

Design of Two Channel Stable IIR Perfect Reconstruction Filter Banks

Xi ZHANG[†] and Toshinori YOSHIKAWA[†], Members

SUMMARY In this paper, a novel method is proposed for designing two channel biorthogonal filter banks with general IIR filters, which satisfy both the perfect reconstruction and causal stable conditions. Since the proposed filter banks are structurally perfect reconstruction implementation, the perfect reconstruction property is still preserved even when all filter coefficients are quantized. The proposed design method is based on the formulation of a generalized eigenvalue problem by using Remez multiple exchange algorithm. Then, the filter coefficients can be computed by solving the eigenvalue problem, and the optimal solution is easily obtained through a few iterations. One design example is presented to demonstrate the effectiveness of the proposed method.

key words: biorthogonal filter bank, structurally perfect reconstruction, stable IIR filter, eigenvalue problem

1. Introduction

Two channel perfect reconstruction (PR) filter banks have been used in different applications of signal processing [1],[2]. The theory and design of FIR PR filter banks have been well established in recent years [1]–[4]. In this paper, we will consider design of two channel biorthogonal PR filter banks using IIR filters that satisfy the causal stable condition. Design of two channel causal IIR PR filter banks have been discussed in [5],[6] and [9]. In [5] and [9], the proposed PR filter banks are based on general IIR filters. However, the PR property is not preserved when filter coefficients are quantized since the structurally PR implementation is not considered. Furthermore, the proposed design methods are time-consuming and the resulting frequency responses are poor. In [6], an efficient structurally PR implementation has been proposed, where for IIR case, allpass filters are used. However, there is a bump of approximately 4 dB at $\omega = \pi/2$, and the magnitude errors in stopband of lowpass and highpass filters cannot be controlled separately, because the same allpass filter is used twice in both the analysis and synthesis filters.

In this paper, we propose a new design method for two channel biorthogonal IIR PR filter banks that satisfy the causal stable condition. We adopt the structurally perfect reconstruction implementation proposed in [6], and use general IIR filters rather than allpass

filters of [6]. Use of general IIR filters will provide more freedom in the design. Then, we can expect to suppress the bump around $\omega = \pi/2$ caused when the allpass filter is used, and control arbitrarily the magnitude errors of lowpass and highpass filters. To obtain the optimal solution in the Chebyshev sense, we apply Remez multiple exchange algorithm and formulate the design problem in the form of a generalized eigenvalue problem [7],[8]. Therefore, the filter coefficients can be computed by solving the eigenvalue problem to get the positive minimum eigenvalue, and the optimal solution is easily obtained through a few iterations. Finally, we present one design example to demonstrate the effectiveness of the proposed method.

2. Biorthogonal IIR PR Filter Banks

In two channel filter banks shown in Fig. 1, assume that $H_0(z), H_1(z)$ are analysis filters, and $G_0(z), G_1(z)$ are synthesis filters. It is well-known that the relationship of input $X(z)$ and output $Y(z)$ of the filter banks is

$$Y(z) = \frac{1}{2} \{H_0(z)G_0(z) + H_1(z)G_1(z)\}X(z) + \frac{1}{2} \{H_0(-z)G_0(z) + H_1(-z)G_1(z)\}X(-z). \quad (1)$$

Hence, the perfect reconstruction condition is

$$\begin{cases} H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K-1} \\ H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \end{cases}, \quad (2)$$

where K is integer. To eliminate completely the aliasing errors, the synthesis filters $G_0(z)$ and $G_1(z)$ must be chosen as

$$\begin{cases} G_0(z) = H_1(-z) \\ G_1(z) = -H_0(-z) \end{cases}. \quad (3)$$

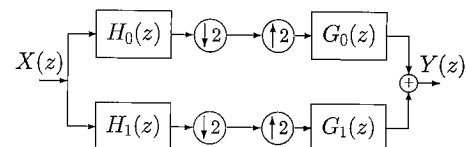


Fig. 1 Two channel filter bank.

Manuscript received December 8, 1997.

Manuscript revised February 16, 1998.

[†]The authors are with the Department of Electrical Engineering, Nagaoka University of Technology, Nagaoka-shi, 940-2188 Japan.

Then, the perfect reconstruction condition of Eq. (2) becomes

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-2K-1}. \quad (4)$$

In [6], the analysis filters $H_0(z)$ and $H_1(z)$ are composed by

$$\begin{cases} H_0(z) = \frac{1}{2}\{z^{-2N-1} + A(z^2)\} \\ H_1(z) = z^{-2M} - B(z^2)H_0(z) \\ \quad = z^{-2M} - \frac{B(z^2)}{2}\{z^{-2N-1} + A(z^2)\}, \end{cases} \quad (5)$$

where N and M are integers. Therefore, the perfect reconstruction condition of Eq. (4) is satisfied, where $K = N + M$. The structurally perfect reconstruction implementation proposed in [6] is shown in Fig. 2. In the implementation shown in Fig. 2, it can be seen that the perfect reconstruction property is still preserved even when all filter coefficients are quantized. In [6], two cases are considered by using linear phase FIR filters and stable IIR filters. For IIR case, since $A(z)$ and $B(z)$ employ the same allpass filter, there exists a bump of approximately 4 dB at $\omega = \pi/2$, and the magnitude errors in stopband of lowpass and highpass filters cannot be controlled separately, no matter how the allpass filter is designed. In this paper, we use general IIR filters rather than allpass filters, i.e.,

$$A(z) = \frac{\sum_{i=0}^{L_1} a_i z^{-i}}{\sum_{i=0}^{L_2} b_i z^{-i}}, \quad (6)$$

$$B(z) = \frac{\sum_{i=0}^{L_3} c_i z^{-i}}{\sum_{i=0}^{L_4} d_i z^{-i}}, \quad (7)$$

where L_1, L_2, L_3, L_4 are integers, the filter coefficients

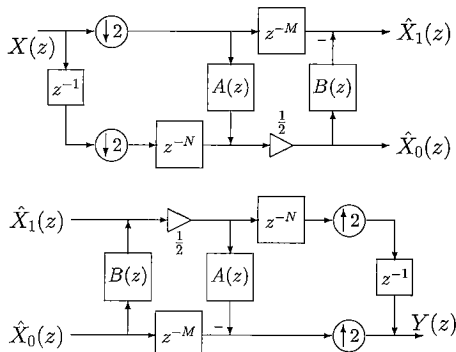


Fig. 2 Structurally perfect reconstruction implementation.

a_i, b_i, c_i, d_i are real, and $b_0 = d_0 = 1$. Using general IIR filters will provide more freedom in the design, hence, we can expect to suppress the bump around $\omega = \pi/2$ and control arbitrarily the magnitude errors of lowpass and highpass filters.

3. Design of IIR PR Filter Banks

In this section, we describe design of IIR PR filter banks based on eigenvalue problem by using Remez multiple exchange algorithm [7],[8].

3.1 Desired Frequency Responses

From Eq. (5), we have

$$\begin{aligned} H_0(z) &= \frac{z^{-2N-1}}{2} \left\{ 1 + \frac{A(z^2)}{z^{-2N-1}} \right\} \\ &= \frac{z^{-2N-1}}{2} \{1 + \hat{A}(z^2)\}, \end{aligned} \quad (8)$$

where

$$\hat{A}(z^2) = z^{2N+1} A(z^2). \quad (9)$$

Therefore, the desired frequency response of $\hat{A}(z^2)$ is

$$\begin{cases} \hat{A}_d(e^{j2\omega}) = 1 & (0 \leq \omega \leq \omega_p) \\ \hat{A}_d(e^{j2\omega}) = -1 & (\omega_s \leq \omega \leq \pi) \end{cases}, \quad (10)$$

where ω_p, ω_s are the passband and stopband edge frequencies respectively, and $\omega_p + \omega_s = \pi$. From Eq. (9), it can be obtained that

$$\hat{A}(e^{j2(\pi-\omega)}) = -\hat{A}^*(e^{j2\omega}), \quad (11)$$

where x^* denotes the complex conjugate of x , thus the desired frequency response of $A(z)$ becomes

$$A_d(e^{j\omega}) = e^{-j(N+\frac{1}{2})\omega} \quad (0 \leq \omega \leq 2\omega_p). \quad (12)$$

Since $H_0(e^{j\omega}) = 0$ in the stopband $[\omega_s, \pi]$, it can be seen from Eq. (5) that $H_1(z) = z^{-2M}$, i.e., the magnitude of $H_1(z)$ is 1 and the phase is linear phase $-2M\omega$ in the band $[\omega_s, \pi]$. In the band $[0, \omega_p]$, $H_0(z) = z^{-2N-1}$, ideally, then,

$$\begin{aligned} H_1(z) &= z^{-2M} - B(z^2)z^{-2N-1} \\ &= z^{-2M} \{1 - \hat{B}(z^2)\}, \end{aligned} \quad (13)$$

where

$$\hat{B}(z^2) = z^{2(M-N)-1} B(z^2). \quad (14)$$

To force the magnitude of $H_1(z)$ to be equal to 0 in the band $[0, \omega_p]$, the desired frequency response of $\hat{B}(z^2)$ is

$$\hat{B}_d(e^{j2\omega}) = 1 \quad (0 \leq \omega \leq \omega_p), \quad (15)$$

thus the desired frequency response of $B(z)$ becomes

$$B_d(e^{j\omega}) = e^{-j(M-N-\frac{1}{2})\omega} \quad (0 \leq \omega \leq 2\omega_p). \quad (16)$$

Therefore, the design problem of the filter banks will become the complex Chebyshev approximation of $A(z)$ and $B(z)$.

3.2 Design of $H_0(z)$

Here, we consider design of $H_0(z)$, i.e., $A(z)$. First, we define an error function between the frequency response and the desired frequency response of $A(z)$ as

$$E_a(\omega) = \frac{A(e^{j\omega}) - A_d(e^{j\omega})}{e^{-j(N+\frac{1}{2})\omega}} = \hat{A}(e^{j\omega}) - 1, \quad (17)$$

and then describe design method of $A(z)$.

3.2.1 Formulation

To obtain the optimal solution in the Chebyshev sense, we apply Remez multiple exchange algorithm. First, we select $(J + 1)$ extremal frequencies ω_i in the band $[0, 2\omega_p]$ as

$$2\omega_p = \omega_0 > \omega_1 > \dots > \omega_J \geq 0, \quad (18)$$

where $J = \lfloor (L_1 + L_2 + 1)/2 \rfloor$, and $\lfloor x \rfloor$ denotes the integer part of x . Note that we must choose $\omega_J > 0$ when $(L_1 + L_2)$ is even, and $\omega_J = 0$ when $(L_1 + L_2)$ is odd. Then, we can formulate $E_a(\omega)$ as follows;

$$E_a(\omega_i) = \hat{A}(e^{j\omega_i}) - 1 = \delta e^{j\theta(\omega_i)}, \quad (19)$$

where $\delta (> 0)$ is magnitude error to be minimized, and $\theta(\omega_i)$ is phase response at ω_i and can be computed in the previous iteration. Substituting $E_a(\omega)$ of Eq. (17) into Eq. (19), we divide Eq. (19) into the real and imaginary parts as

$$\begin{aligned} \sum_{m=0}^{L_1} a_m \cos\left(N - m + \frac{1}{2}\right)\omega_i - \sum_{m=0}^{L_2} b_m \cos(m\omega_i) \\ = \delta \sum_{m=0}^{L_2} b_m \cos(\theta(\omega_i) - m\omega_i) \end{aligned} \quad (i = 0, 1, \dots, J), \quad (20)$$

$$\begin{aligned} \sum_{m=0}^{L_1} a_m \sin\left(N - m + \frac{1}{2}\right)\omega_i + \sum_{m=0}^{L_2} b_m \sin(m\omega_i) \\ = \delta \sum_{m=0}^{L_2} b_m \sin(\theta(\omega_i) - m\omega_i) \end{aligned} \quad (i = 0, 1, \dots, J_1), \quad (21)$$

where $J_1 = J$ when $(L_1 + L_2)$ is even, and $J_1 = J - 1$ when $(L_1 + L_2)$ is odd, since Eq. (19) has not imaginary part at $\omega_J = 0$. Equations (20) and (21) can be rewritten in the matrix form as

$$PA = \delta QA, \quad (22)$$

where $A = [a_0, a_1, \dots, a_{L_1}, b_0, b_1, \dots, b_{L_2}]^T$,

$$P = \begin{bmatrix} \cos(N + \frac{1}{2})\omega_0 & \dots & -1 & \dots & -\cos L_2\omega_0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \cos(N + \frac{1}{2})\omega_J & \dots & -1 & \dots & -\cos L_2\omega_J \\ \sin(N + \frac{1}{2})\omega_0 & \dots & 0 & \dots & \sin L_2\omega_0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sin(N + \frac{1}{2})\omega_{J_1} & \dots & 0 & \dots & \sin L_2\omega_{J_1} \end{bmatrix} \quad (23)$$

$$Q = \begin{bmatrix} 0 & \dots & \cos \theta(\omega_0) & \dots & \cos(\theta(\omega_0) - L_2\omega_0) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \cos \theta(\omega_J) & \dots & \cos(\theta(\omega_J) - L_2\omega_J) \\ 0 & \dots & \sin \theta(\omega_0) & \dots & \sin(\theta(\omega_0) - L_2\omega_0) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \sin \theta(\omega_{J_1}) & \dots & \sin(\theta(\omega_{J_1}) - L_2\omega_{J_1}) \end{bmatrix} \quad (24)$$

It should be noted that Eq. (22) corresponds to a generalized eigenvalue problem, i.e., δ is an eigenvalue and A is a corresponding eigenvector. Therefore, to minimize the magnitude error δ , we must compute the positive minimum eigenvalue by solving the above eigenvalue problem [7],[8]. Then, the corresponding eigenvector gives a set of filter coefficients. By appropriately selecting the initial extremal frequencies ω_i and its phase $\theta(\omega_i)$, we can apply an iteration procedure to attain the optimal solution in the Chebyshev sense. The selection of the initial extremal frequencies ω_i and its phase $\theta(\omega_i)$ will directly influence convergence of the iteration procedure. In the following, we will discuss how to select the initial extremal frequencies ω_i and its phase $\theta(\omega_i)$.

3.2.2 Selection of Initial Value

In the proposed iteration procedure, arbitrarily selecting an initial extremal frequencies ω_i and its phase $\theta(\omega_i)$ cannot guarantee to converge to the optimal solution. Hence, it is very important how to select the initial value. We assume that there exist $(J_1 + 1)$ frequency points $\bar{\omega}_i$ in the band $[0, 2\omega_p]$ as

$$2\omega_p > \bar{\omega}_0 > \bar{\omega}_1 > \dots > \bar{\omega}_{J_1} \geq 0, \quad (25)$$

so that the error function $E_a(\omega)$ satisfies

$$E_a(\bar{\omega}_i) = \hat{A}(e^{j\bar{\omega}_i}) - 1 = 0, \quad (26)$$

where $\bar{\omega}_{J_1} = 0$ when $(L_1 + L_2)$ is even, and $\bar{\omega}_{J_1} > 0$ when $(L_1 + L_2)$ is odd. A possible choice of $\bar{\omega}_i$ is to

pick these frequency points equally spaced in the band $[0, 2\omega_p]$. Other distributions may also be preferred to decrease number of iterations. Since $b_0 = 1$, Eq. (26) can be rewritten into the real and imaginary parts as

$$\sum_{m=0}^{L_1} a_m \cos\left(N-m+\frac{1}{2}\right)\bar{\omega}_i - \sum_{m=1}^{L_2} b_m \cos(m\bar{\omega}_i) = 1$$

$$(i = 0, 1, \dots, J_1), \quad (27)$$

$$\sum_{m=0}^{L_1} a_m \sin\left(N-m+\frac{1}{2}\right)\bar{\omega}_i + \sum_{m=1}^{L_2} b_m \sin(m\bar{\omega}_i) = 0$$

$$(i = 0, 1, \dots, J-1), \quad (28)$$

which is a set of linear equations. Hence, we can obtain an initial solution of filter coefficients by solving the linear equations of Eqs. (27) and (28). Then, we compute the error function $E_a(\omega)$ by using the obtained filter coefficients, and search for the peak points of $|E_a(\omega)|$ in the band $[0, 2\omega_p]$ to get $(J+1)$ initial extremal frequencies ω_i and its phase $\theta(\omega_i)$. The design algorithm is shown as follows.

3.2.3 Design Algorithm

Procedure {Design Algorithm of Stable IIR PR Filter Banks}

Begin

1. Read L_1, L_2, N and ω_p .
2. Select $(J_1 + 1)$ frequency points $\bar{\omega}_i$ equally spaced in the band $[0, 2\omega_p]$.
3. Solve a set of linear equations of Eqs. (27) and (28) to obtain an initial solution of filter coefficients a_i and b_i .
4. Compute error function $E_a(\omega)$ by using the initial filter coefficients, then search peak frequencies as initial extremal frequencies Ω_i and compute its phase $\theta(\Omega_i)$.

Repeat

5. Set $\omega_i = \Omega_i$ for $i = 0, 1, \dots, J$.
6. Compute P and Q by using Eqs. (23) and (24), then find the positive minimum eigenvalue of Eq. (22) to obtain a set of filter coefficients a_i and b_i .
7. Compute error function $E_a(\omega)$, then search peak frequencies Ω_i and compute its phase $\theta(\Omega_i)$.

Until Satisfy the following condition for a prescribed small constant ϵ :

$$\{|\Omega_i - \omega_i| \leq \epsilon \quad (\text{for } i = 0, 1, \dots, J)\}$$

8. Check stability of $A(z)$ by finding the locations of poles.

End .

3.2.4 Stable Condition

In the above design algorithm, the obtained filter $A(z)$ may not be guaranteed to be stable. The stability of $A(z)$ must be checked in step.8 by computing the locations of the poles. The stability of $A(z)$ are generally dependent on the specifications, i.e., L_1, L_2 and N . When L_1 and L_2 are given, the group delay must be chosen enough large to guarantee the obtained filter to be stable, i.e., $N \geq N_{min}$, where N_{min} is the minimum group delay for the stable filters. In our experience, N_{min} is directly proportional to L_1 and L_2 , in general.

3.3 Design of $H_1(z)$

Here, we consider design of $H_1(z)$, i.e., $B(z)$. We can design $B(z)$ by using the design method of $A(z)$ proposed in 3.2. However, it can be seen from Eq. (5) that the frequency response of $H_1(z)$ maybe not optimal even though the frequency response of $B(z)$ is optimal in the Chebyshev sense. From Eq. (5), we have

$$H_1(z) = z^{-2M} \left\{ 1 - \frac{B(z^2)H_0(z)}{z^{-2M}} \right\}$$

$$= z^{-2M} \{1 - \hat{B}(z^2)\hat{H}_0(z)\}, \quad (29)$$

where

$$\hat{H}_0(z) = \frac{H_0(z)}{z^{-2N-1}} = \frac{1}{2} \{1 + \hat{A}(z^2)\}. \quad (30)$$

To force $H_1(z)$ to have an equiripple response in the stopband $[0, \omega_p]$, we have to optimize the frequency response of $\hat{B}(z^2)\hat{H}_0(z)$ in the Chebyshev sense. We define an error function of $B(z)$ as

$$E_b(\omega) = \hat{H}_0(e^{j\frac{\omega}{2}})\hat{B}(e^{j\omega}) - 1. \quad (31)$$

Then, we can formulate $E_b(\omega)$ as shown in 3.2, i.e.,

$$E_b(\omega_i) = \hat{H}_0(e^{j\frac{\omega_i}{2}})\hat{B}(e^{j\omega_i}) - 1 = \delta e^{j\theta(\omega_i)}, \quad (32)$$

where $\hat{H}_0(e^{j\frac{\omega}{2}})$ can be considered as a weighting function. Similarly, for selection of initial value, Eq. (26) must be rewritten also into

$$E_b(\bar{\omega}_i) = \hat{H}_0(e^{j\frac{\bar{\omega}_i}{2}})\hat{B}(e^{j\bar{\omega}_i}) - 1 = 0. \quad (33)$$

Therefore, the design algorithm is the same as that of $A(z)$. In [6], there exists a bump of approximately 4 dB at $\omega = \pi/2$, since $A(z)$ and $B(z)$ are the same allpass filter. In this paper, we use general IIR filters $A(z)$ and $B(z)$, and appropriately choose the group delay N and M to suppress the bump around $\omega = \pi/2$. See design example in detail.

4. Design Example

In this section, we present one design example to demonstrate the effectiveness of the proposed method.

Example: We consider design of an IIR PR filter bank with $N = 7, M = 14, L_1 = L_3 = 8, L_2 = L_4 = 2$, and $\omega_p = 0.4\pi$. First, $A(z)$ is designed by using the method proposed in 3.2. The complex error of $E_a(\omega)$ in the band $[0, 2\omega_p]$ is shown in Fig. 3, and it is clear that it is optimal in the Chebyshev sense. The magnitude and phase response of $A(z)$ are shown in solid line in Figs. 5 and 6, respectively. Then $B(z)$ is designed by using the method proposed in 3.3. The complex error of $E_b(\omega)$ is shown in Fig. 4 and is optimal also. The magnitude and phase response of $B(z)$ are shown in dotted line in Figs. 5 and 6 also. The resulting magnitude responses of $H_0(z)$ and $H_1(z)$ are shown in Fig. 7. It can be seen that both $H_0(z)$ and $H_1(z)$ have equiripple magnitude responses in the stopband, and $H_1(z)$ has not a bump at $\omega = \pi/2$. It is because $A(z)$ and $B(z)$ are general IIR filters and the magnitude responses are $|A(e^{j\omega})| < 1, |B(e^{j\omega})| < 1$ in the transition band, as shown in Fig. 5. It is seen in Fig. 6 that the phase responses of $A(z)$ and $B(z)$ are approximately linear in the band $[0, 2\omega_p]$, hence both $H_0(z)$ and $H_1(z)$ have approximately linear phase responses in the pass-

band. The phase errors of $H_0(z)$ and $H_1(z)$ are shown in Fig. 8, and it is seen that the phase errors are very small. To obtain a stable $A(z)$, N must be chosen as $N \geq 5$, i.e., $N_{min} = 5$. When $N = 7$, M must be chosen as $M \geq 13$, i.e., $M_{min} = 13$ to obtain a stable $B(z)$. Therefore, the obtained $A(z)$ and $B(z)$ are guaranteed to be stable. In Fig. 7, the magnitude response of $H_1(z)$

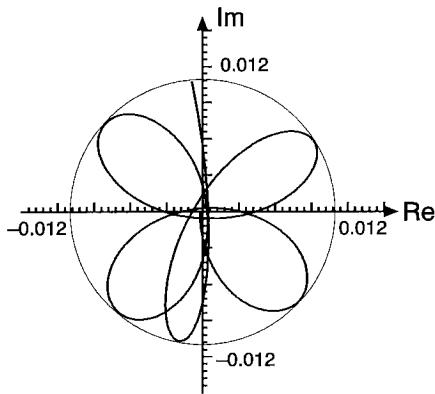


Fig. 3 Complex error trace of $E_a(\omega)$.

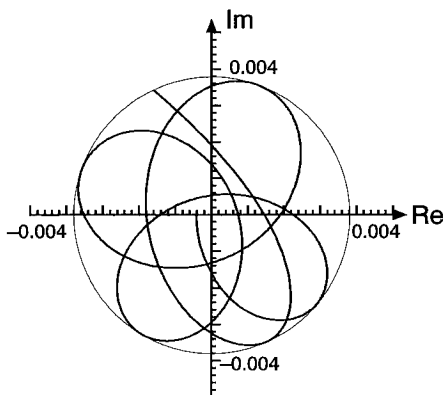


Fig. 4 Complex error trace of $E_b(\omega)$.

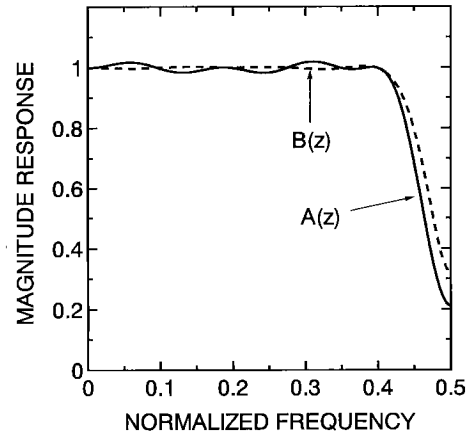


Fig. 5 Magnitude responses of $A(z)$ and $B(z)$.

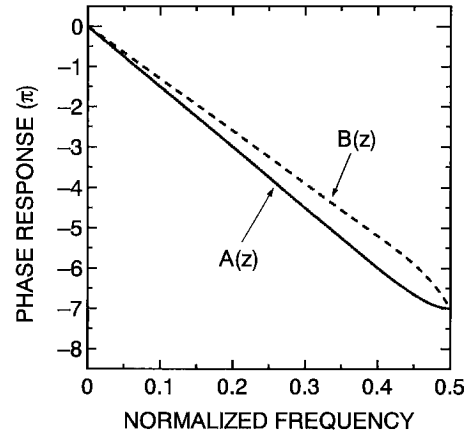


Fig. 6 Phase responses of $A(z)$ and $B(z)$.

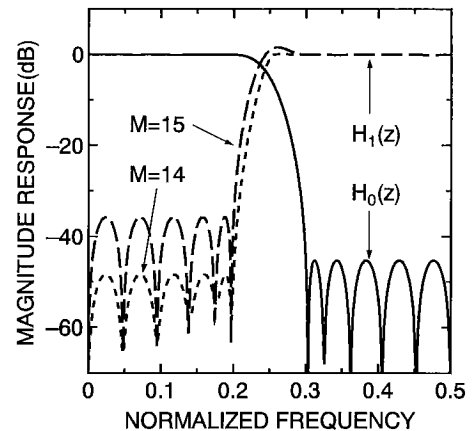


Fig. 7 Magnitude responses of $H_0(z)$ and $H_1(z)$.

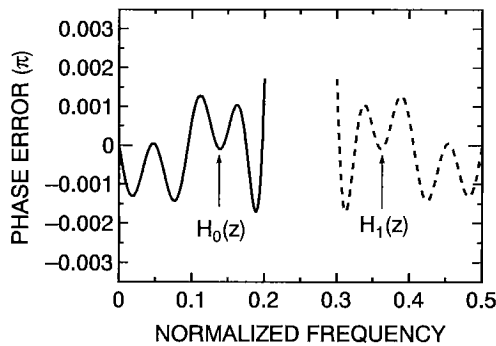


Fig. 8 Phase errors of $H_0(z)$ and $H_1(z)$ in passband.

with $M = 15$ is shown also. It is seen that $H_1(z)$ has a bump around $\omega = \pi/2$ and larger magnitude error in the stopband. Therefore, we conclude that to choose appropriately N and M can suppress the bump around $\omega = \pi/2$ and control the magnitude error in the stopband.

5. Conclusion

In this paper, we have proposed a new method for designing two channel biorthogonal IIR filter banks that satisfy both the perfect reconstruction and causal stable conditions. We have adopted the structurally perfect reconstruction implementation proposed in [6], and used general IIR filters to suppress the bump around $\omega = \pi/2$ caused when allpass filters are used. By using Remez multiple exchange algorithm, we have formulated the design problem of IIR PR filter banks in the form of a generalized eigenvalue problem. Therefore, the filter coefficients can be computed by solving the eigenvalue problem to get the positive minimum eigenvalue, and the optimal solution in the Chebyshev sense is easily obtained through a few iterations. Finally, we have designed one example to demonstrate the effectiveness of the proposed method.

References

- [1] P.P. Vaidyanathan, "Multirate Systems and Filter Banks," Englewood Cliffs, NJ, Prentice Hall, 1993.
- [2] S.K. Mitra and J.F. Kaiser, "Handbook for Digital Signal Processing," John Wiley & Sons, New York, 1993.
- [3] M. Vetterli, "Filter banks allowing perfect reconstruction," Signal Processing, vol.10, no.3, pp.219–244, 1986.
- [4] P.P. Vaidyanathan, "Multirate digital filters, filter banks, polyphase networks, and applications: A tutorial," Proc. IEEE, vol.78, no.1, pp.56–93, Jan. 1990.
- [5] S. Basu, C.H. Chiang, and H.M. Choi, "Wavelets and perfect reconstruction subband coding with causal stable IIR filters," IEEE Trans. Circuit & Syst.–II, vol.42, no.1, pp.24–38, Jan. 1995.
- [6] S.M. Phoong, C.W. Kim, P.P. Vaidyanathan, and R. Ansari, "A new class of two-channel biorthogonal filter banks and wavelet bases," IEEE Trans. Signal Processing, vol.43, no.3, pp.649–665, March 1995.
- [7] X. Zhang and H. Iwakura, "Design of IIR digital filters based on eigenvalue problem," IEEE Trans. Signal Processing, vol.44, no.6, pp.1325–1333, June 1996.
- [8] X. Zhang and H. Iwakura, "Design of IIR Nyquist filters with zero intersymbol interference," IEICE Trans. Fundamentals, vol.E79–A, no.8, pp.1139–1144, Aug. 1996.
- [9] M. Okuda, T. Fukuoka, M. Ikehara, and S. Takahashi, "Design of 2-channel perfect reconstruction IIR filter banks with causality," IEICE Trans., vol.J80–A, no.3, pp.454–462, March 1997.



Xi Zhang was born in Changshu, China, on December 23, 1963. He received the B.E. degree in electronics engineering from the Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China, in 1984, the M.E. and Ph.D. degree in communication engineering from the University of Electro-Communications (UEC), Tokyo, Japan, in 1990 and 1993, respectively. He was with the Department of Electronics Engineering, NUAA, in the period of 1984~1987, and the Department of Communications and Systems, UEC, from April 1993 to March 1996, all as a Research Assistant. Since April 1996, he has been with the Department of Electrical Engineering, Nagaoka University of Technology, Niigata, Japan, as an Associate Professor. He was a recipient of the Award of Science and Technology Progress of China in 1987. His research interests are in the areas of digital signal processing, approximation theory and wavelets. Dr. Zhang is a member of the IEEE.



Toshinori Yoshikawa was born in Kagawa, Japan, on June 20, 1948. He received the B.E., M.E. and Doctor of Engineering degrees from Tokyo Institute of Technology, Tokyo, Japan, in 1971, 1973 and 1976, respectively. From 1976 to 1983, he was with Saitama University engaging in research works on signal processing and its software development. Since 1983, he has been with Nagaoka University of Technology, Niigata, Japan, where he is currently a Professor. His main research area is digital signal processing. Dr. Yoshikawa is a member of the IEEE Computer Society.