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Lossy to lossless image coding based on wavelets using a complex allpass filter

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Wavelet-based image coding has been adopted in the international standard JPEG 2000 for its efficiency. It is well-known that the orthogonality and symmetry of wavelets are two important properties for many applications of signal processing and image processing. Both can be simultaneously realized by the wavelet filter banks composed of a complex allpass filter, thus, it is expected to get a better coding performance than the conventional biorthogonal wavelets. This paper proposes an effective implementation of orthonormal symmetric wavelet filter banks composed of a complex allpass filter for lossy to lossless image compression. First, irreversible real-to-real wavelet transforms are realized by implementing a complex allpass filter for lossy image coding. Next, reversible integer-to-integer wavelet transforms are proposed by incorporating the rounding operation into the filtering processing to obtain an invertible complex allpass filter for lossless image coding. Finally, the coding performance of the proposed orthonormal symmetric wavelets is evaluated and compared with the D-9/7 and D-5/3 biorthogonal wavelets. It is shown from the experimental results that the proposed allpass-based orthonormal symmetric wavelets can achieve a better coding performance than the conventional D-9/7 and D-5/3 biorthogonal wavelets both in lossy and lossless coding.

Keywords: Orthonormal symmetric wavelet; complex allpass filter; lossy to lossless image coding; invertible implementation.

AMS Subject Classification: 22E46, 53C35, 57S20

1. Introduction

Wavelet-based image coding has been extensively studied and adopted in the international standard JPEG 2000.[8,18] In the wavelet-based image coding, two-band perfect reconstruction (PR) filter banks play a very important role. The analysis and synthesis filters are required to have exactly linear phase responses (corresponding to symmetric wavelet bases), allowing us to use the symmetric extension method to accurately handle the boundaries of images. The wavelet filter banks should also be orthonormal to avoid redundancy between the subband images. Therefore, the orthogonality and symmetry are two important properties of wavelets for many applications of signal processing and image processing. Unfortunately, it is widely appreciated that there are no nontrivial orthonormal symmetric wavelets using FIR filters, except for the Haar wavelet which is not continuous.[5] To achieve a better coding performance, a reasonable regularity is necessary for wavelet bases.[1] Therefore, at least one of the above-mentioned conditions has to be given up to get more regularity than the Haar wavelet. For example, the D-9/7 and D-5/3 wavelets supported by JPEG 2000 part 1 are biorthogonal, not orthogonal. Therefore, various classes of orthogonal wavelet filter banks with approximately linear phase responses and biorthogonal wavelet filter banks with exactly linear phase responses have been proposed by using FIR filters in Refs. 9, 2, 10, 15, 16, 17, 19, 20, 6 and 3 and IIR filters in Refs. 14, 7, 10, 12, 4 and 23. On the other hand, it is known that IIR wavelet filter banks can produce the orthonormal symmetric wavelet bases.[7] A class of IIR orthonormal symmetric wavelet filter banks has been proposed by using allpass filters in Refs. 7, 11, 13, 22 and 21. In Ref. 21, the proposed orthonormal symmetric wavelet filter banks are composed of a single complex allpass filter. In this paper, we will apply the orthonormal symmetric wavelet filter banks composed of a complex allpass filter to image compression. Since the filter coefficients are real-valued, irreversible real-to-real wavelet transform can be easily realized, and used only for lossy image coding. However, a reversible integer-to-integer wavelet transform is necessary for lossless image coding. As shown in Refs. 16, 17, 6 and 3, the rounding operation must be incorporated into the processing in order to realize reversible integer-to-integer wavelet transform.

In this paper, we discuss how to realize the orthonormal symmetric wavelet filter banks composed of a single complex allpass filter for lossy to lossless image coding. First, we propose an effective realization of irreversible real-to-real wavelet transforms for lossy image coding by efficiently implementing the complex allpass filter. Next, by incorporating the rounding operation into the filtering processing, we give an invertible implementation of the complex allpass filter to realize the reversible integer-to-integer wavelet transforms for lossless image coding. Finally, we apply the proposed orthonormal symmetric wavelet filter banks into image compression, and investigate its coding performance by using JPEG 2000 reference software (part 5) JJ2000 (Java) version 5.1 provided in Ref. 8. The coding results are compared with the D-9/7 and D-5/3 biorthogonal wavelets supported by JPEG 2000 part 1. It can be seen from the experimental results that the proposed allpass-based orthonormal symmetric wavelet filters can achieve a better lossy to lossless coding performance than the conventional D-9/7 and D-5/3 biorthogonal wavelets.

This paper is organized as follows. The orthonormal symmetric wavelet filter banks composed of a single complex allpass filter are briefly introduced in Sec. 2. In Sec. 3, an effective implementation of the orthonormal symmetric wavelet filter banks is given to obtain irreversible real-to-real wavelet transforms for lossy image coding. In Sec. 4, the reversible integer-to-integer wavelet transforms are proposed by realizing an invertible complex allpass filter for lossless image coding. In Sec. 5, the evaluation and comparison of the coding performance between the proposed allpass-based orthonormal symmetric wavelet filter banks and the conventional D-9/7 and D-5/3 biorthogonal wavelets are shown. Finally, Sec. 6 contains a conclusion.

2. Orthonormal Symmetric Wavelet Filter Banks

It is well-known in Ref. 5 that wavelet bases can be generated by two-band PR filter banks $H(z)$ and $G(z)$, where $H(z)$ and $G(z)$ are the transfer functions of lowpass and highpass filters, respectively, which are defined by the z transform of filter impulse responses. The orthonormal filter banks $H(z)$ and $G(z)$ must satisfy

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 2, \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 2, \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0. \end{cases} \tag{2.1}$$

Moreover, $H(z)$ and $G(z)$ must have exactly linear phase responses also, if symmetric wavelets are needed. In the case of FIR filters, only orthonormal symmetric wavelet filter is the Haar wavelet. In Ref. 21, a class of IIR orthonormal symmetric wavelet filter banks have been proposed by using a single complex allpass filter, as shown in Fig. 1, i.e.

$$\begin{cases} H(z) = \dfrac{1}{\sqrt{2}}\{A(z) + A^{\dagger}(z)\}, \\ G(z) = \dfrac{z^{-1}}{\sqrt{2}j}\{A(z) - A^{\dagger}(z)\}, \end{cases} \tag{2.2}$$

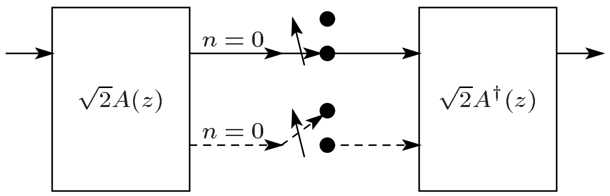

Fig. 1. Wavelet filter banks composed of a single complex allpass filter.

where $A(z)$ is a complex allpass filter of order $2N$ and defined by

$$A(z) = e^{j\eta} \frac{a_0 z^N + ja_1 z^{N-1} + \cdots + ja_1 z^{1-N} + a_0 z^{-N}}{a_0 z^N - ja_1 z^{N-1} + \cdots - ja_1 z^{1-N} + a_0 z^{-N}}, \tag{2.3}$$

where $j^2 = -1, \eta = \pm\frac{\pi}{4}$ or $\pm\frac{3\pi}{4}, a_n$ are a set of real-valued filter coefficients and $a_0 = 1$. Moreover, $A^\dagger(z)$ has a set of coefficients that are complex conjugate with ones of $A(z)$. It has been proven in Ref. 21 that $H(z)$ and $G(z)$ in Eq. (2.2) satisfy the condition of orthonormality in Eq. (2.1) and have exactly linear phase responses.

In the following, we first derive the frequency responses of $H(z)$ and $G(z)$. Let $\theta(\omega)$ be the phase response of $A(z)$, we have from Eq. (2.3),

$$\theta(\omega) = \eta + 2\varphi(\omega), \tag{2.4}$$

where if N is even,

$$\varphi(\omega) = \tan^{-1} \frac{\sum_{n=0}^{N/2-1} a_{2n+1} \cos(N-2n-1)\omega}{\frac{a_N}{2} + \sum_{n=0}^{N/2-1} a_{2n} \cos(N-2n)\omega} \tag{2.5}$$

and if N is odd,

$$\varphi(\omega) = \tan^{-1} \frac{\frac{a_N}{2} + \sum_{n=1}^{(N-1)/2} a_{2n-1} \cos(N-2n+1)\omega}{\sum_{n=0}^{(N-1)/2} a_{2n} \cos(N-2n)\omega}. \tag{2.6}$$

The frequency responses of $H(z)$ and $G(z)$ are then given by

$$\begin{cases} H(e^{j\omega}) = \sqrt{2}\cos\theta(\omega), \\ G(e^{j\omega}) = e^{-j\omega}\sqrt{2}\sin\theta(\omega), \end{cases} \tag{2.7}$$

which have exactly linear phase responses and satisfy the following power-complementary relation;

$$|H(e^{j\omega})|^2 + |G(e^{j\omega})|^2 = 2. \tag{2.8}$$

Therefore, the design problem of $H(z)$ and $G(z)$ becomes the phase approximation of the complex allpass filter $A(z)$, and has been discussed in Ref. 21. In Ref. 21, the closed-form solution for this class of orthonormal symmetric wavelet filter banks having the maximally flat magnitude responses has been given by

$$a_n = \begin{cases} \binom{2N}{n} & (n : \text{even}), \\ -\binom{2N}{n} \tan\frac{\eta}{2} & (n : \text{odd}). \end{cases} \tag{2.9}$$

As a design example, the magnitude responses of the maximally flat filters $H(z)$ and $G(z)$ with $N = 1, 2, 3, 4$ are shown in Fig. 2, and the scaling and wavelet functions

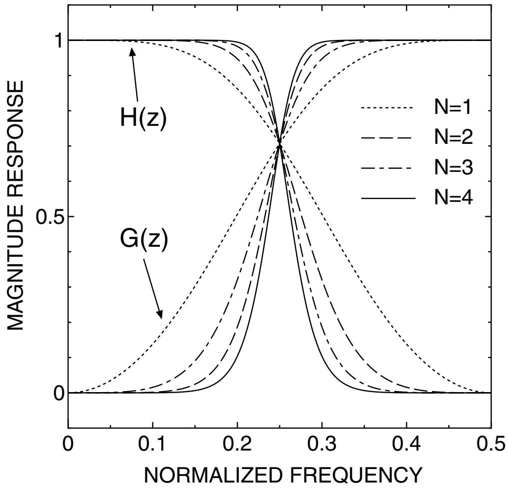

Fig. 2. Magnitude responses of the maximally flat filters $H(z)$ and $G(z)$.

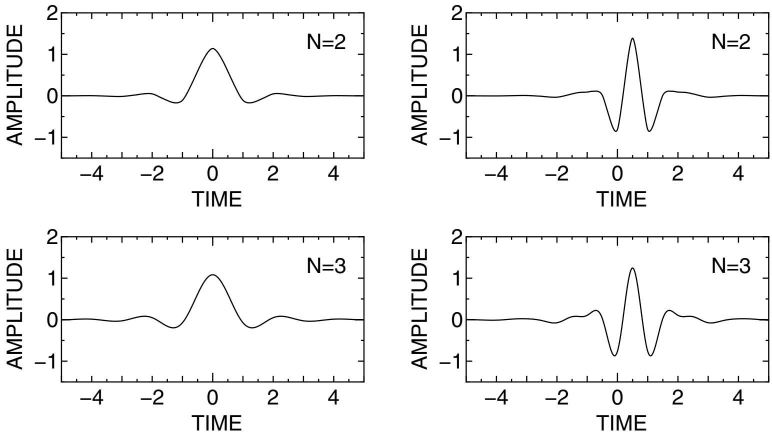

Fig. 3. Scaling and wavelet functions with $N = 2, 3$.

generated from $H(z)$ and $G(z)$ with $N = 2, 3$ are given in Fig. 3. Moreover, the design method of the orthonormal symmetric wavelet filter banks with equiripple magnitude responses has also been proposed in Ref. 21 by using the Remez exchange algorithm. See Ref. 21 in detail.

3. Irreversible Wavelet Transforms

In this section, we present an effective realization of irreversible real-to-real wavelet transforms by using a complex allpass filter. We assume that input signal is real-valued and of length M, where M is even. Since $H(z)$ and $G(z)$ have exactly linear phase responses, the input signal is extended to a periodic signal by employing symmetric extension at the boundaries. It is seen in Fig. 1 that the real-valued input signal will be filtered with the complex allpass filter $A(z)$, then the lowpass and highpass components are obtained as real- and imaginary-parts of the output signal. Since the filter coefficients are symmetric, i.e. $a_n = a_{2N-n}$, $A(z)$ can be divided into the causal-stable subfilter $A_S(z)$ and its inverse $A_S(z^{-1})$ as follows;

$$\sqrt{2}A(z) = (1 \pm j)A_S(z)A_S(z^{-1}). \tag{3.1}$$

Note that $\eta = \pm\frac{\pi}{4}$ or $\pm\frac{3\pi}{4}$, then $\sqrt{2}e^{j\eta} = 1 \pm j$. All poles of $A_S(z)$ are inside the unit circle, and its transfer function is given by

$$A_S(z) = \frac{z^{-N} + \sum_{k=1}^{N} j^k \gamma_k z^{k-N}}{1 + \sum_{k=1}^{N} j^{-k} \gamma_k z^{-k}}, \tag{3.2}$$

where γ_k are real. $A_S(z^{-1})$ is realized by reversing its input signal, filtering it with the causal-stable filter $A_S(z)$, and then re-reversing the output signal. Therefore, $A(z)$ can be realized only by using $A_S(z)$ twice, as shown in Fig. 4.

For the causal-stable filter $A_S(z)$ with input $p(n)$ and output $q(n)$, its input-output relation is given by

$$q(n) = p(n-N) + \sum_{k=1}^{N} j^k \gamma_k [\,p(n+k-N) - (-1)^k q(n-k)]. \tag{3.3}$$

It is seen in Fig. 1 that only even-indexed real-part and odd-indexed imaginary-part of output of $A(z)$ are necessary in analysis filters. Thus, we need not to know all of the output. Let $p(n) = p_r(n) + jp_i(n)$ and $q(n) = q_r(n) + jq_i(n)$, where the indices r and i mean the real and imaginary parts, respectively. From Eq. (3.3), the even-indexed real-part $q_r(2n)$ and odd-indexed imaginary-part $q_i(2n+1)$ of the output can be obtained by

$$\begin{aligned} q_r(2n) = p_r(2n-N) &+ \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k}[p_r(2n+2k-N) - q_r(2n-2k)] \\ &+ \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1}[p_i(2n+2k-N-1) + q_i(2n-2k+1)], \end{aligned} \tag{3.4}$$

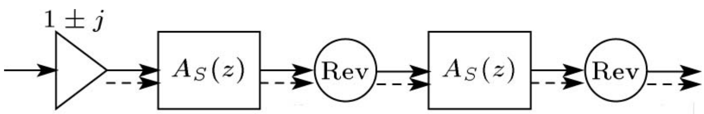

Fig. 4. Implementation of complex allpass filter $A(z)$.

$$q_i(2n+1) = p_i(2n-N+1) + \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k}[p_i(2n+2k-N+1) - q_i(2n-2k+1)] - \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1}[p_r(2n+2k-N) + q_r(2n-2k+2)], \tag{3.5}$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. It can be seen in Eqs. (3.4) and (3.5) that for getting $q_r(2n)$ and $q_i(2n+1)$, only the even-indexed real-part $p_r(2n)$ and odd-indexed imaginary-part $p_i(2n+1)$ of input signal $p(n)$ are necessary if N is even, while the odd-indexed real-part $p_r(2n+1)$ and even-indexed imaginary-part $p_i(2n)$ if N is odd. For example, when $N = 2$, we have

$$q_r(2n) = p_r(2n-2) - \gamma_2[p_r(2n) - q_r(2n-2)] - \gamma_1[p_i(2n-1) + q_i(2n-1)], \tag{3.6}$$

$$q_i(2n+1) = p_i(2n-1) - \gamma_2[p_i(2n+1) - q_i(2n-1)] + \gamma_1[p_r(2n) + q_r(2n)]. \tag{3.7}$$

Thus this second-order complex allpass filter $A_S(z)$ can be realized as shown in Fig. 5. This means that the down-sampling operation can be done before implementing $A_S(z)$. Therefore, the computational complexity can be reduced by firstly decimating the input signal and then filtering it. It is also noted in Eqs. (3.4) and (3.5) that only N real multipliers are needed per output sample.

Since the input signal is periodic as using the symmetric extension, some initial values are needed for starting the processing in Eqs. (3.4) and (3.5). For example, to get $q_r(0)$ we need to know $q_i(-1), q_r(-2), q_i(-3), \ldots, q_{r(i)}(-N)$ beforehand. It is known that although IIR filters have infinite impulse responses in theory, the impulse responses of the stable filters will become very small beyond

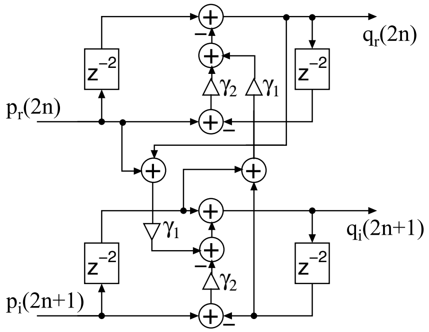

Fig. 5. Implementation of second-order complex allpass filter $A_S(z)$.

a certain interval in practice. Therefore, truncating the impulse response to be of finite length will give little influence. We can calculate the initial values $q_i(-1), q_r(-2), q_i(-3), \ldots, q_{r(i)}(-N)$ by using the truncated impulse response. This is equivalent to truncating the input signal before the given point. We use the latter to obtain the initial values $q_i(-1), q_r(-2), q_i(-3), \ldots, q_{r(i)}(-N)$ in this paper. The synthesis filters can be also realized in the same way, and then is omitted here.

4. Reversible Wavelet Transform

In this section, we discuss reversible integer-to-integer wavelet transforms for lossless coding. In most of the cases, $A_S(z)$ has floating point filter coefficients. Although the input images are matrices of integer values, the filtered output no longer consists of integers. For lossless compression, it is necessary to make an invertible mapping from an integer input to an integer wavelet coefficient. To obtain an integer output, we incorporate the rounding operation into Eqs. (3.4) and (3.5) as follows;

$$
\begin{aligned}
q_r(2n) = p_r(2n-N) + \Bigg\lfloor &\sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k} [p_r(2n+2k-N) \\
&- q_r(2n-2k)] + \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1} [p_i(2n+2k-N-1) \\
&+ q_i(2n-2k+1)] + 0.5 \Bigg\rfloor,
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
q_i(2n+1) = p_i(2n-N+1) + \Bigg\lfloor &\sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k} [p_i(2n+2k-N+1) \\
&- q_i(2n-2k+1)] - \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1} [p_r(2n+2k-N) \\
&+ q_r(2n-2k+2)] + 0.5 \Bigg\rfloor.
\end{aligned} \tag{4.2}
$$

Therefore, we can obtain the integer output $q_r(2n)$ and $q_i(2n+1)$ for $n = 0, 1, \ldots, M/2-1$ from the integer input $p(n)$ by using Eqs. (4.1) and (4.2), starting from the initial values described in Sec. 3.

To recover the integer $p(n)$ from the integer $q(n)$, we revise Eqs. (4.1) and (4.2) and have

$$\begin{aligned} p_r(2n-N) = q_r(2n) - \Bigg\lfloor \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k}[p_r(2n+2k-N) - q_r(2n-2k)] \\ + \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1}[p_i(2n+2k-N-1) \\ + q_i(2n-2k+1)] + 0.5 \Bigg\rfloor, \end{aligned} \tag{4.3}$$

$$\begin{aligned} p_i(2n-N+1) = q_i(2n+1) - \Bigg\lfloor \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} (-1)^k \gamma_{2k}[p_i(2n+2k-N+1) \\ - q_i(2n-2k+1)] - \sum_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} (-1)^k \gamma_{2k-1}[p_r(2n+2k-N) \\ + q_r(2n-2k+2)] + 0.5 \Bigg\rfloor. \end{aligned} \tag{4.4}$$

It is clear that if all of $q_r(2n), q_i(2n+1)$ and some of $p_r(2n-N), p_i(2n-N+1)$, e.g., $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$ for even N, are known *a priori*, we can exactly reconstruct the integer $p_r(2n-N)$ and $p_i(2n-N+1)$ for $n = M/2 - 1, M/2-2, \ldots, N/2$.

For example, an invertible implementation of complex allpass filter $A_S(z)$ with $N = 2$ is given in Fig. 6, where the analysis processing is in Fig. 6(a), and synthesis

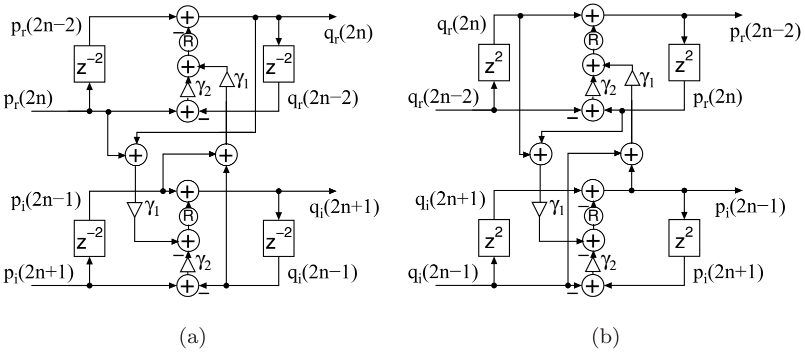

Fig. 6. Invertible implementation of second-order complex allpass filter, (a) analysis and (b) synthesis.

in Fig. 6(b). It is seen in Fig. 6 that the rounding operation Ⓡ is inserted after the multipliers, thus reversible integer-to-integer transforms can be realized. Note that the filtering is done in reverse order for analysis and synthesis.

It is known in lossless coding that $q_r(2n)$ and $q_i(2n+1)$ are transmitted to the decoder without loss. However, it is difficult to perfectly recover the integer $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$ from $q_r(2n)$ and $q_i(2n+1)$ only. This is because the rounding error has occurred in Eqs. (4.1) and (4.2) and has been included in $q(n)$. Thus, the information $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$ must be transmitted to the decoder as a side information. By using the side information transmitted, we can realize the reversible integer-to-integer wavelet transforms with Eqs. (4.1)–(4.4). The side information includes N pixels for each column or line of the image. Since the complex allpass filter of lower-order is used in image coding, N is small in general. Moreover, we can calculate the prediction values of $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$ from $q_r(2n)$ and $q_i(2n+1)$ in the same way as calculating the initial values in Sec. 3, and then transmit the difference between the prediction and actual values to the decoder, instead of the actual values of $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$. The difference of both is relatively small, since it is due to the rounding error in Eqs. (4.1) and (4.2). In the practical experiment, we found that this difference has an entropy of about 1–2 bpp. Therefore, the amount of the side information needed to be transmitted is very small. For example, when $N = 2$, the side information of only 2–4 bits are needed for $A_S(z)$ to filter one line or column of the image. In the decoder, we calculate the prediction values of $p_i(M-1), p_r(M-2), \ldots, p_r(M-N)$ from $q_r(2n)$ and $q_i(2n+1)$ in the same way as in the encoder, and add the difference transmitted to it to get the actual values. Therefore, the original $p(n)$ can be recovered by using Eqs. (4.3) and (4.4).

5. Image Coding Application

In this section, we investigate the coding performance of the proposed orthonormal symmetric wavelets composed of a complex allpass filter with the maximally flat magnitude responses. The maximally flat wavelet filters produce the maximum numbers of vanishing moments of wavelets,[5] which potentially influence the coding performance.[1] The filter order is chosen to $N = 1$–4. The filter coefficients of the complex allpass filter with $N = 1$–4 given in Ref. 21 have been used. JPEG 2000 reference software (part 5) JJ2000 (Java) version 5.1 provided in Ref. 8 has been used to evaluate the coding performance. Eight images (Barbara, Boat, Goldhill, Lena, Man, Mandrill, Pepper and Zelda) of size 512×512, 8 bpp have been used as test images, and the decomposition level of the wavelet transform is set to 6.

5.1. *Irreversible wavelet transform*

We examine the lossy coding performance of the irreversible real-to-real wavelet transform proposed in Sec. 3, and compare the coding performance with the D-9/7 irreversible real-to-real wavelet transform supported by JPEG 2000 part 1.

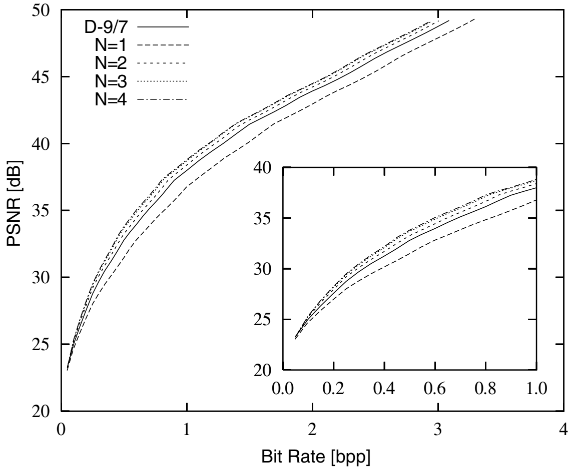

Fig. 7. Lossy coding performance of irreversible wavelet transform for image Barbara.

The distortion is measured by the peak signal to noise ratio (PSNR) between the original image and lossy image reconstructed at the given bit rate, where bit rate means the average number of bits per pixel (bpp). The lossy coding results for images Barbara and Lena are given in Figs. 7 and 8, respectively. It is seen in Fig. 7 that when $N > 1$, the orthonormal symmetric wavelets composed of a complex allpass filter have a better lossy coding performance than the D-9/7 wavelet for image Barbara, while the almost same results are obtained for image Lena, as shown in Fig. 8. For example, at 0.5 bpp for image Barbara, the proposed wavelet filter with $N = 3$ has the PSNR of 33.671 dB, while D-9/7 is 32.765 dB. When N is

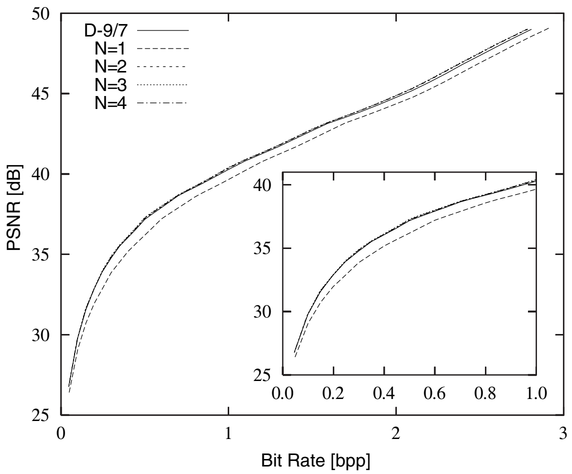

Fig. 8. Lossy coding performance of irreversible wavelet transform for image Lena.

further increased, we get little improvement, however, the computational complexity becomes higher.

5.2. *Reversible wavelet transform*

We investigate the lossy and lossless coding performance of the reversible integer-to-integer wavelet transform proposed in Sec. 4, and compare the coding performance with the D-5/3 reversible integer-to-integer wavelet transform supported by JPEG 2000 part 1.

5.2.1. *Lossless coding performance*

We have investigated the lossless coding performance for eight test images. The lossless coding results with the comparison with the D-5/3 wavelet are given in Table 1. For each image, the best result has been highlighted. It is seen in Table 1 that the orthonormal symmetric wavelets composed of a complex allpass filter with $N = 3$ have the best average lossless coding performance. Although there are two images getting the best results for the D-5/3 wavelet, the proposed orthonormal symmetric wavelets with $N = 2$ and $N = 3$ have obtained the best lossless coding performance for three images, respectively.

5.2.2. *Lossy coding performance*

We examine the lossy coding performance of the reversible integer-to-integer wavelet transform. The side information was not used in lossy coding. The lossy coding results for images Barbara and Lena are shown in Figs. 9 and 10, respectively. It is seen in Figs. 9 and 10 that at a lower bit rate, the proposed orthonormal symmetric wavelets with $N > 1$ have a better lossy coding performance than the D-5/3 wavelet, while when $N = 1$, the results are almost same. However, the orthonormal symmetric wavelets have a poor lossy coding performance at a higher bit rate. This is because the rounding error has a relatively larger influence on the lossy coding performance than the quantization error at a higher bit rate.

Table 1. Lossless coding results: Bit Rate (bpp).

Image	D-5/3	$N = 1$	$N = 2$	$N = 3$	$N = 4$
Barbara	4.695	4.758	4.545	4.503	**4.501**
Boat	4.438	4.527	**4.431**	4.434	4.451
Goldhill	**4.871**	4.942	4.886	4.886	4.894
Lena	4.348	4.440	**4.335**	**4.335**	4.346
Man	**4.730**	4.840	4.740	4.750	4.770
Mandrill	6.149	6.204	6.127	**6.121**	6.122
Pepper	4.653	4.705	**4.649**	4.663	4.679
Zelda	4.019	4.090	3.972	**3.965**	3.975
Average	4.738	4.813	4.711	**4.707**	4.717

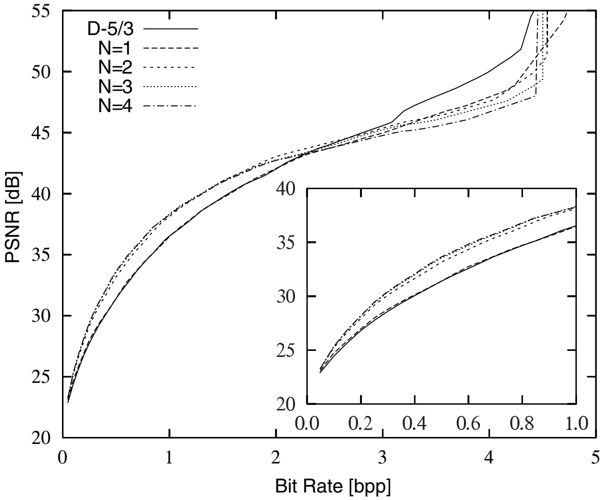

Fig. 9. Lossy coding performance of reversible wavelet transform for image Barbara.

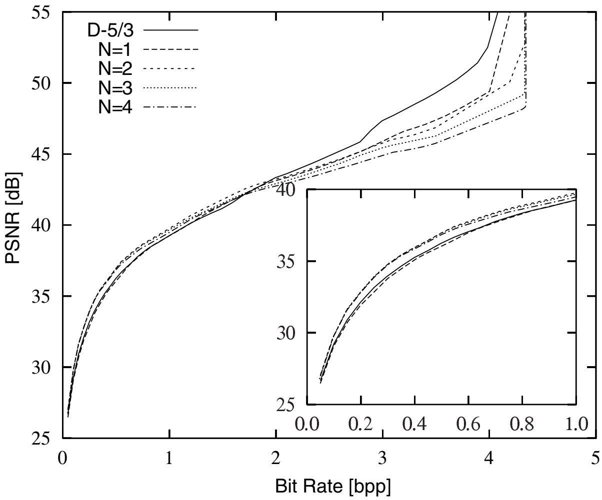

Fig. 10. Lossy coding performance of reversible wavelet transform for image Lena.

To improve the lossy coding performance at a higher bit rate, we use real-to-real inverse transform at the decoder, instead of integer-to-integer inverse transform in Eqs. (4.3) and (4.4). That is, the rounding operation is removed from Eqs. (4.3) and (4.4). The difference of lossy coding performance between the real-to-real and integer-to-integer inverse transforms are given in Figs. 11 and 12, respectively. It is seen in Figs. 11 and 12 that unlike the D-5/3 wavelet, the proposed orthonormal symmetric wavelets can obtain an improvement up to 1.5 dB.

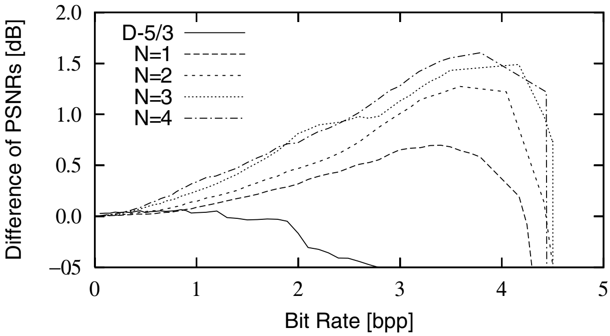

Fig. 11. Lossy coding performance improvement for image Barbara.

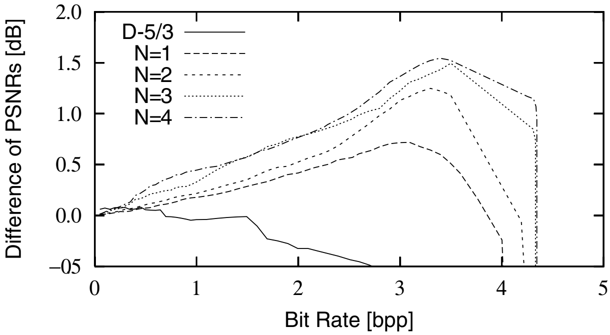

Fig. 12. Lossy coding performance improvement for image Lena.

6. Conclusions

In this paper, we have proposed an effective realization of orthonormal symmetric wavelets composed of a single complex allpass filter for lossy to lossless image coding. First, we have realized the irreversible real-to-real wavelet transform by efficiently implementing the complex allpass filter. Then, we have presented the invertible implementation of the complex allpass filter by incorporating the rounding operation into the filtering processing to realize the reversible integer-to-integer wavelet transform. Finally, we have investigated the coding performance of the proposed allpass-based orthonormal symmetric wavelets by using JPEG 2000 reference software (part 5) JJ2000 (Java) version 5.1, and compared the lossy and lossless coding performance with the D-9/7 and D-5/3 wavelets. It have been shown from the experimental results that the proposed allpass-based orthonormal symmetric wavelets can achieve a better lossy to lossless coding performance than the conventional D-9/7 and D-5/3 wavelets.

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