Design of two-channel IIR linear phase PR filter banks

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Abstract

In this paper, a novel method is proposed for designing two-channel biorthogonal perfect reconstruction filter banks with exact linear phase using noncausal IIR filters. Since the structurally perfect reconstruction implementation is adopted, the proposed filter banks are guaranteed to be a perfect reconstruction even when all filter coefficients are quantized. From the view point of wavelets, design of biorthogonal IIR linear phase filter banks with an additional flatness constraint is considered. The proposed design method is based on the formulation of a generalized eigenvalue problem by using Remez exchange algorithm. Hence, the filter coefficients can be obtained by solving the eigenvalue problem to compute the positive minimum eigenvalue, and the optimal solution in the Chebyshev sense is easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified. Some design examples are presented to demonstrate the effectiveness of the proposed method. © 1999 Published by Elsevier Science B.V. All rights reserved.

Zusammenfassung

In diesem Beitrag wird eine neue Methode zum Entwurf zweikanaliger biorthogonaler Rekonstruktionsfilterbänke mit exakt linearer Phase durch Verwendung nichtkausaler IIR-Filter vorgestellt. Da strukturell perfekte Rekonstruktionsimplementierung angenommen wurde, sind die vorgestellten Filterbänke garantiert perfekte Rekonstruktionen, selbst wenn alle Filterkoeffizienten quantisiert sind. Ausgehend von Wavelets wird der Entwurf biorthogonaler IIR Filterbänke linearer Phase mit zusätzlicher Glattheitsforderung betrachtet. Die vorgeschlagene Entwurfsmethode basiert auf der Formulierung eines verallgemeinerten Eigenwertproblems durch Ausnutzung des Remez-Algorithmus. Die filterkoeffizienten können durch Lösung des Eigenwertproblems und Berechnung des kleinsten positiven Eigenwertes erreicht werden. Die im Tschebyschew-Sinne optimale Lösung kann leicht durch einige Iterationen erreicht werden. Die vorgestellte Prozedur ist numerisch effizient und die Glattheitsbedingung kann beliebig definiert werden. Einige Entwurfsbeispiele werden zur Demonstration der Effektivität der vorgestellten Methode präsentiert. © 1999 Published by Elsevier Science B.V. All rights reserved.

Résumé

Dans cet article, nous proposons une nouvelle méthode de conception de bancs de filtres à reconstruction parfaite, à phase linéaire exacte, biorthogonaux à deux canaux, en utilisant des filtres IIR non-causaux. Comme nous adoptons l'implémentation à reconstruction structurellement parfaite, il est garanti que les bancs de filtres proposés sont à reconstruction parfaite même quand tous les coefficients des filtres sont quantifiés. Du point de vue des ondelettes, nous

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considérons la conception de bancs de filtres IIR biorthogonaux à phase linéaire avec une contrainte supplémentaire de platitude. La méthode de conception proposée repose sur la formulation d'un problème aux valeurs propres généralisé, en utilisant un algorithme d'échange de Remez. Donc, les coefficients des filtres peuvent être obtenus par la résolution du problème des valeurs propres pout calculer la valeur propre positive minimale, et la solution optimale au sens de Chebyshev est obtenue aisément par quelques itérations. La procédure proposée est efficace en termes de temps de calcul, et la contrainte de platitude peut être spécifiée de façon arbitraire. Quelques exemples de conception sont présentés pour démontrer l'efficacité de la méthode proposée. © 1999 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Two-channel perfect reconstruction (PR) filter banks have been used in different applications of signal processing, such as subband coding of speech and image signals, transmultiplexers, and voice privacy systems [1–17]. The theory and design of FIR PR filter banks have been well established in recent years [2–8,10–17]. The PR filter banks include two cases: orthonormal and biorthogonal. For orthonormal case, the FIR PR filter banks, except Haar function, cannot possess exact linear phase responses that are desired in some applications of image signal processing. Thus, biorthogonal PR filter banks are proposed to obtain exact linear phase responses. Design of biorthogonal FIR linear phase PR filter banks has been also discussed in the previous works [2,3,5,7]. However, compared with IIR filters, FIR filters generally require higher-order filters for meeting the same magnitude specification. Hence, using IIR filters will result in computational savings [12]. In this paper, we will consider the design of biorthogonal PR filter banks with exact linear phase responses using IIR filters. Although causal IIR filters can possess only approximately linear phase responses, we can obtain an exact linear phase by using noncausal IIR filters, which can be decomposed into causal and anticausal parts to implement, and then anticausal part can be realized by using time reversal for finite length inputs. For example, in the subband image coding systems, the generalized circular convolution and symmetric extension methods can be used to treat the boundaries of images [12]. In most designs, the PR property of filter banks cannot be preserved generally when the filter coefficients are quantized. To force the PR condition to be satisfied

regardless of what the filter coefficients are, the structurally PR implementation will be required. In [7], an efficient structurally PR implementation has been proposed, where for the FIR case, exact linear phase filters are used, and for the IIR case, causal allpass filters are used.

In this paper, we propose a new method for designing two-channel biorthogonal linear phase PR filter banks by using noncausal IIR filters. We adopt the structurally PR implementation proposed in [7], thus the obtained IIR filter banks still satisfy PR condition even when all filter coefficients are quantized. It is well known [1,6,7,9–11,17] that wavelet bases can be generated from PR filter banks. Then, synthesis of wavelet bases has been reduced to design PR filter banks. From the regularity condition of wavelets, a flatness constraint is required to impose on the PR filter banks. In this paper, from the view point of wavelets, we consider the design of IIR linear phase PR filter banks with an additional flatness constraint. By using Remez exchange algorithm, we formulate the design problem in the form of a generalized eigenvalue problem [18,19]. Thus, by solving the eigenvalue problem to compute the positive minimum eigenvalue, we can get a set of filter coefficients as the corresponding eigenvector. Therefore, the optimal filter coefficients with an equiripple response can be easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified.

This paper is organized as follows. Section 2 describes an efficient structurally PR implementation. Section 3 presents a design method of IIR linear phase PR filter banks based on eigenvalue problem by using Remez exchange algorithm. Section 4 shows two design examples to demonstrate

the effectiveness of the proposed method. Conclusions are given in Section 5.

2. Structurally PR filter banks

In two-channel filter banks shown in Fig. 1, assume that $H_0(z)$ and $H_1(z)$ are analysis filters, and $G_0(z)$ and $G_1(z)$ are synthesis filters. It is well known that the relationship of input $X(z)$ and output $Y(z)$ of the filter banks is

$$Y(z) = \tfrac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\}X(z)$$
$$+ \tfrac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\}X(-z). \tag{1}$$

Then the perfect reconstruction condition is

$$H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K-1},$$
$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0, \tag{2}$$

where K is integer. By using the polyphase matrix description,

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{bmatrix}\begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}$$
$$= \boldsymbol{H}(z^2)\begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} G_{00}(z^2) & G_{01}(z^2) \\ G_{10}(z^2) & G_{11}(z^2) \end{bmatrix}$$
$$= \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^{\mathrm{T}}\boldsymbol{G}(z^2), \tag{4}$$

the perfect reconstruction condition of Eq. (2) becomes

$$\boldsymbol{G}(z)\boldsymbol{H}(z) = \frac{z^{-K}}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{z^{-K}}{2}\boldsymbol{I}, \tag{5}$$
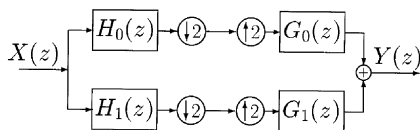
where $\boldsymbol{I}$ is the identity matrix. It is well known that

$$\begin{bmatrix} z^{-N} & 0 \\ -A(z) & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ A(z) & z^{-N} \end{bmatrix} = z^{-N}\boldsymbol{I}, \tag{6}$$

$$\begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix}\begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix} = z^{-M}\boldsymbol{I}, \tag{7}$$

where N and M are integers, and $A(z)$ and $B(z)$ are arbitrary transfer functions. If we constitute $\boldsymbol{H}(z)$ and $\boldsymbol{G}(z)$ as

$$\boldsymbol{H}(z) = \begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \tfrac{1}{2}A(z) & \tfrac{1}{2}z^{-N} \end{bmatrix}, \tag{8}$$

$$\boldsymbol{G}(z) = \begin{bmatrix} \tfrac{1}{2}z^{-N} & 0 \\ -\tfrac{1}{2}A(z) & 1 \end{bmatrix}\begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix}, \tag{9}$$

then the perfect reconstruction condition of Eq. (5) is satisfied regardless of what $A(z)$ and $B(z)$ are, i.e., the PR filter banks can be still obtained even when the filter coefficients of $A(z)$ and $B(z)$ are quantized. Note that $K = N + M$. The structurally PR implementation proposed in [7] is shown in Fig. 2. Hence, the design problem of the filter bank will become design of the analysis or synthesis filters. From Eqs. (8) and (9), the transfer functions of analysis and synthesis filters are

$$H_0(z) = z^{-2M} - B(z^2)H_1(z),$$
$$H_1(z) = \tfrac{1}{2}\{z^{-2N-1} + A(z^2)\} \tag{10}$$

and

$$G_0(z) = \tfrac{1}{2}\{z^{-2N-1} - A(z^2)\} = -H_1(-z),$$
$$G_1(z) = z^{-2M} + B(z^2)G_0(z) = H_0(-z), \tag{11}$$

respectively. In the following, we will consider the design of the analysis filters $H_0(z)$ and $H_1(z)$.
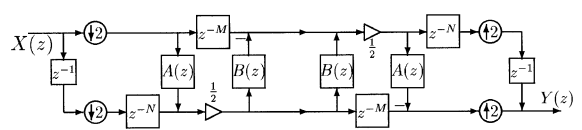

Fig. 1. Two-channel filter bank.

Fig. 2. Structurally perfect reconstruction implementation.

3. Design of IIR linear phase PR filter banks

In this section, we describe design of IIR linear phase PR filter banks based on eigenvalue problem by using Remez exchange algorithm. Here, we use general IIR filters $A(z)$ and $B(z)$, that is,

$$A(z) = \frac{\sum_{i=0}^{L_1} a_i z^{-i}}{\sum_{i=0}^{L_2} b_i z^{-i}}, \qquad (12)$$

$$B(z) = \frac{\sum_{i=0}^{L_3} c_i z^{-i}}{\sum_{i=0}^{L_4} d_i z^{-i}}, \qquad (13)$$

where L_1, L_2, L_3, L_4 are integers, a_i, b_i, c_i, d_i are real coefficients, and $b_0 = d_0 = 1$.

3.1. Desired magnitude responses

From Eq. (10), we have

$$
\begin{aligned}
H_1(z) &= \tfrac{1}{2} z^{-2N-1}\{1 + z^{2N+1} A(z^2)\} \\
&= \tfrac{1}{2} z^{-2N-1}\{1 + \hat{A}(z^2)\},
\end{aligned} \qquad (14)
$$

where

$$\hat{A}(z) = z^{N+1/2} A(z) = z^{N+1/2} \frac{\sum_{i=0}^{L_1} a_i z^{-i}}{\sum_{i=0}^{L_2} b_i z^{-i}}. \qquad (15)$$

To obtain the exact linear phase, the filter coefficients of $A(z)$ must be symmetric, that is,

$$
\begin{aligned}
a_i &= a_{L_1-i}, \quad i = 0,1,\ldots,L_1, \\
b_i &= b_{L_2-i}, \quad i = 0,1,\ldots,L_2.
\end{aligned} \qquad (16)
$$

When L_2 is odd, $A(z)$ will have a pole at $z = -1$, i.e., $\sum_{i=0}^{L_2}(-1)^i b_i = 0$. Hence, L_2 must be even. To force $\hat{A}(z)$ to be zero phase, we have $N + \tfrac{1}{2} - \tfrac{1}{2}L_1 + \tfrac{1}{2}L_2 = 0$, and L_1 and L_2 must satisfy

$$
\begin{aligned}
L_1 &= 2I_1 + 1, \\
L_2 &= 2I_2, \\
L_1 - L_2 &= 2N + 1,
\end{aligned} \qquad (17)
$$

where I_1 and I_2 are integers. Then, $\hat{A}(z)$ becomes zero phase, and its frequency response is given by

$$\hat{A}(e^{j\omega}) = \frac{\sum_{i=0}^{I_1} a_i \cos(I_1 - i + \tfrac{1}{2})\omega}{\tfrac{1}{2}b_{I_2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega}. \qquad (18)$$

Therefore, $H_1(z)$ has an exact linear phase, and its magnitude response is

$$|H_1(e^{j\omega})| = \tfrac{1}{2}\{1 + \hat{A}(e^{j2\omega})\}. \qquad (19)$$

To force $H_1(z)$ to be lowpass, the desired magnitude response of $\hat{A}(z^2)$ is

$$
\begin{aligned}
\hat{A}_d(e^{j2\omega}) &= 1, \quad 0 \leqslant \omega \leqslant \omega_p, \\
\hat{A}_d(e^{j2\omega}) &= -1, \quad \omega_s \leqslant \omega \leqslant \pi,
\end{aligned} \qquad (20)
$$

where ω_p and ω_s are the passband and stopband edge frequencies, respectively, and $\omega_p + \omega_s = \pi$. From Eq. (18), we can see that

$$\hat{A}(e^{j(2\pi-\omega)}) = -\hat{A}(e^{j\omega}), \qquad (21)$$

thus the desired magnitude response is reduced to

$$\hat{A}_d(e^{j\omega}) = 1, \quad 0 \leqslant \omega \leqslant 2\omega_p. \qquad (22)$$

From Eq. (10), we have

$$
\begin{aligned}
H_0(z) &= z^{-2M}\{1 - z^{2M} H_1(z) B(z^2)\} \\
&= z^{-2M}\{1 - z^{2N+1} H_1(z)\hat{B}(z^2)\} \\
&= z^{-2M}\{1 - \tfrac{1}{2}(1 + \hat{A}(z^2))\hat{B}(z^2)\},
\end{aligned} \qquad (23)
$$

where

$$\hat{B}(z) = z^{M-N-1/2} B(z) = z^{M-N-1/2}\frac{\sum_{i=0}^{L_3} c_i z^{-i}}{\sum_{i=0}^{L_4} d_i z^{-i}}. \qquad (24)$$

Similarly, to force $\hat{B}(z)$ to be zero phase, the following conditions must be satisfied:

$$
\begin{aligned}
c_i &= c_{L_3-i}, \quad i = 0,1,\ldots,L_3, \\
d_i &= d_{L_4-i}, \quad i = 0,1,\ldots,L_4
\end{aligned} \qquad (25)
$$

and

$$
\begin{aligned}
L_3 &= 2I_3 + 1, \\
L_4 &= 2I_4, \\
L_3 - L_4 &= 2(M - N) - 1,
\end{aligned} \qquad (26)
$$

where I_3 and I_4 are integers. Hence, the frequency response of $\hat{B}(z)$ is given by

$$\hat{B}(e^{j\omega}) = \frac{\sum_{i=0}^{I_3} c_i \cos(I_3 - i + \tfrac{1}{2})\omega}{\tfrac{1}{2}d_{I_4} + \sum_{i=0}^{I_4-1} d_i \cos(I_4 - i)\omega}. \qquad (27)$$

Then $H_0(z)$ has an exact linear phase and its magnitude response is

$$|H_0(e^{j\omega})| = 1 - |H_1(e^{j\omega})|\hat{B}(e^{j2\omega}). \tag{28}$$

Since $|H_1(e^{j\omega})| = 0$ in $[\omega_s,\pi]$, it is clear from Eq. (28) that $|H_0(e^{j\omega})| = 1$, that is, the band $[\omega_s,\pi]$ is the passband of $H_0(z)$. In $[0,\omega_p]$, $|H_1(e^{j\omega})| = 1$, ideally, to force $|H_0(e^{j\omega})| = 0$, then the desired magnitude response of $B(z)$ must be

$$\hat{B}_d(e^{j\omega}) = 1, \quad 0 \leqslant \omega \leqslant 2\omega_p. \tag{29}$$

Therefore, the design problem of the filter banks will become approximation of $A(z)$ and $B(z)$. Note that $A(z)$ and $B(z)$ are noncausal due to the symmetric conditions of Eqs. (16) and (25). $A(z)$ and $B(z)$ must be decomposed into causal and anticausal parts to implement, then the anticausal part can be realized by using time reversal for finite length inputs, such as image signals.

3.2. Design of maximally flat filters

We consider design of $H_1(z)$, i.e., $A(z)$. First, we define an error function $E_a(\omega)$ between the desired and actual magnitude responses of $A(z)$ as

The design purpose is to find a set of filter coefficients a_i and b_i to minimize $E_a(\omega)$ in the band $[0,2\omega_p]$.

It is well known [1,6,7,9–11,17] that wavelet bases can be generated from PR filter banks, then the synthesis of wavelet bases are reduced to design PR filter banks. From the regularity condition of wavelets, the PR filter banks are required to satisfy certain flatness constraints, that is,

$$\left.\frac{\partial^k |H_1(e^{j\omega})|}{\partial \omega^k}\right|_{\omega=\pi} = 0, \quad k = 0,1,\ldots,2J_1 + 1, \tag{31}$$

where J_1 is an integer and $0 \leqslant J_1 \leqslant I_1 + I_2$. When $J_1 = I_1 + I_2$, $H_1(z)$ will be a maximally flat filter

and the corresponding wavelet has maximum-order regularity.

From Eqs. (19), (21) and (30), the flatness constraints of Eq. (31) are equivalent to

$$\hat{A}(e^{j0}) = 1,$$
$$\left.\frac{\partial^k \hat{A}(e^{j\omega})}{\partial \omega^k}\right|_{\omega=0} = 0, \quad k = 1,2,\ldots,2J_1 + 1, \tag{32}$$

that is,

$$\left.\frac{\partial^k E_a(\omega)}{\partial \omega^k}\right|_{\omega=0} = 0, \quad k = 0,1,\ldots,2J_1 + 1. \tag{33}$$

Substituting Eq. (30) into Eq. (33), we get

$$\frac{1}{2}b_{I_2} + \sum_{i=0}^{I_2-1} b_i - \sum_{i=0}^{I_1} a_i = 0,$$
$$\sum_{i=0}^{I_2-1} b_i(I_2 - i)^{2k} - \sum_{i=0}^{I_1} a_i\left(I_1 - i + \frac{1}{2}\right)^{2k} = 0,$$
$$k = 1,2,\ldots,J_1. \tag{34}$$

When the maximally flat filters are needed, we can solve the $(I_1 + I_2 + 1)$ linear equations of Eq. (34) to obtain the filter coefficients, since $J_1 = I_1 + I_2$ and $b_0 = 1$. Therefore, the maximally flat filters can be easily obtained.

$$E_a(\omega) = 1 - \hat{A}(e^{j\omega}) = \frac{\frac{1}{2}b_{I_2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega - \sum_{i=0}^{I_1} a_i \cos(I_1 - i + \frac{1}{2})\omega}{\frac{1}{2}b_{I_2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega}. \tag{30}$$

3.3. Design of filters with given flatness

It is well known that the maximally flat filters are poorly selective. However, frequency selectivity is also thought of as a useful property for many applications. It is known in [10] that frequency selectivity and regularity somewhat contradict each other. For this reason, we consider design of IIR filters that have the best-possible frequency selectivity for a given flatness constraint. Assume that the flatness constraints of Eq. (31) are required where $J_1 < I_1 + I_2$. We want to obtain an equiripple response by using the remaining degrees of freedom. First, we select $(I_1 + I_2 - J_1 + 1)$ extremal frequencies ω_i in $[0,2\omega_p]$ as follows:

$$2\omega_p = \omega_0 > \omega_1 > \cdots > \omega_{(I_1+I_2-J_1)} > 0. \tag{35}$$

By using Remez exchange algorithm, we then formulate $E_a(\omega)$ as

$$E_a(\omega_i) = 1 - \hat{A}(e^{j\omega_i}) = (-1)^i \delta, \qquad (36)$$

where δ (> 0) is the magnitude error, and the denominator polynomial of $E_a(\omega)$ must satisfy

$$\frac{1}{2}b_{I_2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega \neq 0, \quad \text{for all } \omega. \qquad (37)$$

Substituting Eq. (30) into Eq. (36), we rewrite Eqs. (34) and (36) in the matrix form as

$$PA = \delta QA, \qquad (38)$$

where $A = [a_0, a_1, \ldots, a_{I_1}, b_0, b_1, \ldots, b_{I_2}]^T$, and the matrices P and Q are given by

$$P = \begin{bmatrix} -1 & \cdots & -1 & 1 & \cdots & 1 & \frac{1}{2} \\ -(I_1 + \frac{1}{2})^2 & \cdots & -(\frac{1}{2})^2 & I_2^2 & \cdots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -(I_1 + \frac{1}{2})^{2J_1} & \cdots & -(\frac{1}{2})^{2J_1} & I_2^{2J_1} & \cdots & 1 & 0 \\ -\cos(I_1 + \frac{1}{2})\omega_0 & \cdots & -\cos\frac{1}{2}\omega_0 & \cos I_2\omega_0 & \cdots & \cos\omega_0 & \frac{1}{2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\cos(I_1 + \frac{1}{2})\omega_{(I_1+I_2-J_1)} & \cdots & -\cos\frac{1}{2}\omega_{(I_1+I_2-J_1)} & \cos I_2\omega_{(I_1+I_2-J_1)} & \cdots & \cos\omega_{(I_1+I_2-J_1)} & \frac{1}{2} \end{bmatrix}, \qquad (39)$$

$$Q = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & \cos I_2\omega_0 & \cdots & \cos\omega_0 & \frac{1}{2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & (-1)^{I_1+I_2-J_1}\cos I_2\omega_{(I_1+I_2-J_1)} & \cdots & (-1)^{I_1+I_2-J_1}\cos\omega_{(I_1+I_2-J_1)} & \frac{1}{2}(-1)^{I_1+I_2-J_1} \end{bmatrix}. \qquad (40)$$

It should be noted that Eq. (38) is a generalized eigenvalue problem, i.e., δ is an eigenvalue, and A is a corresponding eigenvector [18]. It is known in [18] that to obtain a solution that satisfies Eq. (37), we only need to find the eigenvector corresponding to the positive minimum eigenvalue. In this design problem, we have found that the positive minimum eigenvalue is the absolute minimum one. Therefore, a set of filter coefficients can be easily obtained. In order to achieve an equiripple magnitude response, we make use of an iteration procedure to get the optimal filter coefficients. The design algorithm is shown as follows.

3.4. Design algorithm

Procedure { Design Algorithm of IIR Linear Phase Filters }
Begin
1. Read L_1, L_2, J_1 and ω_p.
2. Select initial extremal frequencies Ω_i ($i = 0, 1, \ldots, I_1 + I_2 - J_1$) equally spaced in the band $[0, 2\omega_p]$.
Repeat
3. Set $\omega_i = \Omega_i$ ($i = 0, 1, \ldots, I_1 + I_2 - J_1$).
4. Compute P, Q by using Eqs. (39) and (40), then find the positive minimum eigenvalue of Eq. (38) to obtain the filter coefficients a_i and b_i that satisfies Eq. (37).

5. Search the peak frequencies of $E_a(\omega)$ within the band $[0, 2\omega_p]$, and store these frequencies into the corresponding Ω_i.

Until Satisfy the following condition for the prescribed small constant ε:

$$\left\{ \sum_{i=0}^{I_1+I_2-J_1} |\Omega_i - \omega_i| \leqslant \varepsilon \right\}$$

End .

3.5. Design of $H_0(z)$

We consider design of $H_0(z)$, i.e., $B(z)$. $B(z)$ can be similarly designed by using the design algorithm proposed in Section 3.4. It is seen from Eq. (28) that the magnitude response of $H_0(z)$ is dependent on that of both $A(z)$ and $B(z)$. Even if both $A(z)$ and $B(z)$ have equiripple magnitude responses, we cannot guarantee that the magnitude response of $H_0(z)$ must be equiripple. To achieve an equiripple magnitude response of $H_0(z)$, we define an error function $E_b(\omega)$ as

$$E_b(\omega) = 1 - |H_1(e^{j\omega/2})|\hat{B}(e^{j\omega}), \qquad (41)$$

and then use Remez exchange algorithm to formulate $E_b(\omega)$ as

$$E_b(\omega_i) = 1 - |H_1(e^{j\omega_i/2})|\hat{B}(e^{j\omega_i}) = (-1)^i\delta, \qquad (42)$$

where $|H_1(e^{j\omega/2})|$ can be considered to be a weighting function. Hence, $H_0(z)$ will have an equiripple magnitude response in stopband. Similarly, $H_0(z)$ is also required to satisfy a given flatness constraint, that is,

$$\left. \frac{\partial^k |H_0(e^{j\omega})|}{\partial\omega^k} \right|_{\omega=0} = 0, \quad k = 0, 1, \dots, 2J_2 + 1, \qquad (43)$$

where J_2 is an integer. Since the flatness degree of $H_0(z)$ is decided by the lower one of flatness between $A(z)$ and $B(z)$, then $J_2 \leqslant J_1$. From Eq. (28), the flatness constraint of Eq. (43) is equivalent to

$$\hat{B}(e^{j0}) = 1,$$

$$\left. \frac{\partial^k \hat{B}(e^{j\omega})}{\partial\omega^k} \right|_{\omega=0} = 0, \quad k = 1, 2, \dots, 2J_2 + 1. \qquad (44)$$

Then we can get the linear equations similar to Eq. (34). Therefore, we can formulate the design problem of $B(z)$ in the form of the eigenvalue problem, as shown in Section 3.3. The design algorithm is the same as that in Section 3.4.

4. Design examples

In this section, we present two design examples to demonstrate the effectiveness of the proposed method.

Example 1. We consider design of the minimax filter bank shown in Example 3.2 of [7] with $\omega_p = 0.4\pi$ and $\omega_s = 0.6\pi$ for comparison purposes. In [7], $A(z)$ was a FIR linear phase filter of order 11, and $B(z)$ was the same as $A(z)$. By using the coefficients shown in [7], we computed the magnitude response of $H_0(z)$ and $H_1(z)$, which are shown in Fig. 4 in the dotted line. The stopband attenuation of $H_0(z)$ and $H_1(z)$ were 35.6 and 45.2 dB, respectively. Note that the resulting magnitude responses are different from that shown in Fig. 5 of [7]. We cannot obtain the same result by using the coefficients given in [7]. We use the proposed method to design the IIR linear phase PR filter bank with the same specification. The order of $A(z)$ needs to be $L_1 = 3$ and $L_2 = 2$ only. The magnitude response of $A(z)$ is shown in Fig. 3, and that of $H_1(z)$ is shown in Fig. 4 in the solid line. It is clear that $H_1(z)$ has an equiripple response and the stopband attenuation 45 dB. To control the magnitude error of
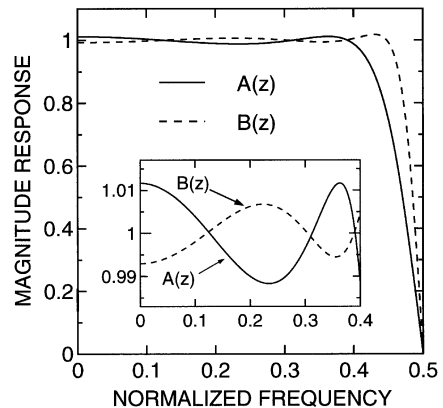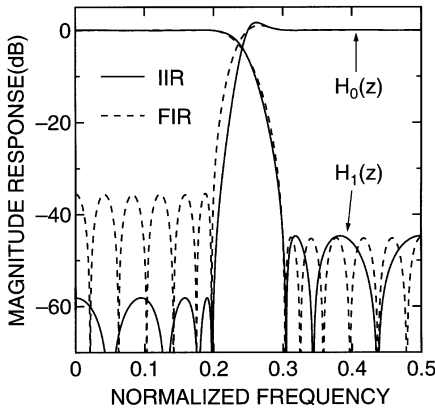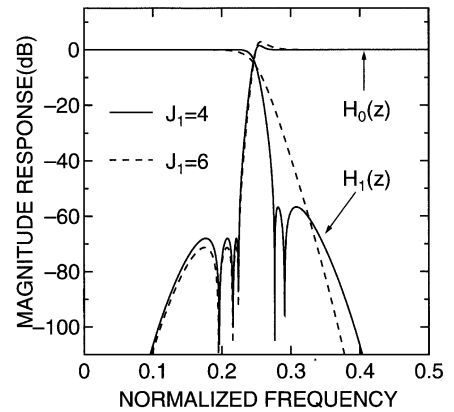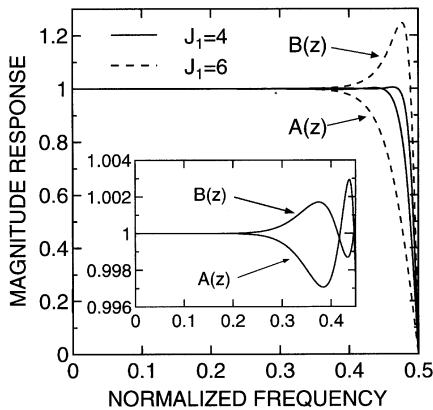

Fig. 3. Magnitude responses of $A(z)$ and $B(z)$ in Example 1.

Fig. 4. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 1.

Fig. 6. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 2.

Fig. 5. Magnitude responses of $A(z)$ and $B(z)$ in Example 2.

$L_2 = 6$ and $L_3 = 9$, $L_4 = 6$. The magnitude responses of $A(z)$ and $B(z)$ are shown in Fig. 5, and that of $H_0(z)$ and $H_1(z)$ are shown in Fig. 6 in the solid line. The stopband attenuation of $H_0(z)$ and $H_1(z)$ are 68.0 and 56.7 dB, respectively. It is clear that the equiripple responses with given flatness are obtained. We have also designed a filter bank with $J_1 = 6$ and $J_2 = 4$. The magnitude responses of $A(z)$ and $B(z)$ are shown in Fig. 5, and that of $H_0(z)$ and $H_1(z)$ are shown in Fig. 6 in the dotted line also. It is seen that $H_1(z)$ is a maximally flat filter.

5. Conclusions

In this paper, we have proposed a new method for designing two-channel IIR linear phase PR filter banks. Since we have adopted the structurally PR implementation proposed in [7], the PR condition is still satisfied even when all filter coefficients are quantized. From the view point of wavelets, we have shown the design of IIR linear phase PR filter banks with an additional flatness constraint. By using Remez exchange algorithm, we have formulated the design problem in the form of a generalized eigenvalue problem. Therefore, by solving the eigenvalue problem to compute the positive minimum eigenvalue, a set of filter coefficients can be obtained as the corresponding eigenvector. The optimal filter coefficients with an equiripple response can be easily obtained through a few

$H_0(z)$ in stopband, we use $B(z)$ different from $A(z)$. The order of $B(z)$ is $L_3 = 3$ and $L_4 = 4$, and the magnitude response is shown in Fig. 3 in the dotted line. The magnitude response of $H_0(z)$ is shown in Fig. 4. It is clear that it is equiripple and the stopband attenuation is 58.1 dB. The overall group delay of this IIR filter bank is 1 sample, while that of the FIR filter bank of [7] is 33 samples. However, the proposed IIR filter banks need some extra storages for realizing the anticausal recursive filtering.

Example 2. We consider the design of an IIR linear phase PR filter bank with a given flatness. The design specification is $J_1 = J_2 = 4$, $\omega_p = 0.45\pi$ and $\omega_s = 0.55\pi$. The order of $A(z)$ and $B(z)$ are $L_1 = 7$,

iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified.

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