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Online acoustic feedback mitigation with improved noise-reduction performance in active noise control systems

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Abstract: The main focus of this paper is the acoustic feedback path neutralisation during online operation of the single-channel active noise control (ANC) systems. Invasive techniques, in which additive auxiliary noise is injected for online feedback path modelling (FBPM), degrade the noise-reduction performance of ANC systems. In the existing methods for online FBPM, additive auxiliary noise with fixed variance is injected during all operating conditions of ANC systems. In this paper, a scheduling strategy is proposed to have time-varying gain for additive auxiliary noise. The time-varying gain is computed on the basis of (i) the convergence status of the FBPM filter, and (ii) the power of the interference term in the error signal of the FBPM filter. Simulation results show that, compared to the existing methods, the proposed method can better achieve the conflicting requirements of fast convergence of the FBPM filter, and reduced steady-state power of the residual error at the error microphone. For sake of completeness, appendix presents typical simulation results when both the secondary path and feedback path are simultaneously identified during online operation of ANC systems.

1 Introduction

A block diagram of single-channel feedforward active noise control (ANC) systems with fixed feedback path neutralisation (FBPN) for duct applications is shown in Fig. 1 [1]. Here $r(n)$ is the primary noise from the noise source, $y_f(n)$ is the acoustic feedback coupling signal, $c(n)$ is the corrupted reference signal picked-up by the reference microphone, $d(n)$ is the primary noise at the error microphone, $y_s(n)$ is the anti-noise signal to cancel $d(n)$, $e(n)$ is the residual error signal picked-up by the error microphone, $P(z)$ is the primary path transfer function from the reference microphone to the error microphone, $F(z)$ is the feedback path transfer function from the loudspeaker to the reference microphone, $\hat{F}(z)$ is the FBPN filter, and $S(z)$ is the secondary path transfer function from output of ANC filter $W(z)$ to the error microphone. Due to the robustness and ease of implementation of filtered-X-least-mean-square (FXLMS) algorithm, it is widely used for adaptation of ANC filter $W(z)$. For stable operation of ANC system, the reference signal is required to be filtered through the model of secondary path $S(z)$, $\hat{S}(z)$, and hence the name FXLMS algorithm [1]. The output signal of $W(z)$, $y(n)$, is propagated by the cancelling loudspeaker. The signal $y(n)$ goes downstream through $S(z)$ to generate the cancelling signal $y_s(n)$ which combines with $d(n)$ to generate the residual error signal $e(n)$ being picked-up by the error microphone. Unfortunately, the output signal $y(n)$ propagates upstream via feedback path $F(z)$ and corrupts the signal $r(n)$. This is called acoustic feedback, and the objective of $\hat{F}(z)$ is to model the characteristics of $F(z)$ and hence provide the FBPN.

Considering Fig. 1 and assuming that the filter $\hat{F}(z)$ is not present, the residual error in z-domain can be written as

$$\begin{aligned} E(z) &= P(z)R(z) + S(z)Y(z) \\ &= P(z)R(z) + S(z)\frac{W(z)R(z)}{1 - W(z)F(z)} \end{aligned} \tag{1}$$

where $R(z)$ denotes the z-transform of the primary noise signal at the reference microphone, $r(n)$. It is clear from (1) that ANC system may become unstable if $W(z)F(z) = 1$ at some frequencies. There are various types of strategies that have been reported in the literature to solve the problem of the acoustic feedback in the ANC systems. These include (i) directional (array of) microphones and speakers [2, 3], (ii) non-acoustic sensors such as tachometer to acquire the reference signal [4, 5], (iii) adaptive feedback ANC employing only the error microphone [6, 7], (iv) adaptive IIR filter based acoustic feedback compensation [8, 9], (v) fixed FBPN filter (obtained through off-line modelling) [1, 10], and (vi) adaptive FBPN using an FIR filter [11–17]. The structure-based approaches mentioned in (i)–(iii) are either expensive or have limited applicabilities. Among the signal processing-based approaches (iv)–(vi), the IIR filter-based methods have an inherent problem of stability.

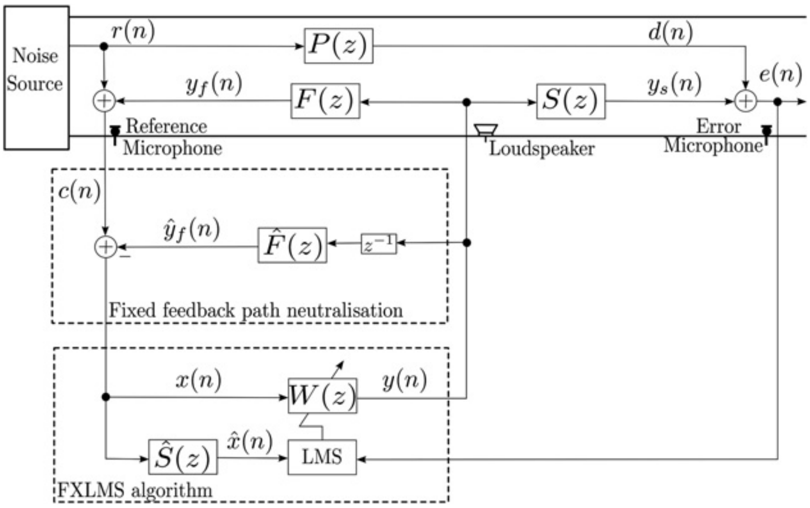

Fig. 1 *Block diagram of single-channel feedforward ANC systems with fixed feedback path neutralisation*

Moreover, the IIR filter may converge to a local minimum. The FBPN based on fixed filter $\hat{F}(z)$ (as shown in Fig. 1) offers a simplest solution, where $\hat{F}(z)$ may be obtained offline. In actual practice the acoustic path $F(z)$ may be time-varying, therefore, the main focus of this paper is to investigate the adaptive FBPN using FIR filter. A brief overview of existing FIR filter-based methods is given below.

In [11] and [12], Kuo investigated the FIR filter-based methods for online feedback path modelling and neutralisation (FBPMN). The problems with Kuo's method are that: (1) it works only for predictable noise sources, and (ii) a proper selection of decorrelation delay is needed. Later Akhtar *et al*. proposed a new structure for online FBPMN [13–15]. The advantage of Akhtar's method is that in comparison with Kuo's method, improved performance is realised both for narrowband and broad band input signals. Furthermore, no decorrelation delay is needed and computational cost is lower than Kuo's method. In [16], Akhtar's method is slightly modified, and in [16] and [17] a variable step size (VSS) is used for the FBPMN filter to improve the convergence of $\hat{F}(z)$.

In Akhtar's method, the step-size variation is such that it is minimum at the start-up of ANC system and increases to a maximum value as the ANC system converges. For LMS-based adaptive filter with input $v(n)$, the excess mean-square-error (MSE) is given by [1]

$$\xi_{\text{excess}} \simeq 0.5\mu L P_{\text{v}} \xi_{\min} \tag{2}$$

where μ is the step-size parameter, L is the tap-weight length, P_v is the power of input signal, and $\xi_{\min}$ is the minimum MSE corresponding to Weiner solution of an adaptive filter. It is easy to conclude from (2) that the large value of step-size in the steady-state results in large excess MSE, therefore in Akhtar's method the proper selection of maximum value of step-size parameter for the FBPMN filter is very important. In the proposed method fixed step-size is used for the FBPMN filter and its convergence is controlled by varying the input signal power. In [11–17], additive auxiliary noise with fixed variance is used in all operating conditions. The additive auxiliary noise contributes to the residual error, and therefore degrades the noise-reduction performance of ANC system. The problems with the existing methods for online FBPMN sets the motivation for the proposed method.

In the proposed method, a fixed step-size is used for the FBPMN filter and convergence is controlled by varying the input signal power. Essentially, an auxiliary-noise-power (ANP) scheduling strategy is employed that can better achieve the conflicting requirements of fast convergence of the FBPMN filter and improved noise-reduction performance at the steady-state.

The rest of the paper is organised as follows. Section 2 explains the proposed method and details the computational complexity comparison, Section 3 describes the simulation results, and Section 4 gives the concluding remarks. Finally, the Appendix presents a few typical results for simultaneous online FBPM and secondary path modelling (SPM).

2 Proposed method

A block diagram of the proposed method is shown in Fig. 2, where online FBPMN is achieved by additive auxiliary noise $v_g(n)$ being modelled as white Gaussian noise (WGN). The same signal $v_g(n)$ can also be employed for online SPM. However, in this paper we mainly concentrate on online FBPMN. Assuming that $W(z)$ is an FIR filter, the output $y(n)$ is given as

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_{\text{w}}(n) \tag{3}$$

where $\boldsymbol{w}(n) = [w_0(n), w_1(n), \ldots, w_{L_{\text{w}}-1}(n)]^T$ is the tap-weight vector at iteration n, L_{w} is the tap-weight length, and $\boldsymbol{x}_{\text{w}}(n) = [x(n), x(n-1), \ldots, x(n-L_{\text{w}}+1)]^T$ is the reference signal vector of $W(z)$. The weight update equation for ANC filter $W(z)$ is given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \mu_{\text{w}} e(n)\hat{\boldsymbol{x}}(n) \tag{4}$$

where μ_{w} is the step-size parameter, and $\hat{\boldsymbol{x}}(n) = \left[\hat{x}(n), \hat{x}(n-1), \ldots \hat{x}\left(n-L_{\text{w}}+1\right)\right]^T$ is the filtered reference signal vector. The reference signal $x(n)$ filtered through $\hat{S}(z)$ is computed as

$$\hat{x}(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}_{\text{s}}(n) \tag{5}$$

where $\hat{\boldsymbol{s}}(n) = [\hat{s}_0(n), \hat{s}_1(n), \ldots, \hat{s}_{L_{\text{s}}-1}(n)]^T$ is the tap-weight

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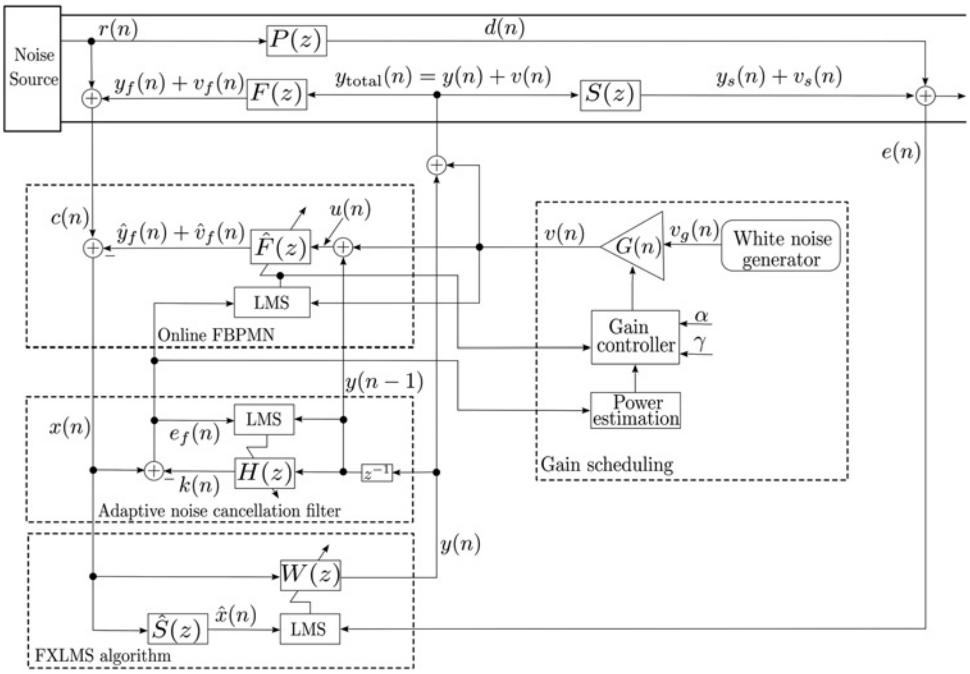

Fig. 2 *Proposed method for online feedback path modelling and neutralisation*

vector at iteration n, $\boldsymbol{x}_s(n)=[x(n), x(n-1), \ldots, x(n-L_s+1)]^T$, and L_s is the tap-weight length of $\hat{S}(z)$. The residual error signal $e(n)$ picked-up by the error microphone is given by

$$e(n) = d(n) + y_s(n) + v_s(n) \tag{6}$$

where $d(n)=p(n)*r(n)$, $p(n)$ is the impulse response of $P(z)$, $*$ represents the convolution operation; $y_s(n)=s(n)*y(n)$, $s(n)$ is the impulse response of $S(z)$; and $v_s(n)=s(n)*v(n)$ denotes the contribution of the additive auxiliary noise at the error microphone, where $v(n)$ is the additive auxiliary WGN. The additive auxiliary noise $v(n)$ is injected into the ANC system for online FBPM (can be used for SPM as well). This internally generated signal is added with the ANC filter output $y(n)$. The sum $y_{\text{total}}(n)=y(n)+v(n)$ is used to derive the secondary source (loudspeaker). The signal generated by the loudspeaker will not only travel downstream to cancel $d(n)$, but will also propagate upstream through $F(z)$, and a corrupted reference signal $c(n)$ is picked-up by the reference microphone given as

$$c(n) = r(n) + y_f(n) + v_f(n) \tag{7}$$

where $y_f(n)=f(n)*y(n)$, $f(n)$ is the impulse response of $F(z)$; and $v_f(n)=f(n)*v(n)$ denotes the contribution of the additive auxiliary noise at the reference microphone. The output of the FBPMN filter $\hat{F}(z)$ is subtracted from $c(n)$ to compute the reference signal $x(n)$ as

$$x(n) = c(n) - \hat{y}_f(n) - \hat{v}_f(n) = (r(n) + y_f(n) - \hat{y}_f(n)) + v_f(n) - \hat{v}_f(n) \tag{8}$$

where the signal $\hat{y}_f(n) + \hat{v}_f(n)$ (estimate of acoustic feedback signal $y_f(n)+v_f(n)$) is the output of $\hat{F}(z)$, and is computed as

$$\hat{y}_f(n) + \hat{v}_f(n) = \hat{\boldsymbol{f}}^T(n)\boldsymbol{u}_f(n) = \hat{\boldsymbol{f}}^T(n)\left[\boldsymbol{y}_f(n) + \boldsymbol{v}_f(n)\right] \tag{9}$$

where $\hat{\boldsymbol{f}}(n) = \left[\hat{f}_0(n), \hat{f}_1(n), \ldots, \hat{f}_{L_f-1}(n)\right]^T$ is the tap-weight vector at iteration n, L_f is the tap-weight length, $\boldsymbol{u}_f(n)=[u(n), u(n-1), \ldots, u(n-L_f+1)]^T$ is the input signal vector, and $\boldsymbol{y}_f(n)=[y(n-1), y(n-2), \ldots, y(n-L_f)]^T$ and $\boldsymbol{v}_f(n)=[v(n), v(n-1), \ldots, v(n-L_f+1)]^T$, respectively, are the input signal vectors of $\hat{F}(z)$ corresponding to ANC filter output $y(n)$ and additive auxiliary noise $v(n)$. The signal $x(n)$ acts as the desired response for adaptive noise cancellation (ADNC) filter $H(z)$, and the filter $H(z)$ is adapted using LMS algorithm as

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu_h e_f(n)\boldsymbol{y}_h(n) \tag{10}$$

where $\boldsymbol{h}(n) = \left[h_0(n), h_1(n), \ldots, h_{L_h-1}(n)\right]^T$ is the tap-weight vector at iteration n, L_h is the tap-weight length, μ_h is the step-size parameter, $\boldsymbol{y}_h(n)=[y(n-1), y(n-2), \ldots, y(n-L_h)]^T$ is the input signal vector, and $e_f(n)$ is the error signal of $H(z)$, being computed as

$$e_f(n) = \left[r(n) + y_f(n) - \hat{y}_f(n) - k(n)\right] + \left[v_f(n) - \hat{v}_f(n)\right] \tag{11}$$

where $k(n)$ is the output of $H(z)$, being computed as

$$k(n) = \boldsymbol{h}^T(n)\boldsymbol{y}_h(n) \tag{12}$$

The same signal $e_f(n)$ also acts as the error signal for FBPMN filter $\hat{F}(z)$; however, the term $r(n) + y_f(n) - \hat{y}_f(n)$ in $x(n)$ acts as an interference for adaptation of $\hat{F}(z)$. The function of the supporting filter $H(z)$ is to remove the interference term $r(n) + y_f(n) - \hat{y}_f(n)$ from $x(n)$. Assuming that $H(z)$ converges $\Rightarrow k(n) \rightarrow r(n) + y_f(n) - \hat{y}_f(n) \Rightarrow e_f(n) = v_f(n) - \hat{v}_f(n)$. Using $e_f(n)$, the FBPMN filter $\hat{F}(z)$ is adapted as

$$\hat{\boldsymbol{f}}(n+1) = \hat{\boldsymbol{f}}(n) + \mu_f e_f(n)\boldsymbol{v}_f(n) \tag{13}$$

where μ_f is the step-size parameter, and $\boldsymbol{v}_f(n)=[v(n), v(n-1), \ldots v(n-L_f+1)]^T$ is the input signal vector of $\hat{F}(z)$. As stated earlier, the variance of the additive auxiliary noise is made variable by employing a time-varying gain $G(n)$

$$v(n) = G(n)v_g(n) \tag{14}$$

where $v_g(n)$ is a zero-mean unit variance WGN.

2.1 Computation of time-varying gain G(n)

Additive auxiliary noise $v(n)$ injected in ANC systems for online FBPMN (can be used for SPM as well) contributes to the residual error, which should be minimised. In order to improve the noise-reduction performance, the variance of additive auxiliary noise $v(n)$ is made variable. When ANC system is far from the steady-state, additive auxiliary noise $v(n)$ with large variance is injected to have fast convergence of the FBPMN filter. At this stage the large contribution of additive auxiliary noise is masked by the large value of primary residual noise at the error microphone, $d(n) + y_s(n)$. After the convergence of ANC system the term $d(n) + y_s(n)$ becomes small, and is not able to mask the additive auxiliary noise at the error microphone. The gain scheduling strategy is such that, with the convergence of ANC system, it will reduce the contribution of additive auxiliary noise to the residual error.

As the error signal of the FBPMN filter, $e_f(n)$, is time-varying (has decreasing trend) in nature, therefore the gain $G(n)$ in the proposed method is computed by making $\left\{P_{(\hat{y}_f+\hat{v}_f)}(n) - P_{\hat{y}_f}(n)\right\}$ to be equal to the power of $e_f(n-1)$, and is given mathematically as

$$P_{(\hat{y}_f+\hat{v}_f)}(n) - P_{\hat{y}_f}(n) = P_{e_f}(n-1) \tag{15}$$

From Fig. 2, the term $P_{(\hat{y}_f+\hat{v}_f)}(n)$ in (15) can be written as

$$P_{(\hat{y}_f+\hat{v}_f)}(n) = \|\hat{\boldsymbol{f}}(n)\|^2\left[E\left[(y(n-1))^2\right] + G^2(n)E\left[(v_g(n))^2\right]\right] \tag{16}$$

where $E[\cdot]$ denotes the mathematical expectation, $\|\cdot\|$ denotes the euclidean norm, $E[(y(n-1))^2] \simeq P_y(n-1)$, and $v_g(n)$ is a zero-mean, unit variance WGN. Similarly the term $P_{\hat{y}_f}(n)$ in (15) can be written as

$$P_{\hat{y}_f}(n) = \|\hat{\boldsymbol{f}}(n)\|^2 P_y(n-1) \tag{17}$$

Substituting the value of $P_{(\hat{y}_f+\hat{v}_f)}(n)$ and $P_{\hat{y}_f}(n)$, respectively, from (16) and (17) in (15), and solving for the time-varying gain $G(n)$ we get

$$G(n) = \sqrt{\frac{P_{e_f}(n-1)}{\|\hat{\boldsymbol{f}}(n)\|^2}} \tag{18}$$

where the power of the error signal $P_{e_f}(n)$ can be estimated online using a low pass estimator as

$$P_{e_f}(n) = \lambda P_{e_f}(n-1) + (1-\lambda)e_f^2(n) \tag{19}$$

where $0.9 < \lambda < 1$ is a forgetting factor. In the proposed method fixed step-size is used for the FBPMN filter, and the convergence is controlled by varying its input signal power i.e $P_v(n)$. The gain computed by (18) very quickly drops to a low value, thus resulting in a very small input signal power of the FBPMN filter. This will result in freezing of the convergence of the FBPMN filter even if $\hat{F}(z)$ is far from $F(z)$. In order to avoid this problem the gain is filtered and is given by

$$G(n) = \alpha G(n-1) + \gamma \cdot \sqrt{\frac{P_{e_f}(n-1)}{\|\hat{\boldsymbol{f}}(n)\|^2}} \tag{20}$$

where α and γ controls the decay rate and the steady-state value of the time-varying gain $G(n)$. The parameter α is like a forgetting factor and varies between $0.99 < \alpha < 1$ while $\gamma > 0$ is typically selected as a very small value.

The proposed ANP scheduling strategy for online FBPMN has the following advantages.

- In (11), the term $r(n) + y_f(n) - \hat{y}_f(n) - k(n)$ acts as an interference for the FBPMN filter. The value of gain $G(n)$ in the proposed method has no upper bound, and it can increase in accordance with the power of interference term $P_{[r(n)+y_f(n)-\hat{y}_f(n)-k(n)]}$. This will eliminate the requirement of minimum value of step-size (in Akhtar's method) for the FBPMN filter due to large interference term $P_{[r(n)+y_f(n)-\hat{y}_f(n)-k(n)]}$.
- The large variance of additive auxiliary noise, at the start-up of ANC system or if there is some perturbation in the acoustic paths after ANC system converges, results in fast convergence of the FBPMN filter and reduced interference in the input signal of ANC filter $W(z)$. This improves the convergence of ANC filter, and hence results in fast convergence of primary residual noise power, $E[(d(n) + y_s(n))^2]$.
- The low variance of additive auxiliary noise $v(n)$ at the steady-state improves the noise-reduction performance of ANC systems.

2.2 Computational complexity analysis

The detailed computational complexity analysis, in terms of number of additions/subtractions and multiplications required per iteration, for Kuo's [12], Akhtar's [17] and the proposed methods is given in Table 1. In addition to the computations shown in Table 1, Akhtar's method requires one division per iteration in computing VSS [17], and the proposed method requires one division and one square root operation per iteration in computing time-varying gain $G(n)$ (20). In Kuo's method fixed step-size parameters are used for adaptive filters, and separate filters are used for FBPM and FBPN. The Kuo's method has high computational complexity compared to Akhtar's method. In Akhtar's method the action of FBPM and FBPN filter is combined into a single FBPMN filter. Although, in Akhtar's method computing the VSS for the FBPMN filter involves some computations, however, the overall computational complexity of Akhtar's method is lower than Kuo's method. In the proposed method gain scheduling strategy is used, therefore the computational cost of the proposed method is higher than Akhtar's method but it is almost the same as that of Kuo's method. For clarity of presentation, the numerical values of the number of computations needed are given in Table 1 for two different scenarios of filter tap-weight lengths.

3 Simulation results and discussion

In this section, simulation results are presented to compare the performance of the proposed method with Kuo's [12], and

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Table 1 Detailed computational complexity analysis for online FBPMN of various methods discussed in the paper

Mathematical expressions					
S. no	To compute	$\times$	$+$	$\div$	$\sqrt{}$
1.	$\hat{y}_f(n) = \hat{\boldsymbol{f}}^T(n)\boldsymbol{x}_y(n)$	L_f	L_f-1		
2.	$c'(n) = c(n) - \hat{y}_f(n)$	–	1		
3.	$y_h(n) = \boldsymbol{h}^T(n)\boldsymbol{x}_h(n)$	L_h	L_h-1		
4.	$e_h(n) = c'(n) - y_h(n)$	–	1		
5.	$x(n) = c'(n) - e_h(n)$	–	1		
6.	$\hat{v}_f(n) = \hat{\boldsymbol{f}}^T(n)\boldsymbol{v}(n)$	L_f	L_f-1		
7.	$e_f(n) = e_h(n) - \hat{v}_f(n)$	–	1		
8.	$\hat{x}_s(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}(n)$	L_s	L_s-1		
9.	$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}(n)$	L_w	L_w-1		
10.	$x_{in}(n) = y(n) + v(n)$	–	1		
11.	$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu_h e_h(n)\boldsymbol{x}_h(n)$	L_h+1	L_h		
12.	$\hat{\boldsymbol{f}}(n+1) = \hat{\boldsymbol{f}}(n) + \mu_f e_f(n)\boldsymbol{v}(n)$	L_f+1	L_f		
13.	$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \mu_w e(n)\hat{\boldsymbol{x}}_s(n)$	L_w+1	L_w		
Total computations in Kuo's method[a]	$3L_f+2L_h+2L_w+L_s+3$	$3L_f+2L_h+2L_w+L_s$			
14.	$x_f(n) = y(n-1) + v(n)$	–	1		
15.	$\hat{y}_f(n) + \hat{v}_f(n) = \hat{\boldsymbol{f}}^T(n)\boldsymbol{x}_f(n)$	L_f	L_f-1		
16.	$x(n) = c(n) - (\hat{y}_f(n) + \hat{v}_f(n))$	–	1		
17.	$e_f(n) = x(n) - y_h(n)$	–	1		
18.	$P_{e_f}(n)$ using (19)	3	1		
19.	$P_x(n)$ using (19)	3	1		
20.	$\rho(n) = \dfrac{P_{e_f}(n)}{P_x(n)}$	–	–	1	
21.	$\mu_s(n) = \rho(n)\mu_{s_{min}} + (1-\rho(n))\mu_{s_{max}}$	2	2		
22.	$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu_h e_f(n)\boldsymbol{x}_h(n)$	L_h+1	L_h		
23.	$\hat{\boldsymbol{f}}(n+1) = \hat{\boldsymbol{f}}(n) + \mu_f(n)e_f(n)\boldsymbol{v}(n)$	L_f+1	L_f		
Total computations in Akhtar's method[b]	$2L_f+2L_h+2L_w+L_s+11$	$2L_f+2L_h+2L_w+L_s+4$	1		
24.	$v(n) = G(n)v_g(n)$	1	–		
25.	$\|\|\hat{\boldsymbol{f}}(n)\|\|^2$	L_f	L_{f-1}		
26.	$G(n)$ using (20)	2	1	1	1
Total computations in the proposed method[c]	$3L_f+2L_h+2L_w+L_s+9$	$3L_f+2L_h+2L_w+L_s+1$	1	1	
Numerical examples					
Example 1 ($L_f = L_w = 32$, $L_h = L_s = 16$) :					
	Kuo's method	211	208	–	–
	Akhtar's method	187	180	1	–
	Proposed method	217	209	1	1
Example 2 ($L_f = L_w = L_h = L_s = 1024$) :					
	Kuo's method	8195	8192	–	–
	Akhtar's method	7179	7172	1	–
	Proposed method	8201	8193	1	1

Akhtar's [17] methods. The performance comparison is carried out on the basis of following performance measures.

- Relative modelling error of the feedback path being defined as

$$\Delta F(n)(\mathrm{dB}) = 10\log_{10}\frac{\|\boldsymbol{f}(n) - \hat{\boldsymbol{f}}(n)\|^2}{\|\boldsymbol{f}(n)\|^2} \tag{21}$$

- MSE in the reference signal $\Delta X(n)$ being defined as

$$\Delta X(n)(\mathrm{dB}) = 10\log_{10}\left(E\left[(x(n) - r(n))^2\right]\right) \tag{22}$$

- Mean-noise-reduction (MNR) at the error microphone

$$\mathrm{MNR}(n)(\mathrm{dB}) = 10\log_{10}E\frac{\left[e^2(n)\right]}{E\left[d^2(n)\right]} \tag{23}$$

- MSE at the error microphone without additive auxiliary noise

$$\mathrm{MSE}_{\mathrm{ideal}}(n)(\mathrm{dB}) = 10\log_{10}(E[(d(n) + y_s(n))^2]) \tag{24}$$

Using data from [1], the acoustic paths $P(z)$, $S(z)$ and $F(z)$ are modelled as FIR filters of tap-weight lengths 48, 16 and

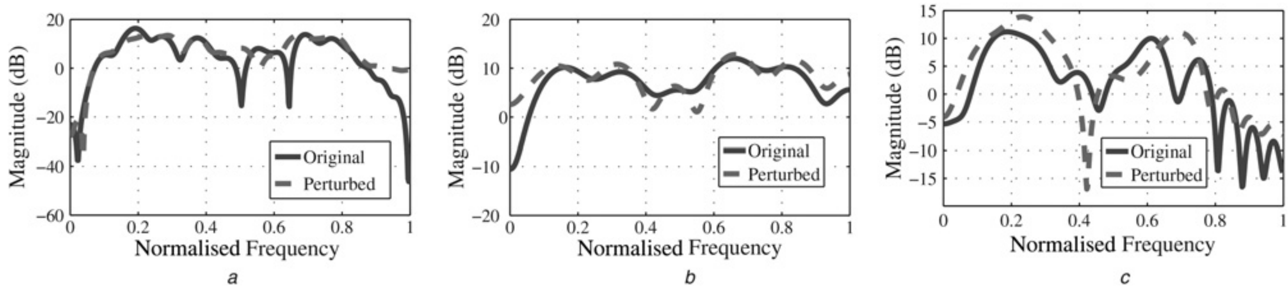

Fig. 3 *Magnitude response of*

a Primary path $P(z)$
b Secondary path $S(z)$
c Feedback path $F(z)$

32, respectively. The magnitude response of the acoustic paths $P(z)$, $S(z)$ and $F(z)$ are shown in Fig. 3 (solid line). The adaptive filters $W(z)$, $\hat{F}(z)$ and $H(z)$ are selected as FIR filters of tap-weight length 32, 32 and 16, respectively. In all methods, -5 dB offline modelling is used for $\hat{F}(z)$. The ANC filter $W(z)$ and $H(z)$ are initialised by null vectors. The modelling excitation signal is a zero-mean WGN with variance 0.05 for Kuo's and Akhtar's methods, and variance 1 for the proposed method. The sampling frequency of 2 kHz is used. All the simulation results are averaged over 20 independent realizations.

3.1 Case 1: multi-tonal input

The primary noise signal at the reference microphone, $r(n)$ is a multi-tonal input with frequencies 100, 150, 300, 400 and 450 Hz. The variance of the input signal $r(n)$ is 2 and a zero-mean WGN with variance 0.002 is added to $r(n)$ to account for measurement noise. The step-size parameters are experimentally adjusted for fast and stable convergence for various methods. The step-size parameter for $W(z)$ and $H(z)$ is $\mu_w = 3 \times 10^{-5}$ and $\mu_h = 5 \times 10^{-4}$, respectively, for all methods. The step-size parameter for $\hat{F}(z)$ is selected as Kuo's method, $\mu_f = 5 \times 10^{-3}$, proposed method, $\mu_f = 1 \times 10^{-3}$ and Akhtar's method, $\mu_{f_{min}} = 3 \times 10^{-4}$, $\mu_{f_{max}} = 5 \times 10^{-3}$, respectively. The rest of the parameters are adjusted as $\Delta = 32$, $\alpha = 0.9992$ and $\gamma = 2 \times 10^{-3}$.

The simulation results for Case 1 are shown in Figs. 4 and 5. In all plots of Figs. 4 and 5, the jump at $n = 30\,000$ shows a perturbation in the acoustic paths. The magnitude response of the perturbed acoustic paths are shown in Fig. 3 (dashed line).

- Fig. 4*a* shows the plot of time-varying gain $G(n)$ in the proposed method. When ANC system is far from the steady-state or when there is a perturbation in the acoustic paths, the value of the gain $G(n)$ is large. The large value of the gain results in fast convergence of the FBPMN filter, and therefore quickly reduces the interference for the input signal of $W(z)$. After the convergence of ANC system the value of the gain $G(n)$ is small. This will reduce the contribution of additive auxiliary noise to the residual error, thus improves the noise-reduction performance.
- The curves for relative modelling error $\Delta F(n)$ as defined in (21), are shown in Fig. 4*b*. As stated earlier fast convergence of the FBPMN filter is desirable to quickly neutralise the acoustic feedback path effect, and hence to reduce the interference in the input signal of $W(z)$. It is clear from Fig. 4*b* that the proposed method gives fast convergence of the FBPMN filter. The fast convergence is due to large variance of additive auxiliary noise at the start-up of ANC system. Since in Kuo's and Akhtar's methods, no gain scheduling is used therefore additive auxiliary noise with small variance is injected, otherwise it will degrade the noise-reduction performance of ANC system.
- The curves for $\Delta X(n)$ are shown in Fig. 4*c*. It is clear from Fig. 4*c* that ANC system without FBPN filter results in a large interference in the input signal of $W(z)$, and becomes unstable after the acoustic paths perturbation at $n = 30000$. In the proposed method fast convergence of the FBPM filter efficiently neutralise the acoustic feedback path coupling effect, thus results in reduced interference in the input signal of $W(z)$ compared to Kuo's and Akhtar's methods.
- The curves for MNR performance are shown in Fig. 5*a*. It is clear that for ANC system without FBPN filter, large

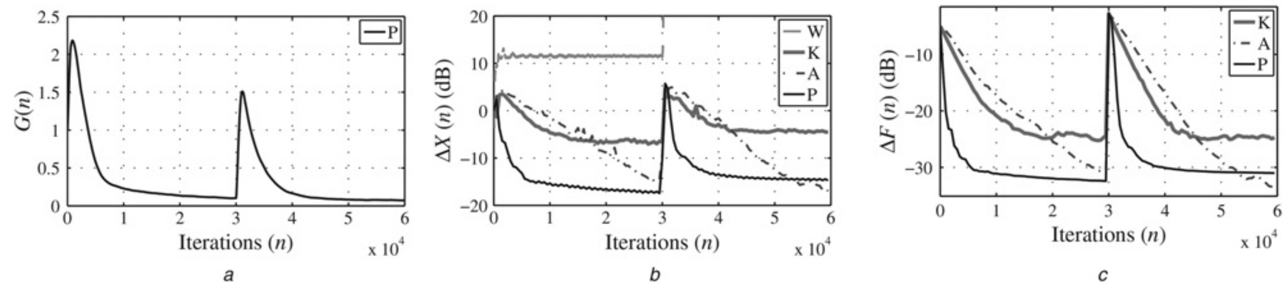

Fig. 4 *Simulation results in Case 1 for multi-tonal input*

a Time-varying gain, $G(n)$
b Relative modelling error, $\Delta F(n)$ (dB)
c MSE in the reference signal, $\Delta X(n)$ (dB)
W, Without FBPN; K, Kuo's method; A, Akhtar's method; P, Proposed method

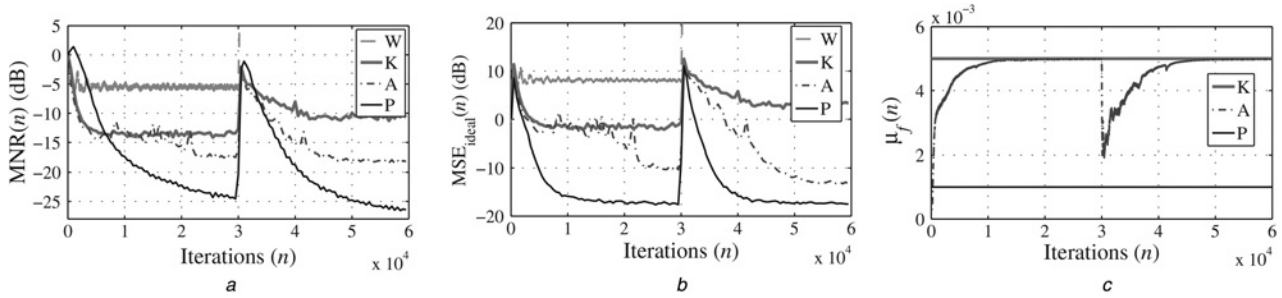

Fig. 5 *Simulation results in Case 1 for multi-tonal input*

a MNR at the error microphone, MNR(n)
b MSE at the error microphone without additive auxiliary noise, $\mathrm{MSE}_{\mathrm{ideal}}(n)$
c Step-size parameter, $\mu_f(n)$
W, Without FBPN; K, Kuo's method; A, Akhtar's method; P, Proposed method

interference in the input signal of $W(z)$ affects the overall convergence of ANC system, and therefore results in large $E[e^2(n)]$. The proposed method results in improved noise-reduction performance at the steady-state compared to Kuo's and Akhtar's methods. The improved noise-reduction performance is due to small contribution of $E[(d(n)+y_s(n))^2]$ (see Fig. 5*b*) and $E[v_s^2(n)]$ (due to gain scheduling) in $E[e^2(n)]$.

- Fig. 5*b* shows the curves for $\mathrm{MSE}_{\mathrm{ideal}}(n)$ at the error microphone without additive auxiliary noise contribution, $E[(d(n)+y_s(n))^2]$. In case of ANC systems without FBPN filter, no additive auxiliary noise is injected, therefore $E[e^2(n)] = E[(d(n)+y_s(n))^2]$. The absence of FBPN filter results in large interference in the input signal of $W(z)$, therefore $W(z)$ is not able to generate the desired anti-noise signal $y_s(n)$ at the error microphone. As stated earlier, in the proposed method the fast convergence of the FBPMN filter results in reduced interference in the input signal of $W(z)$, therefore $E[d(n)+y_s(n)^2]$ converges quickly towards minimum value.
- The value of step-size for FBPMN filter in the proposed method is different from that of Kuo's and Akhtar's methods. This is to compensate for different input variance of additive auxiliary noise $v(n)$ in these methods. Fig. 5*c* shows the step-size variation for the FBPMN filter $\hat{F}(z)$. In Kuo's and the proposed method fixed step-size is used, while in Akhtar's method VSS is used. In the proposed method the value of step-size for the FBPMN filter is smaller than Kuo's method, but still the proposed method gives fast convergence of the FBPMN filter. This is due to large variance of additive auxiliary noise in the proposed method compared to Kuo's method. As stated earlier, in Akhtar's method initially a small value of step-size is used due to large value of interference term in the error signal of the FBPMN filter. In the later stage, the step-size increases accordingly as the interference decreases.

3.2 Case 2: broad-band input

The reference signal is obtained by filtering WGN signal, with variance 2, through a bandpass filter of order 128 with a passband of [100 500] Hz. The step-size parameters are experimentally adjusted for fast and stable convergence for various methods. The step-size parameter for $W(z)$ and $H(z)$ is $\mu_w = 1\times10^{-4}$ and $\mu_h = 5\times10^{-4}$, respectively, for all methods. The step-size for $\hat{F}(z)$ is $\mu_f = 5\times10^{-3}$ in Kuo's and the proposed method. In Akhtar's method the minimum and maximum value of the step-size parameter for the FBPMN filter is $\mu_{f_{\min}} = 4\times10^{-3}$ and $\mu_{f_{\max}} = 8\times10^{-3}$, respectively. The rest of the parameters are adjusted as $\Delta = 32$, $\alpha = 0.999$ and $\gamma = 1\times10^{-4}$.

Figs. 6 and 7 shows the simulation results for Case 2. It is clear form Figs. 6 and 7 that similar behaviour as in Case 1 is observed for Akhtar's method and the proposed methods. The ANC system without FBPN become unstable for all realizations. For broad-band input, Kuo's method achieves almost same modelling accuracy as by Akhtar's and the proposed methods. However, for broad-band inputs the predictor used in Kuo's structure results in large interference for the the input signal of $W(z)$. The ANC filter $W(z)$ is therefore not able to generate the desired anti-noise signal at the error microphone, thus affecting the overall convergence behaviour of ANC system.

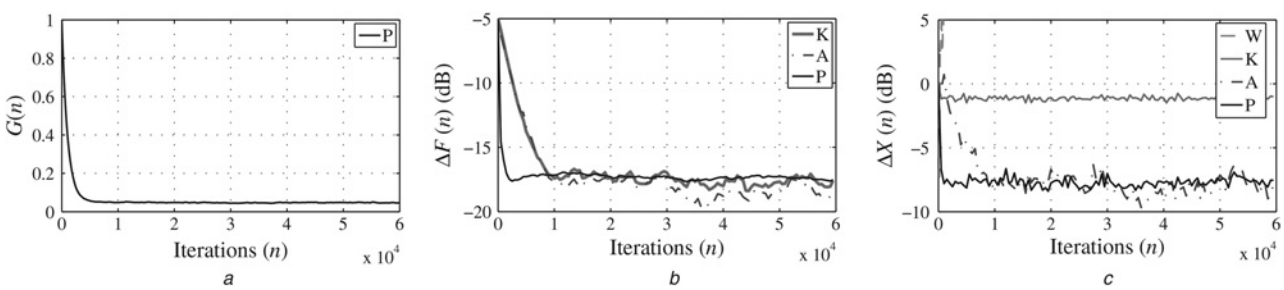

Fig. 6 *Simulation results in Case 2 for broad-band input*

a Time-varying gain, $G(n)$
b Relative modelling error, $\Delta F(n)$ (dB)
c MSE in the reference signal, $\Delta X(n)$ (dB)
W, Without FBPN; K, Kuo's method; A, Akhtar's method; P, Proposed method

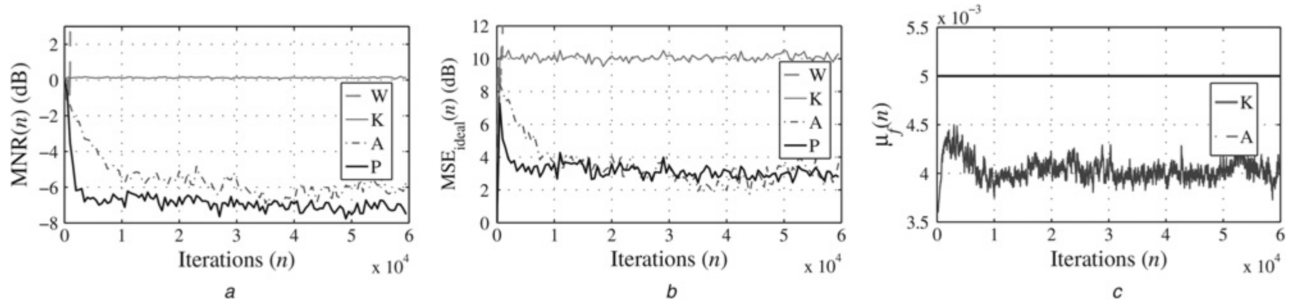

Fig. 7 *Simulation results in Case 2 for broad-band input*

a MNR at the error microphone, MNR(n)
b MSE at the error microphone without additive auxiliary noise, $\mathrm{MSE}_{\mathrm{ideal}}(n)$
c Step-size parameter, $\mu_f(n)$
W, Without FBPN; K, Kuo's method; A, Akhtar's method; P, Proposed method

4 Conclusion

In this paper, a gain scheduling strategy is proposed to vary ANP. The gain varies: (i) in accordance with how far the FBPM filter is from $F(z)$, and (ii) in accordance with the power of interference term in the error signal of the FBPM filter. At the start-up of ANC system the gain is large, therefore additive auxiliary noise with large variance is injected. This results in fast convergence of the FBPM filter, and hence reduced interference in the input signal of ANC filter. In the steady-state the gain is reduced to a very small value, thus additive auxiliary noise with small variance is injected. This results in improved noise-reduction performance at the steady-state. In order to track the variations in the secondary path, the idea of gain scheduling for FBPM and SPM is combined to have a time-varying gain for simultaneous online FBPM and SPM. The simulation results verify the effectiveness of the proposed method.

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6 Appendix 1

6.1 Simultaneous online modelling of feedback and secondary paths

In actual practice the secondary path is time-varying, therefore both the FBPM and SPM filters must be simultaneously updated during online operation of the ANC systems. The objective of this appendix is to consider the ANP scheduling strategy for simultaneous online FBPM

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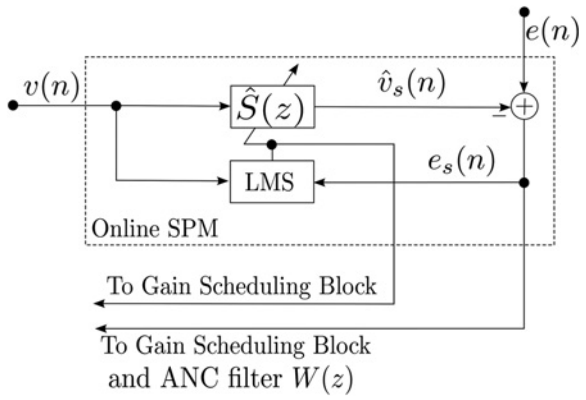

Fig. 8 *Block diagram for online secondary path modelling with additive auxiliary noise power scheduling*

and SPM. The block diagram shown in Fig. 8 is integrated with the block diagram of Fig. 2 for simultaneous online FBPM and SPM. From Fig. 8 the error signal $e_s(n)$ of SPM filter is computed as

$$e_s(n) = e(n) - \hat{v}_s(n) \tag{25}$$

where

$$\hat{v}_s(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{v}_s(n) \tag{26}$$

where $\boldsymbol{v}_s(n) = [v(n), v(n-1), \ldots, v(n - L_s + 1)]^T$ is the input signal vector of $\hat{S}(z)$. The weight update equation for SPM filter is given by

$$\hat{\boldsymbol{s}}(n+1) = \hat{\boldsymbol{s}}(n) + \mu_s e_s(n)\boldsymbol{v}_s(n) \tag{27}$$

where μ_s is the step-size parameter for SPM filter. The error signal of SPM filter $e_s(n)$ is also used as an error signal of ANC filter, and thus the weight update equation for ANC filter $W(z)$ is modified as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \mu_w e_s(n)\hat{\boldsymbol{x}}(n) \tag{28}$$

Recently we have proposed a gain scheduling strategy for online SPM [18]. The expression for the time-varying gain $G(n)$ is obtained by making the power $P_{v_s}(n)$ to be equal to the power $P_{e_s}(n-1)$. In the case of ANC systems the signal $v_s(n)$ is not accessible, therefore the following condition

$$P_{\hat{v}_s}(n) = P_{e_s}(n-1) \tag{29}$$

is forced, where $P_{\hat{v}_s}(n)$ is given by

$$P_{\hat{v}_s}(n) = G^2(n)||\hat{\boldsymbol{s}}(n)||^2 E\left[v_g^2(n)\right] = G^2(n)||\hat{\boldsymbol{s}}(n)||^2 \tag{30}$$

Solving (29) and (30) for gain $G(n)$, gives

$$G(n) = \sqrt{\frac{P_{e_s}(n-1)}{||\hat{\boldsymbol{s}}(n)||^2}} \tag{31}$$

where $P_{e_s}(n-1)$ is the power of the error signal of SPM filter (can be estimated using low pass estimator of type (19)). By combining (18) and (31), the overall time-varying gain $G(n)$ for simultaneous online FBPM and SPM is given as

$$G(n) = \alpha G(n-1) + \gamma \cdot \max\left\{\sqrt{\frac{P_{e_f}(n-1)}{||\hat{\boldsymbol{f}}(n)||^2}}, \sqrt{\frac{P_{e_s}(n-1)}{||\hat{\boldsymbol{s}}(n)||^2}}\right\} \tag{32}$$

where $\max\{\cdot\}$ is the maximum operator that will select the argument with maximum value. It is worth mentioning that the FBPM and SPM has no direct relationship, and hence, the max operator is employed to ensure the convergence of both the FBPM and the SPM filters.

6.2 Simulation results

Up to the best knowledge of authors, the other researchers have considered online FBPM [11–17] and SPM [18–25] as separate problems, and there is no work considering simultaneous adaptation of $\hat{F}(z)$ and $\hat{S}(z)$. Here we present a few typical results for the proposed method for simultaneous online FBPM and SMP, and a detailed treatment is beyond the scope of the present work. We repeat the experiment for a multi tonal noise source as considered in Case 1. The length of SPM filter and the step-size parameter, respectively, are selected as $L_s = 16$ and $\mu_s = 1 \times 10^{-3}$, and the rest of the simulation conditions and parameters are the same as in Case 1. As done for FBPM filter $\hat{F}(z)$, offline modelling with modelling accuracy of -5 dB is used to initialise the SPM filter $\hat{S}(z)$. Besides the performance measures as described in Section 3, the relative modelling error of the secondary path $S(z)$ is computed as

$$\Delta S(n)(\text{dB}) = 10\log_{10}\frac{||\boldsymbol{s}(n) - \hat{\boldsymbol{s}}(n)||^2}{||\boldsymbol{s}(n)||^2} \tag{33}$$

The simulation results in Figs. 9 and 10 show the performance

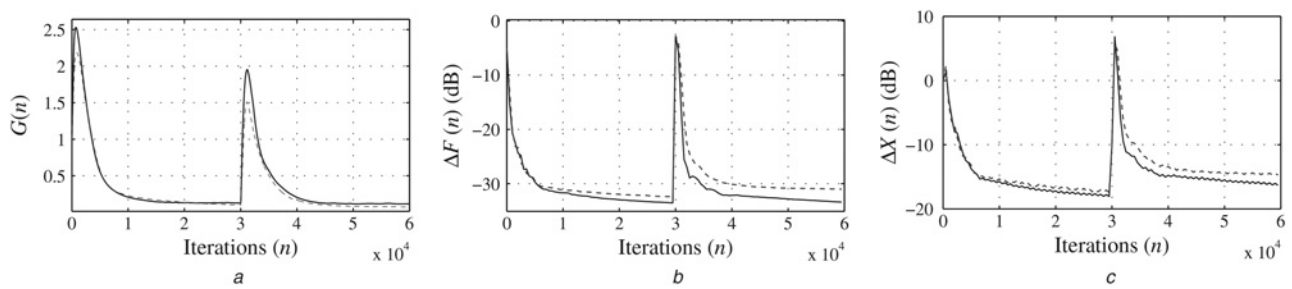

Fig. 9 *Simulation results for online FBPM (dotted line) and simultaneous online FBPM and SPM (solid line)*

a Time-varying gain, $G(n)$
b Relative modelling error, $\Delta F(n)$ (dB)
c MSE in the reference signal, $\Delta X(n)$ (dB)

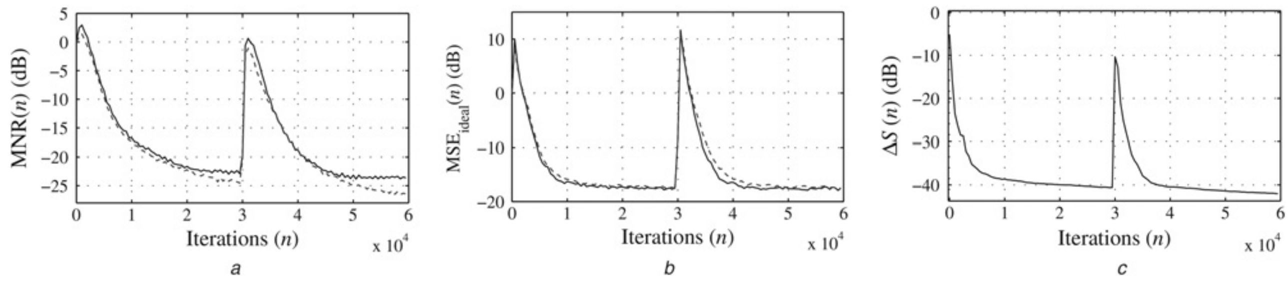

Fig. 10 *Simulation results for online FBPM (dotted line) and simultaneous online FBPM and SPM (solid line)*

a MNR at the error microphone, MNR(n)
b MSE at the error microphone without additive auxiliary noise, $\mathrm{MSE}_{\mathrm{ideal}}(n)$
c Relative modelling error, $\Delta S(n)$ (dB)

of the proposed method for online FBPM, and for simultaneous online FBPM and SPM, where we observe almost similar performance as in Case 1. Fig. 9*a* shows the plot (solid line) of the time-varying $G(n)$ computed using (32) for simultaneous online FBPM and SPM. The value of the $G(n)$ is large at the start-up of ANC system, or when there is a perturbation in the acoustic paths. This results in fast convergence of the FBPM filter and the SPM filter. In the steady-state, the gain $G(n)$ is reduced to a small value, and hence improves the noise-reduction performance. Fig. 10*c* shows the plot of relative modelling error for secondary path. In the proposed method the online SPM can results in $\Delta S(n)$ as low as –40 dB before and after the acoustic paths perturbation at $n = 30000$.

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