PAPER

Recursive Orthonormal Wavelet Bases with Vanishing Moments

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SUMMARY This paper presents a new method for constructing orthonormal wavelet bases with vanishing moments based on general IIR filters. It is well-known that orthonormal wavelet bases can be generated by paraunitary filter banks. Then, synthesis of orthonormal wavelet bases can be reduced to design of paraunitary filter banks. From the orthonormality and regularity of wavelets, we derive some constraints to IIR filter banks, and investigate relations between the constrained filter coefficients and its zeros and poles. According to these relations, we can apply Remez exchange algorithm in stopband directly, and formulate the design problem in the form of an eigenvalue problem. Therefore, a set of filter coefficients can be easily computed by solving the eigenvalue problem, and the optimal filter coefficients with an equiripple response can be obtained after applying an iteration procedure. The proposed procedure is computationally efficient, and the number of vanishing moments can be arbitrarily specified.

key words: *orthonormal wavelet, paraunitary filter bank, IIR filter, eigenvalue problem*

1. Introduction

Wavelets have received considerable attention in various fields of applied mathematics, signal processing, multiresolution theory, and so on during past several years. The connection between continuous-time wavelets and discrete filter banks was originally investigated by Daubechies, and is now well understood [1]–[11]. Wavelet bases can be generated by perfect reconstruction two-band filter bank solutions. In this paper, we consider a paraunitary filter bank, which, when iterated, generates orthonormal wavelet bases. Paraunitary filter banks can be implemented using finite impulse response (FIR) or infinite impulse response (IIR) filters. The case of FIR filters, which lead to compactly supported wavelets, has been examined in detail in [2],[9]–[11]. In this paper we also restrict ourselves to IIR filters, which lead to more general wavelets of infinite support [5]. Design of IIR paraunitary filter banks composed of allpass filters have been discussed in [7] and [8] also. By using allpass filters, the filter bank can be implemented by fewer multipliers. However, the transfer function produced by parallel connection of two allpass filters is limited, and the design methods in [7] and [8] cannot be used to generate wavelet bases with specified vanishing moments, where the vanishing moment implies that the generated wavelet varies smoothly in time, and is useful for compression of smooth functions by wavelet transforms and so on [2]–[6].

In this paper, we propose a new method for constructing recursive orthonormal wavelet bases with vanishing moments based on general IIR filters. Since synthesis of orthonormal wavelet bases has been reduced to design of paraunitary filter banks, we only require to consider design of IIR paraunitary filter banks with an additional flatness constraint. First of all, we derive some constraints to IIR filter banks from the orthonormality and regularity of wavelets, and investigate relations between the constrained filter coefficients and its zeros and poles. According to these relations, we can find that the magnitude response of the product filter is antisymmetric between passband and stopband. Therefore, we can apply Remez exchange algorithm in stopband directly, and formulate the design problem in the form of an eigenvalue problem [12],[13]. By solving the eigenvalue problem to compute the absolute minimum eigenvalue, we can get a set of filter coefficients as the corresponding eigenvector. Then, the optimal filter coefficients with an equiripple response can be easily obtained after applying an iteration procedure. The proposed procedure is computationally efficient, and the number of vanishing moments of wavelets can be arbitrarily specified. Finally, we present some design examples to demonstrate the validity of the proposed procedure.

2. Wavelets and Filter Banks

Assume that $\psi(t)$ is a basic wavelet function, the wavelet transform (WT) to a signal $f(t) (f \in \mathbf{L}^2(\mathbf{R}))$ is defined as

$$F_W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-b}{a} \right) dt, \qquad (1)$$

where x^* denotes the complex conjugate of x, and the dilation/contraction and translation parameters are $a \in \mathbf{R}^+$, $b \in \mathbf{R}$. When discretized, $a = 2^{-k}$ and $b = 2^{-k}m$ (k, m: integer), in general. The wavelet function $\psi(t)$ is generally complex function, we con-

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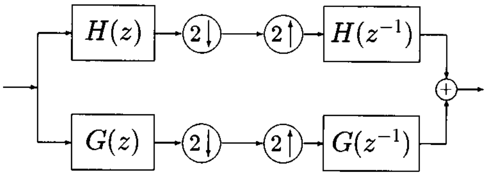

Fig. 1 Paraunitary filter bank (noncausal).

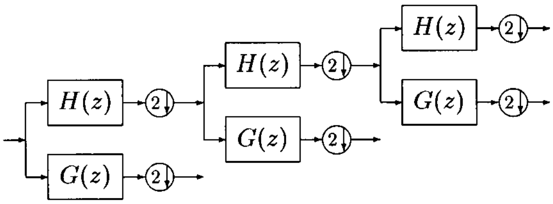

Fig. 2 Multiple stage filter bank.

sider only the real function $\psi(t)$ in this paper. In the following, $\psi(t)$ is restricted to be real.

It is well-known [1]–[6] that orthonormal wavelet bases can be generated by a paraunitary filter bank $\{H(z), G(z)\}$ shown in Fig. 1. In Fig. 1, $H(z)$ is a lowpass filter, and $G(z)$ is highpass. When the filter bank of Fig. 1 is iterated on the lowpass branch at each step of decomposition, as shown in Fig. 2, the limit function of the impulse responses produce a scaling function $\phi(t)$ and wavelet function $\psi(t)$. Assume that $\hat{\phi}(\omega), \hat{\psi}(\omega)$ are the Fourier transforms of $\phi(t)$ and $\psi(t)$, respectively, the scaling and wavelet function are related with the filter bank $\{H(z), G(z)\}$ in frequency domain as follows [4]–[6];

$$\begin{cases} \hat{\phi}(\omega) = H(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) = \prod_{k=1}^{\infty} H(e^{j2^{-k}\omega}) \\ \hat{\psi}(\omega) = G(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) \end{cases}. \quad (2)$$

From the orthonormality of wavelets, the filter bank has to satisfy the following constraints [4]–[6];

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 1 \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 1 \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0 \end{cases}. \quad (3)$$

Here, we define the product filter as

$$P(z) = H(z)H(z^{-1}). \quad (4)$$

It is known from Eq. (3) that $P(z)$ is a half-band filter. Hence, we consider design of the real half-band filter $P(z)$ expressed in the form [13]

$$P(z) = \frac{1}{2} + \frac{\sum_{n=0}^{N} a_{2n+1}\left[z^{2n+1} + z^{-(2n+1)}\right]}{b_0 + \sum_{m=1}^{M} b_{2m}\left[z^{2m} + z^{-2m}\right]}, \quad (5)$$

where N, M are integers, the filter coefficients a_i and b_i are real, and $b_0 = 1$. Note that since N and M can be arbitrarily selected, the transfer function is more general than one composed of allpass filters in [7] and [8]. It can be seen that $P(z)$ has symmetric filter coefficients. Hence, its zeros occur on the unit circle or in mirror-image pairs, while the poles occur in mirror-image pairs. If all zeros on the unit circle are double zeros, we can decompose zeros and poles of $P(z)$ to get a stable $H(z)$. Assume that the numerator and denominator of $P(z)$ are $N(z)N(z^{-1})$ and $D(z^2)D(z^{-2})$ respectively, and $D(z^2)$ has all poles of $P(z)$ inside the unit circle, then the stable $H(z)$ is

$$H(z) = \frac{N(z)}{D(z^2)}, \quad (6)$$

we can construct

$$G(z) = \pm z^{-(2J+1)}\frac{N(-z^{-1})}{D(z^2)}, \quad (7)$$

where $J = \max\{N, M\}$. It is clear from Eqs. (6) and (7) that the constraints of Eq. (3) are satisfied, then only one filter, i.e., lowpass filter $H(z)$, has to be designed. Therefore, the design problem will become design of $P(z)$ whose zeros on the unit circle must be double zeros. Note that $H(z^{-1})$ and $G(z^{-1})$ in Fig. 1 are antistable since $H(z)$ and $G(z)$ are stable. When the inverse wavelet transform is required in some applications, we must consider implementation of antistable filters. See [7],[8] in detail.

Although the filter bank is never iterated to infinity in practice, it is required that the limit function exists and is regular, i.e., continuous, possibly with several continuous derivatives [4]–[6]. The simplest regularity condition for filter design is a flatness constraint on the magnitude response at $\omega = \pi$. Kth-order flatness is obtained if $H(z)$ contains K zeros located at $z = -1$. Then, we have

$$\left.\frac{d^k H(e^{j\omega})}{d\omega^k}\right|_{\omega=\pi} = 0 \quad (k = 0, 1, \ldots, K-1), \quad (8)$$

and

$$\int_{-\infty}^{\infty} t^k \psi(t)dt = 0 \quad (k = 0, 1, \ldots, K-1), \quad (9)$$

which means that the generated wavelet will have K consecutive vanishing moments. K vanishing moments imply that both the wavelet and the filter spectrum have more smoothness. This property is potentially useful in some practical applications, e.g., for compression of smooth functions by wavelet transforms and so on [2]–[6]. Of course, frequency selectivity is also thought of as a useful property for many applications. However, regularity and frequency selectivity somewhat contradict each other. For this reason, we consider design of an IIR filter that has the best possible frequency selectivity for a specified number of vanishing moments.

3. Design of IIR Filter Banks

In this section, we describe design of IIR paraunitary filter banks with an additional flatness constraint based on eigenvalue problem [12],[13].

3.1 Property

Before designing the filter bank, we investigate the property of the product filter $P(z)$. $P(z)$ of Eq. (5) can be rewritten as

$$P(z) = \frac{a_0 + \sum_{n=1}^{L} a_n \left[z^n + z^{-n}\right]}{b_0 + \sum_{m=1}^{M} b_{2m} \left[z^{2m} + z^{-2m}\right]}, \tag{10}$$

where $L = \max\{2M, 2N+1\}$,

$$a_{2n} = \frac{1}{2} b_{2n} \quad (n = 0, 1, \ldots, M), \tag{11}$$

and when $N > M$,

$$a_{2n} = 0 \quad (n = M+1, M+2, \ldots, N), \tag{12}$$

when $N < M-1$,

$$a_{2n+1} = 0 \quad (n = N+1, N+2, \ldots, M-1). \tag{13}$$

Note that when $N = M$ or $N = M-1$, there is no coefficient of $a_n = 0$ in Eq. (10). It is seen from Eq. (10) that $P(z)$ has total $2L$ zeros and $4M$ poles, where $2I_1 (I_1 = L - N - M - 1)$ zeros are used for satisfying the condition of Eq. (12) or (13), and all poles are used for satisfying the condition of Eq. (11). Therefore, there are only $2I_2 (I_2 = M + N + 1)$ independent zeros to be optimized.

We can obtain the magnitude response of $P(z)$ from Eq. (5) by

$$P(e^{j\omega}) = \frac{1}{2} + \frac{2\sum_{n=0}^{N} a_{2n+1} \cos(2n+1)\omega}{b_0 + 2\sum_{m=1}^{M} b_{2m} \cos(2m)\omega}. \tag{14}$$

and from Eq. (3),

$$P(e^{j\omega}) + P(e^{j(\pi-\omega)}) \equiv 1, \tag{15}$$

which means that the magnitude response of $P(z)$ is antisymmetric to $(\frac{\pi}{2}, \frac{1}{2})$, then the ripple in passband $[0, \omega_p]$ is equal to one in stopband $[\omega_s, \pi]$, where $\omega_p + \omega_s = \pi$. Therefore, we only require to approximate the stopband response by locating $2I_2$ independent zeros.

3.2 Maximally Flat Filters

To obtain the maximum number of vanishing moments, we have to design a maximally flat filter. Hence, all $2I_2$ independent zeros are required to locate at $z = -1$, that is, $K = I_2$. Then, the numerator polynomial of $P(z)$ will be

$$N(z)N(z^{-1}) = (1+z)^{I_2}(1+z^{-1})^{I_2} Q(z), \tag{16}$$

where

$$Q(z) = q_0 + \sum_{n=1}^{I_1} q_n [z^n + z^{-n}]. \tag{17}$$

Compared with Eq. (10), we have

$$a_n = \sum_{i=-N_1}^{I_1} c_{n-i} q_i, \tag{18}$$

where $N_1 = \min\{I_1, I_2 - n\}$, and

$$c_i = \frac{2I_2!}{(I_2 - i)!(I_2 + i)!}. \tag{19}$$

Note that $q_i = q_{-i}$ and $c_i = c_{-i}$. When $N > M$, from Eq. (12), we get

$$\sum_{i=-N_1}^{I_1} c_{2n-i} q_i = 0 \quad (n = M+1, \ldots, N), \tag{20}$$

and when $N < M-1$, from Eq. (13),

$$\sum_{i=-N_1}^{I_1} c_{2n+1-i} q_i = 0 \quad (n = N+1, \ldots, M-1). \tag{21}$$

Due to $b_0 = 1$, we have $a_0 = 1/2$ from Eq. (11), that is,

$$a_0 = \sum_{i=-I_1}^{I_1} c_i q_i = \frac{1}{2}. \tag{22}$$

Then, we can obtain a set of filter coefficients q_i by solving the linear equations of Eqs. (22) and (20) when $N > M$, or Eq. (22) when $N = M$ or $N = M-1$, or Eqs. (22) and (21) when $N < M-1$. The filter coefficients a_i and b_i can be computed by using Eqs. (18) and (11). Hence, design of the maximally flat filter is finished. From the obtained filter coefficients, we construct $H(z)$ and $G(z)$ by decomposing zeros and poles of $P(z)$ as shown in Eqs. (6) and (7), and then generate a scaling function $\phi(t)$ and wavelet function $\psi(t)$. See [4],[8] in detail.

3.3 Filters with Given Vanishing Moments

It is known that the maximally flat filters, generating regular wavelets with the maximum number of vanishing moments, are poorly selective. Here, we consider design of an IIR filter that has the best possible frequency selectivity under K consecutive vanishing moments are given. Since $H(z)$ has K zeros located at $z = -1$, then the numerator polynomial of $P(z)$ can be expressed as

$$N(z)N(z^{-1}) = (1+z)^K(1+z^{-1})^K Q(z), \tag{23}$$

where

$$Q(z) = q_0 + \sum_{n=1}^{L-K} q_n[z^n + z^{-n}]. \tag{24}$$

Similarly to Eq. (18), we have

$$a_n = \sum_{i=-N_2}^{N_3} c_{n-i}q_i, \tag{25}$$

where $N_2 = \min\{L-K, K-n\}$, $N_3 = \min\{L-K, K+n\}$, and

$$c_i = \frac{2K!}{(K-i)!(K+i)!}. \tag{26}$$

When $N > M$, from Eq. (12),

$$\sum_{i=-N_2}^{N_3} c_{2n-i}q_i = 0 \quad (n = M+1, \ldots, N), \tag{27}$$

and when $N < M-1$, from Eq. (13),

$$\sum_{i=-N_2}^{N_3} c_{2n+1-i}q_i = 0 \quad (n=N+1, \ldots, M-1). \tag{28}$$

It is known that $P(z)$ has total $2I_2$ independent zeros, then K must satisfy $K \leqq I_2$. Therefore, the number of the remaining independent zeros is $2I_3(I_3 = I_2 - K)$, and they are required to locate on the unit circle in order to obtain the best frequency selectivity. The zreos of $P(z)$ on the unit circle, except at $z = \pm 1$, occur in complex conjugate pairs, and are required to be double zeros, therefore, I_3 must be even. Since the magnitude response of $P(z)$ is antisymmetric, we need to optimize the magnitude response in stopband only. To obtain an equiripple stopband response, we apply Remez exchange algorithm in stopband directly, and formulate the condition for $P(e^{j\omega})$ in the form of a generalized eigenvalue problem [12],[13]. First, we select $I_3 + 1$ extremal frequencies ω_i in stopband as follows;

$$\omega_s = \omega_0 < \omega_1 < \cdots < \omega_{I_3} < \pi. \tag{29}$$

Considering that all zeros of $P(z)$ on the unit circle must be double zeros, we formulate $P(e^{j\omega})$ as

$$P(e^{j\omega_i}) = \begin{cases} \delta & (i = 0, 2, \ldots, I_3) \\ 0 & (i = 1, 3, \ldots, I_3 - 1) \end{cases}, \tag{30}$$

where δ is a magnitude error. Substituting Eqs. (10) and (23) into Eq. (30), we can rewrite Eqs. (30) and (27) or (28) or none in the matrix form as

$$\boldsymbol{SQ} = \delta \boldsymbol{TB}, \tag{31}$$

where $\boldsymbol{Q} = [q_0, q_1, \ldots, q_{(L-K)}]^T$, $\boldsymbol{B} = [b_0, b_2, \ldots, b_{2M}]^T$, and the elements of $\boldsymbol{S}$ are

$$S_{ij} = \begin{cases} 1 & (j = 0) \\ 2\cos(j\omega_i) & (j = 1, 2, \ldots, L-K) \end{cases}, \tag{32}$$

when $i = 0, 1, \ldots, I_3$, and

$$S_{ij} = \begin{cases} c_{2i+I_4} & (j = 0) \\ c_{2i+I_4-j} + c_{2i+I_4+j} & (j = 1, \ldots, L-K) \end{cases} \tag{33}$$

when $i = I_3+1, I_3+2, \ldots, L-K$, where $I_4 = 2(M-I_3)$ when $N > M$, and $I_4 = 2(N-I_3)+1$ when $N < M-1$. Note that $c_i = 0$ when $i > K$. The elements of $\boldsymbol{T}$ are

$$T_{ij} = \begin{cases} \dfrac{1}{(2\cos\frac{\omega_i}{2})^{2K}} & (j = 0) \\ \dfrac{2\cos(2j\omega_i)}{(2\cos\frac{\omega_i}{2})^{2K}} & (j = 1, 2, \ldots, M) \end{cases}, \tag{34}$$

when $i = 0, 2, \ldots, I_3$, and

$$T_{ij} = 0 \quad \text{(else)}. \tag{35}$$

From Eqs. (11) and (25), we have

$$\boldsymbol{B} = \boldsymbol{VQ}, \tag{36}$$

where the elements of $\boldsymbol{V}$ are

$$V_{ij} = \begin{cases} 2c_{2i} & (j = 0) \\ 2(c_{2i-j} + c_{2i+j}) & (j = 1, \ldots, L-K) \end{cases} \tag{37}$$

when $i = 0, 1, \ldots, M$. Substituting Eq. (36) into Eq. (31), we can get

$$\boldsymbol{SQ} = \delta \boldsymbol{TVQ}, \tag{38}$$

which corresponds to a generalized eigenvalue problem, i.e., δ is an eigenvalue and $\boldsymbol{Q}$ is a corresponding eigenvector. Therefore, to minimize the magnitude error δ, we compute the absolute minimum eigenvalue by solving the eigenvalue problem of Eq. (38). Then the corresponding eigenvector gives a set of filter coefficients. To force the magnitude response to be equiripple, we apply an iteration procedure to obtain the optimal solution. The design algorithm is shown as follows.

3.4 Design Algorithm

Procedure {Design Algorithm of Wavelet Filters}
Begin

1. Read N, M, K and ω_s.
2. Select initial extremal frequencies $\Omega_i(i = 0, 1, \ldots, I_3)$ equally spaced in stopband.

Repeat

3. Set $\omega_i = \Omega_i$ for $i = 0, 1, \ldots, I_3$.
4. Compute $\boldsymbol{S}, \boldsymbol{T}$ and $\boldsymbol{V}$ by using Eqs. (32)–(35) and (37), then find the absolute minimum eigenvalue of Eq. (38) to obtain the filter coefficients q_i, and compute a_i, b_i by Eqs. (25) and (11).
5. Compute the magnitude response of $P(z)$, and

search the peak frequencies $\Omega_i (i = 0, 1, \ldots, I_3)$ in stopband.

Until Satisfy the following condition for the prescribed small constant ϵ:

$$\{|\Omega_i - \omega_i| \leqq \epsilon \quad (\text{for } i = 0, 1, \ldots, I_3)\}$$

6. Construct $H(z)$ and $G(z)$ by decomposing the poles and zeros of $P(z)$, then generate the scaling function $\phi(t)$ and wavelet function $\psi(t)$ (see [4],[8] in detail).

End.

4. Design Examples

In this section, we present some design examples to demonstrate the validity of the proposed procedure.

Example 1: We consider design of a maximally flat IIR filter with $N = M = 4$. The magnitude response of the designed filter $H(z)$, having minimum phase response, is shown in Fig. 3 in solid line, and the generated scaling and wavelet function are shown in Fig. 4 and Fig. 5, respectively. In Fig. 3, the magnitude responses of two filters with $N = 3$ and $M = 5$ or $N = 5$ and $M = 3$ are shown also. It is seen in Fig. 3 that the magnitude responses of three filters are almost same. This is because three filters have the same flatness at $\omega = 0$ and $\omega = \pi$.

Example 2: We consider design of an IIR paraunitary filter bank with $N = 4$, $M = 5$, $K = 8$ and $\omega_s = 0.6\pi$. We have designed the product filter $P(z)$ by using the proposed procedure, and constructed $H(z)$ that has minimum phase response and $G(z)$. The magnitude response of $H(z)$ is shown in Fig. 6 in solid line, and the generated scaling function and wavelet function are shown in Fig. 7 and Fig. 8, respectively. In Fig. 6, the magnitude responses of two filters with $K = 6$ or

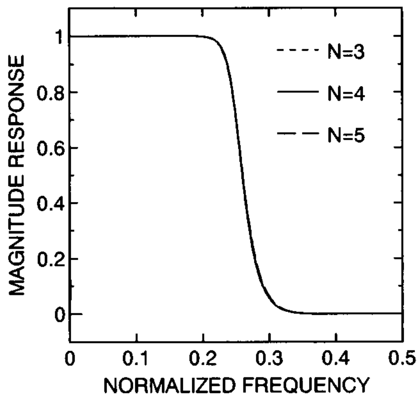

Fig. 3 Magnitude responses of Example 1.

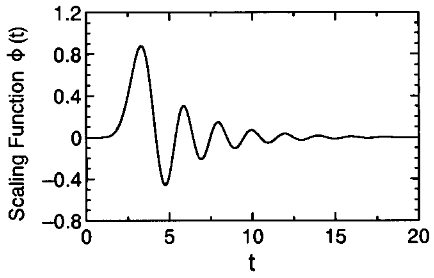

Fig. 4 Scaling function of Example 1.

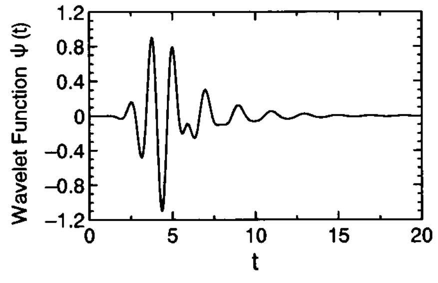

Fig. 5 Wavelet function of Example 1.

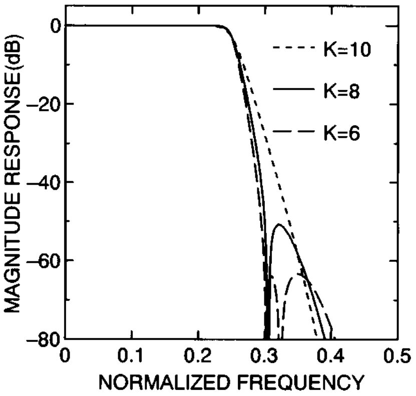

Fig. 6 Magnitude responses of Example 2.

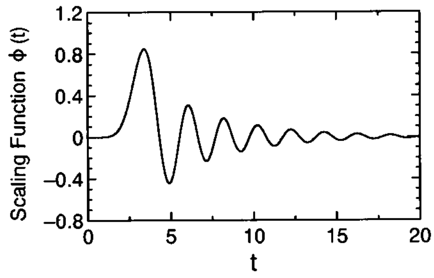

Fig. 7 Scaling function of Example 2.

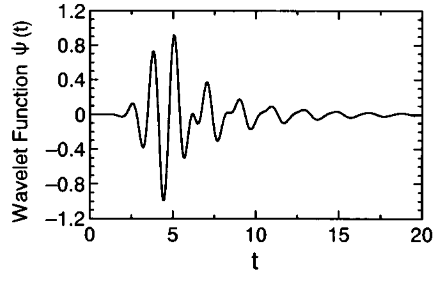

Fig. 8 Wavelet function of Example 2.

$K = 10$ are shown also. It is clear that the magnitude error decreases with a decreasing K.

5. Conclusions

In this paper, we have proposed a new method for constructing recursive orthonormal wavelet bases with vanishing moments. We have derived some constraints to IIR paraunitary filter banks from the orthonormality and regularity of wavelets, and investigated relations between the constrained filter coefficients and its zeros and poles. Therefore, we can apply Remez exchange algorithm in stopband directly, and formulate the design problem in the form of an eigenvalue problem. By solving the eigenvalue problem to compute the absolute minimum eigenvalue, we can get a set of filter coefficients as the corresponding eigenvector. Hence, the optimal filter coefficients with an equiripple response can be easily obtained after applying an iteration procedure. The proposed procedure is computationally efficient, and the number of vanishing moments of wavelets can be arbitrarily specified.

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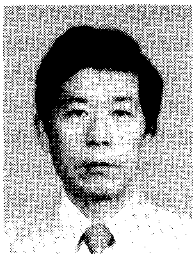

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