = 0 & (i = 1, 2, \cdots, 2K) \end{cases} \tag{15}$$

When the maximally flat filters are needed, i.e., $K = M/2$, we can obtain a closed-form solution given by

$$p_m = \frac{(-1)^m}{2m+1} \frac{\displaystyle\prod_{i=0}^{M}\left(i+\frac{1}{2}\right)^2}{(M-m)!(M+m+1)!}. \tag{16}$$

### 3.3 Filter Design with Given Flatness

It is known that the maximally flat filters are poorly selective. Frequency selectivity is also thought of as a useful property for many applications. However, frequency selectivity and regularity somewhat contradict each other. For this reason, we consider the design of $H(z)$ that has the best possible frequency selectivity for a given degree of flatness. When $0 \leq K < M/2$, we wish to get an equiripple magnitude response by using the remaining degrees of freedom. Firstly, we select $(M-2K+1)$ extremal frequencies $\omega_i$ in the stopband $[\omega_s, \pi]$ as follows;

$$\omega_s = \omega_0 < \omega_1 < \cdots < \omega_{(M-2K)} < \pi. \tag{17}$$

In order to have double complex-conjugate zeros on the unit circle, we must force $P(e^{j\omega}) \geq 0$. Then we apply the Remez exchange algorithm in the stopband and formulate $P(e^{j\omega})$ as

$$\begin{aligned} P(e^{j\omega_i}) &= \frac{1}{2} + 2\sum_{m=0}^{M} p_m \cos(2m+1)\omega_i \\ &= (1+(-1)^i)\,\delta, \end{aligned} \tag{18}$$

where $\delta$ $(> 0)$ is magnitude error. Equation (18) can be rewritten as

$$\sum_{m=0}^{M} p_m \cos(2m+1)\omega_i - \frac{1+(-1)^i}{2}\delta = -\frac{1}{4}. \tag{19}$$

Therefore, a set of filter coefficients $p_m$ can be obtained by solving the linear equations of Eqs. (15) and (19). In order to achieve an equiripple magnitude response, we make use of an iteration procedure to get the optimal solution. The design algorithm is shown as follows.

### 3.4 Design Algorithm

**Procedure** {Design Algorithm for Symmlets}

**Begin**

1. Read $M, K$, and the cutoff frequency $\omega_s$.
2. Select initial extremal frequencies $\Omega_i$ $(i = 0, 1, \cdots, M-2K)$ equally spaced in the stopband $[\omega_s, \pi]$.

**Repeat**

3. Set $\omega_i = \Omega_i$ $(i = 0, 1, \cdots, M-2K)$.
4. Solve Eqs. (15) and (19) to obtain a set of filter coefficients $p_m$.
5. Search for the peak frequencies of $P(e^{j\omega})$ in the stopband, and store these frequencies into the corresponding $\Omega_i$.

**Until** Satisfy the following condition for the prescribed small constant $\epsilon$ ($\epsilon = 10^{-4}$ in general):

$$\left\{ \sum_{i=0}^{M-2K} |\Omega_i - \omega_i| \leq \epsilon \right\}$$

6. Factorize $P(z)$ to construct $H(z)$ and $G(z)$, then generate the scaling function $\phi(t)$ and wavelet function $\psi(t)$.

**End** .

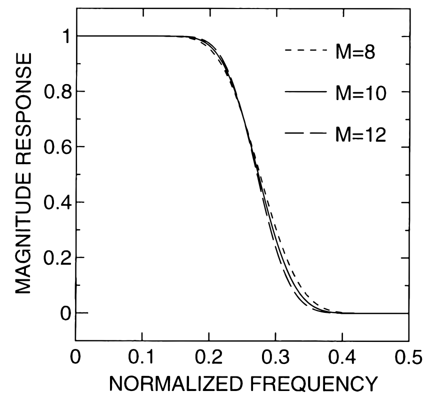


**Fig. 1** Magnitude responses in Example 1.

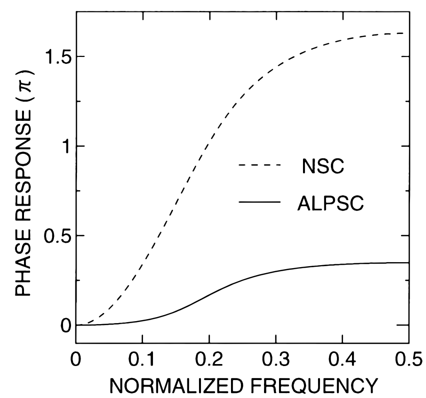


**Fig. 2** Phase responses in Example 1.

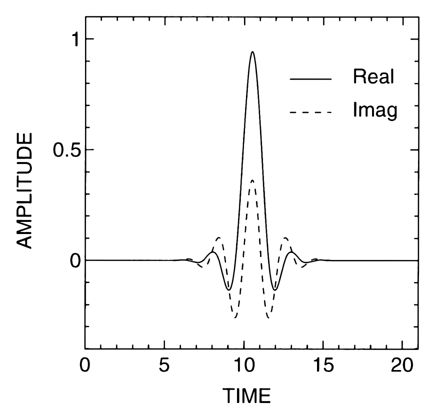


**Fig. 3** Scaling function of ALPSC filter in Example 1.

## 4. Design Examples

In this section, we present two design examples to demonstrate the effectiveness of the proposed method. *Example 1* : We consider the design of the maximally flat filter with $N = 21$, i.e., $M = 10$. By setting $K = 5$, the maximally flat filter is obtained from Eq. (16). By factorizing the resulting $P(z)$, ALPSC and NSC filters are obtained. ALPSC and NSC filters have the same magnitude response, which is shown in Fig. 1 in the solid line. In Fig. 1, the magnitude responses of $M = 8$ $(N = 17)$ and $M = 12$ $(N = 25)$ are also shown for comparison purposes. The phase responses of ALPSC and NSC filters are shown in Fig. 2, with removing linear phase $-N\omega/2$. Note that since $\hat{H}(e^{j\omega})$ is an even function, only the positive frequency part is shown. It is seen that ALPSC filter has a smaller phase error than NSC filter. The scaling and wavelet functions generated by ALPSC and NSC filters are shown in Fig. 3 to Fig. 6, respectively, where the solid and dotted

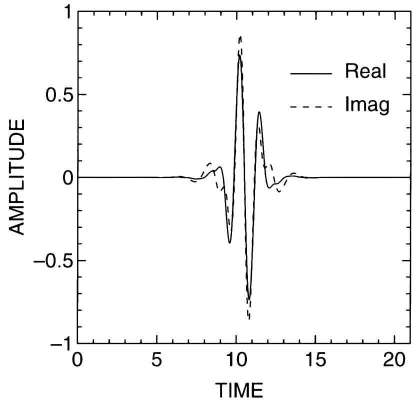


**Fig. 4** Wavelet function of ALPSC filter in Example 1.

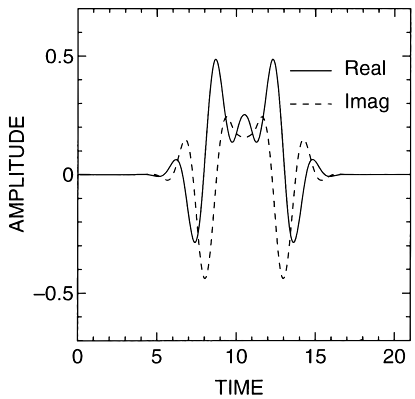


**Fig. 5** Scaling function of NSC filter in Example 1.

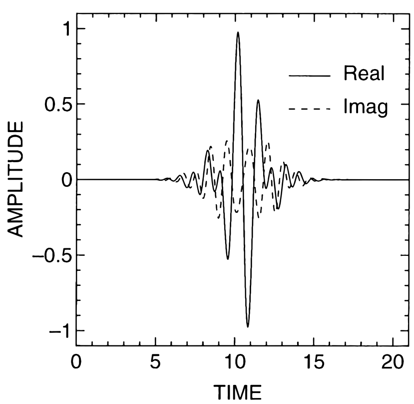


**Fig. 6** Wavelet function of NSC filter in Example 1.

lines denote the real and imaginary parts, respectively. It is clear that the scaling function is symmetric, while the wavelet function is antisymmetric.

*Example 2* : We consider the filter design with $N = 21$ $(M = 10)$, $K = 4$, $\omega_p = 0.4\pi$ and $\omega_s = 0.6\pi$. The filter has been designed by using the proposed method. We then factorized $P(z)$ to obtain ALPSC and NSC filters. Their magnitude and phase responses, with removing linear phase $-N\omega/2$, are shown in Fig. 7 and Fig. 8, respectively. In Fig. 7, the magnitude responses

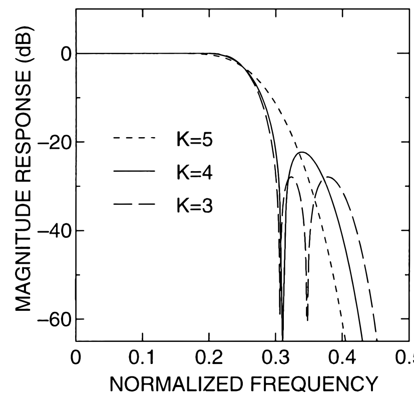


**Fig. 7** Magnitude responses in Example 2.

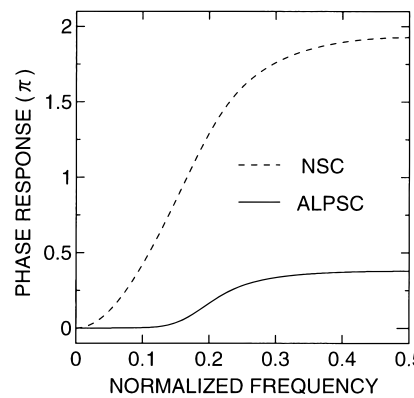


**Fig. 8** Phase responses in Example 2.

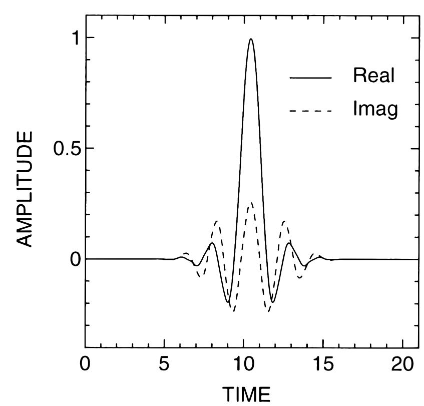


**Fig. 9** Scaling function of ALPSC filter in Example 2.

with $K = 3$ and $K = 5$ are shown also. It is seen that the filter of $K = 5$ is the maximally flat filter, and the magnitude error decreases with a decreasing $K$. It is seen in Fig. 8 that ALPSC filter has a smaller phase error than NSC filter. The scaling and wavelet functions generated by ALPSC and NSC filters are shown in Fig. 9 to Fig. 12, respectively.

## 5. Conclusions

In this paper, a new class of complex-valued compact-

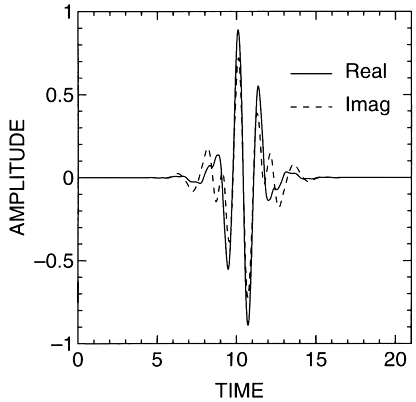


**Fig. 10** Wavelet function of ALPSC filter in Example 2.

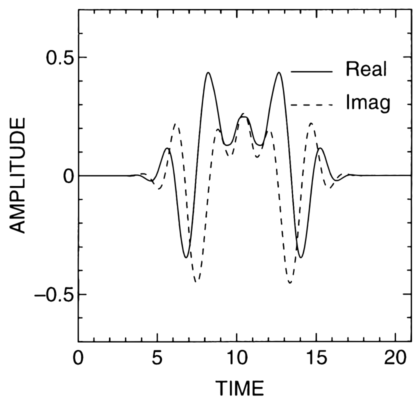


**Fig. 11** Scaling function of NSC filter in Example 2.

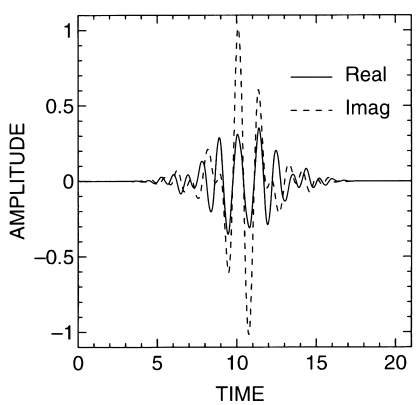


**Fig. 12** Wavelet function of NSC filter in Example 2.

supported orthonormal symmlets have been presented. Firstly, some properties of complex-valued compact-supported orthonormal symmlets are investigated, and relationship between complex-valued symmlets and real-velued half-band filters is shown. Since the construction of complex-valued symmlets can be reduced to the design of real-valued half-band filters, a design method for real-value half-band FIR filters with some flatness requirements has been proposed. For the maximally flat half-band filters, a closed-form solution is given. For the filter design with a given degree of flatness, a set of filter coefficients can be easily computed by solving a set of linear equations, and the optimal solution is obtained through a few iterations, because the efficient Remez exchange algorithm has been applied in the stopband. Finally, some examples have been designed to demonstrate the effectiveness of the proposed method.

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