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Design of orthonormal IIR wavelet filter banks using allpass filters

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Abstract

This paper presents a new method for designing two-band orthonormal IIR wavelet filter banks using allpass filters. It is well known that orthonormal wavelet bases can be generated by paraunitary filter banks. Thus, synthesis of orthonormal wavelet bases can be reduced to the design of paraunitary filter banks. In this paper, two-band orthonormal IIR wavelet filter banks using a parallel connection of two real allpass filters or a complex allpass filter are examined. From the regularity of wavelets, an additional flatness condition is required to impose on the filter banks. Then, the design problem of orthonormal IIR wavelet filter banks with a given flatness condition is discussed. By considering the given flatness condition and using the Remez exchange algorithm, the design problem can be formulated in the form of an eigenvalue problem. Therefore, a set of filter coefficients can be easily gotten by solving the eigenvalue problem to compute the absolute minimum eigenvalue, and the optimal solution with an equiripple response can be obtained after using an iteration procedure. The proposed method is computationally efficient since the efficient Remez exchange algorithm is employed, and the flatness condition can be arbitrarily specified. Some design examples are presented to demonstrate the effectiveness of the proposed method. © 1999 Published by Elsevier Science B.V. All rights reserved.

Zusammenfassung

Diese Arbeit präsentiert eine neue Methode zum Entwurf von zweikanaligen orthonormalen IIR-Wavelet-Filterbänken unter Verwendung von Allpässen. Es ist allgemein bekannt, daß orthonormale Waveletbasen durch paraunitäre Filterbänke erzeugt werden können. Daher kann der Entwurf von orthonormalen Waveletbasen auf den Entwurf von paraunitären Filterbänken zurückgeführt werden. In dieser Arbeit untersuchen wir zweikanalige orthonormale IIR-Wavelet-Filterbänke, die zwei parallel gekoppelte reelle Allpässe oder einen komplexen Allpaß verwenden. Die Regularität der Wavelets erfordert eine zusätzliche Flachheitsbedingung beim Entwurf der Filterbank. Wir diskutieren deshalb das Entwurfsproblem orthonormaler IIR-Wavelet-Filterbänke mit einer gegebenen Flachheitsbedingung. Die Verwendung des Austauschverfahrens von Remez unter Berücksichtigung der gegebenen Flachheitsbedingung führt auf eine Formulierung der Entwurfsaufgabe als Eigenwertproblem. Die Filterkoeffizienten können daher leicht durch Lösen des Eigenwertproblems (Bestimmung des absolut kleinsten Eigenwerts) berechnet werden. Die optimale Lösung mit konstanter Welligkeit erhält man durch Anwendung eines Iterationsverfahrens. Die vorgeschlagene Methode ist recheneffizient, da der effiziente Remez-Austauschalgorithmus verwendet wird, und die Flachheitsbedingung kann beliebig vorgegeben werden. Die Brauchbarkeit des vorgeschlagenen Verfahrens wird durch einige Entwurfsbeispiele demonstriert. © 1999 Published by Elsevier Science B.V. All rights reserved.

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Résumé

Cet article présente une méthode nouvelle pour la conception de bancs de filtres d'ondelettes IIR orthonormaux bi-bande à l'aide de filtres passetout. II est bien connu ques les bases orthonormales d'ondelettes peuvent être générées par des bancs de filtres para-unitaires. De ce fait, la synthèse de bases d'ondelettes orthonormales peut être réduite à la conception de bancs de filtres para-unitaires. Dans cet article, des bancs de filtres d'ondelettes IIR orthonormales bi-bande utilisant la connexion parallèle de deux filtres passe-tout réels ou un filtre passe-tout complexe sont examinés. Du fait de la régularité des ondelettes une condition additionnelle de platitude imposée aux bancs de filtres est requise. Le problème de la conception de bancs de filtres d'ondelettes IIR orthonormales avec une condition de platitude donnée est donc discuté. A l'aide de la condition de platitude et de l'algorithme d'échange de Remez, ce problème de conception peut être formulé sous la forme d'un problème de valeurs propres. De ce fait, un ensemble de coefficients de filtre peut être facilement obtenu par résolution du problème de calcul de la valeur propre minimale en valeur absolue, et la solution optimale présentant un taux d'ondulation constant en réponse peut être obtenue après application d'une procédure itérative. La méthode proposée est efficiente du point de vue calculatoire puisque l'algorithme d'échange de Remez est employé, et que la condition de platitude peut être spécifiée arbitrairement. Quelques exemples de conception sont présentés pour illustrer l'effectivité de la méthode proposée.

Keywords: Orthonormal wavelet; IIR allpass filter; Remez exchange algorithm; Eigenvalue problem

1. Introduction

Wavelets have received considerable attention in various fields of applied mathematics, signal processing, multiresolution theory, and so on during the past several years [1,4,6,12]. The connection between continuous-time wavelets and discrete filter banks was originally investigated by Daubechies, and is now well understood [1,2,4,6,12,14]. Wavelet bases can be generated by perfect reconstruction (PR) filter bank solutions. In this paper, we will consider a two-band paraunitary filter bank, which, when iterated, generates orthonormal wavelet bases. Paraunitary filter banks can be realized using finite impulse response (FIR) or infinite impulse response (IIR) filters. The case of FIR filters, which lead to compactly supported wavelets, has been examined in detail in [1,3,6,8,9,12,14]. In this paper, we also restrict ourselves to IIR filters, which lead to more general wavelets of infinite support [2,17]. It is known in [7,11,13] that IIR filters composed of two real allpass filters or a complex allpass filter can be implemented with low complexity structures that are robust to finite precision effects. Hence such allpass-based IIR filters are more attractive than general IIR filters. In [5,10], design methods for two-band IIR paraunitary filter banks using a parallel connection of two real allpass filters have been proposed. However, only the maximally flat and elliptic filters were described. In addition, design of orthonormal IIR filter banks using a complex allpass filter is still open problem.

In this paper, we propose a new method for designing two-band orthonormal IIR wavelet filter banks with a given flatness condition using a parallel connection of two real allpass filters or a complex allpass filter. From the regularity of wavelets, an additional flatness condition is generally required to impose on the paraunitary filter banks [2,14]. We then consider the design of IIR wavelet filter banks with the best possible frequency selectivity for a given flatness condition. By considering the given flatness condition, we use the Remez exchange algorithm and formulate the design problem in the form of an eigenvalue problem [15–17]. Therefore, we can easily get a set of filter coefficients by solving the eigenvalue problem to compute the absolute minimum eigenvalue, and obtain the optimal solution with an equiripple response through a few iterations. The proposed method is computationally efficient since the efficient Remez exchange algorithm is employed and the interpolation step has been reduced to the computation of only one eigenvalue [16]. Furthermore, the flatness condition can be arbitrarily specified.

This paper is organized as follows. Section 2 describes two-band orthonormal IIR wavelet filter banks using two real allpass filters or a complex allpass filter. Section 3 presents a new method for designing IIR filters composed of two real allpass filters with a given flatness condition based on a generalized eigenvalue problem by using the Remez exchange algorithm. Section 4 describes design of IIR filters using a complex allpass filter with a given flatness condition. Section 5 shows some design examples to demonstrate the effectiveness of the proposed method. Conclusions are given in Section 6.

2. Orthonormal wavelet filter banks

It is well known [1,2,4,6,12,14] that orthonormal wavelet bases can be generated by a two-band paraunitary filter bank $\{H(z),G(z)\}$. Here, let $H(z)$ denote the lowpass filter and $G(z)$ the highpass filter. From the orthonormality of wavelets, the filter bank must satisfy

$$\begin{aligned} &H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 1, \\ &G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 1, \\ &H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0. \end{aligned} \tag{1}$$

In this paper, we first consider IIR filters $H(z)$ and $G(z)$ that are based on a parallel connection of two real allpass filters as shown in Fig. 1, i.e.,

$$\begin{aligned} H(z) &= \tfrac{1}{2}\{A_1(z^2) + z^{-1}A_2(z^2)\}, \\ G(z) &= \tfrac{1}{2}\{A_1(z^2) - z^{-1}A_2(z^2)\}, \end{aligned} \tag{2}$$

where $A_1(z)$ and $A_2(z)$ are real allpass filters, i.e., their coefficients are real. It is clear that $H(z)$ and $G(z)$ of Eq. (2) satisfy the orthonormal condition of Eq. (1). Second, we consider $H(z)$ and $G(z)$ using

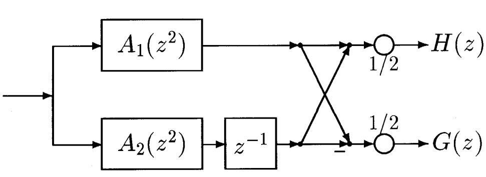

Fig. 1. Filter structure using real allpass filters.

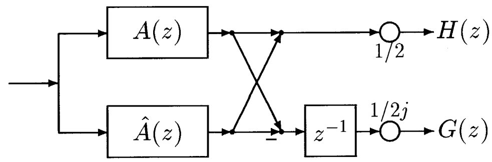

Fig. 2. Filter structure using complex allpass filter.

a complex allpass filter as shown in Fig. 2, i.e.,

$$\begin{aligned} H(z) &= \frac{1}{2}\{A(z) + \hat{A}(z)\}, \\ G(z) &= \frac{z^{-1}}{2\mathrm{j}}\{A(z) - \hat{A}(z)\}, \end{aligned} \tag{3}$$

where $A(z)$ and $\hat{A}(z)$ are Nth-order complex allpass filters, and their coefficients are mutually complex-conjugate. To meet the orthonormal condition of Eq. (1), the constraint that $A(z)$ and $\hat{A}(z)$ must satisfy is

$$A(z) = \pm \mathrm{j}\hat{A}(-z), \tag{4}$$

which shows that if α is a pole of $A(z)$, then $-\alpha^*$ is also a pole of $A(z)$, where α is complex and α^* is complex-conjugate of α. Consequently, $A(z)$ can be expressed as

$$A(z) = \eta z^{-N}\frac{\sum_{n=0}^{N_1} a_{2n}z^{2n} + \mathrm{j}\sum_{n=0}^{N_2} a_{2n+1}z^{2n+1}}{\sum_{n=0}^{N_1} a_{2n}z^{-2n} - \mathrm{j}\sum_{n=0}^{N_2} a_{2n+1}z^{-2n-1}}, \tag{5}$$

where a_n are real, N, N_1 and N_2 are integers, and $\eta = \exp[\pm \mathrm{j}\pi/4]$. When N is even, $N_1 = N/2$ and $N_2 = N/2 - 1$, and when N is odd, $N_1 = N_2 = (N-1)/2$.

It can be seen from Eq. (2) or (3) that the magnitude responses of $H(z)$ and $G(z)$ satisfy the following power-complementary relation:

$$|H(\mathrm{e}^{\mathrm{j}\omega})|^2 + |G(\mathrm{e}^{\mathrm{j}\omega})|^2 = 1, \tag{6}$$

which means that we need to consider design of only one filter $H(z)$. From the regularity of wavelets, it is known [2,14] that an additional flatness condition is required to impose on $H(z)$, i.e.,

$$\left.\frac{\partial^k |H(\mathrm{e}^{\mathrm{j}\omega})|}{\partial \omega^k}\right|_{\omega=\pi} = 0 \quad (k = 0,1,\dots,K-1), \tag{7}$$

where K is integer. Hence, the resulting wavelet function will have K consecutive vanishing moments. This flatness condition can be obtained if $H(z)$ contains K zeros located at $z = -1$. In many applications of signal processing, frequency selectivity is also thought of as a useful property from the viewpoint of signal band-splitting. However, for a given order filter, regularity and frequency selectivity somewhat contradict each other [8]. For this reason, we consider design of $H(z)$ that has the best possible frequency selectivity for a given flatness condition.

3. Design of IIR filters using two real allpass filters

In this section, we consider design of IIR filters $H(z)$ using a parallel sum of two real allpass filters, and describe a new method for designing $H(z)$ with a given flatness condition based on an eigenvalue problem by using the Remez exchange algorithm [15–17].

From Eq. (2), we have

$$H(z) = \tfrac{1}{2}\{A_1(z^2) + z^{-1}A_2(z^2)\} = \tfrac{1}{2}A_1(z^2)\{1 + z^{-1}U(z^2)\}, \tag{8}$$

where $U(z)$ is a Nth-order real allpass filter, and is defined as

$$U(z) = \frac{A_2(z)}{A_1(z)} = z^{-N}\frac{\sum_{n=0}^{N} a_n z^n}{\sum_{n=0}^{N} a_n z^{-n}}, \tag{9}$$

where the filter coefficients a_n are real, and $a_0 = 1$. Let $\theta(\omega)$ be the phase response of $z^{-1}U(z^2)$, i.e.,

$$\exp[\mathrm{j}\theta(\omega)] = \mathrm{e}^{-\mathrm{j}(2N+1)\omega}\frac{\sum_{n=0}^{N} a_n \mathrm{e}^{\mathrm{j}2n\omega}}{\sum_{n=0}^{N} a_n \mathrm{e}^{-\mathrm{j}2n\omega}}, \tag{10}$$

$$\theta(\omega) = 2\tan^{-1}\frac{\sum_{n=0}^{N} a_n \sin\Theta_n(\omega)}{\sum_{n=0}^{N} a_n \cos\Theta_n(\omega)}, \tag{11}$$

where $\Theta_n(\omega) = (2n - N - \frac{1}{2})\omega$. Then, the magnitude response of $H(z)$ is given by

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = \cos\frac{\theta(\omega)}{2} = \frac{\sum_{n=0}^{N} a_n \cos\Theta_n(\omega)}{\sqrt{\{\sum_{n=0}^{N} a_n \sin\Theta_n(\omega)\}^2 + \{\sum_{n=0}^{N} a_n \cos\Theta_n(\omega)\}^2}}. \tag{12}$$

It is clear from Eq. (12) that $|H(\mathrm{e}^{\mathrm{j}\pi})| = 0$. To meet the flatness condition in Eq. (7), the numerator polynomial of Eq. (12) must satisfy

$$\left.\frac{\partial^k\{\sum_{n=0}^{N} a_n \cos\Theta_n(\omega)\}}{\partial\omega^k}\right|_{\omega=\pi} = 0 \quad (k = 0,1,\dots,K-1), \tag{13}$$

where $K = 2M + 1$ is odd, and M is integer. Thus, we can get

$$\sum_{n=0}^{N} a_n\left(2n - N - \frac{1}{2}\right)^{2m-1} = 0 \quad (m = 1,2,\dots,M). \tag{14}$$

Note that M should be such that $0 \leqslant M \leqslant N$. When $M = N$, $H(z)$ becomes the maximally flat filter, and the solution can be obtained by solving only the above linear equations for $a_0 = 1$. When $0 \leqslant M < N$, our aim is to achieve an equiripple magnitude response by using the remaining degree of freedom. It is seen from Eq. (12) that $H(z)$ has the following relation between the passband $[0,\omega_\mathrm{p}]$ and stopband $[\omega_\mathrm{s},\pi]$;

$$|H(\mathrm{e}^{\mathrm{j}\omega})|^2 + |H(\mathrm{e}^{\mathrm{j}(\pi-\omega)})|^2 = 1, \tag{15}$$

where $\omega_\mathrm{p} + \omega_\mathrm{s} = \pi$. This means that only the stopband response needs to be approximated. First, we select $(N - M + 1)$ extremal frequencies ω_i in the stopband $[\omega_\mathrm{s},\pi]$ as follows:

$$\omega_\mathrm{s} = \omega_0 < \omega_1 < \cdots < \omega_{(N-M)} < \pi. \tag{16}$$

By using the Remez exchange algorithm, we formulate $|H(\mathrm{e}^{\mathrm{j}\omega})|$ as

$$|H(\mathrm{e}^{\mathrm{j}\omega_i})| = \cos\frac{\theta(\omega_i)}{2} = (-1)^i\delta_m, \tag{17}$$

where $\delta_m\ (>0)$ is a magnitude error. From Eqs. (12) and (17), we have

$$\frac{\sum_{n=0}^{N} a_n \cos\Theta_n(\omega_i)}{\sum_{n=0}^{N} a_n \sin\Theta_n(\omega_i)} = \coth\{\cos^{-1}(-1)^i\delta_m\} = (-1)^i\delta, \tag{18}$$

where $\delta = \delta_m/\sqrt{1-\delta_m^2}$, and the denominator polynomial must satisfy

$$\sum_{n=0}^{N} a_n \sin\Theta_n(\omega) \neq 0 \quad (\omega_\mathrm{s} \leqslant \omega \leqslant \pi). \tag{19}$$

Then, we rewrite Eqs. (14) and (18) in the matrix form as

$$\boldsymbol{PA} = \delta \boldsymbol{QA}, \tag{20}$$

where $\boldsymbol{A} = [a_0, a_1, \dots, a_N]^{\mathrm{T}}$, and the matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are

$$\boldsymbol{P} = \begin{bmatrix} (-\frac{1}{2} - N) & (\frac{3}{2} - N) & \cdots & (N - \frac{1}{2}) \\ (-\frac{1}{2} - N)^3 & (\frac{3}{2} - N)^3 & \cdots & (N - \frac{1}{2})^3 \\ \vdots & \vdots & \ddots & \vdots \\ (-\frac{1}{2} - N)^{2M-1} & (\frac{3}{2} - N)^{2M-1} & \cdots & (N - \frac{1}{2})^{2M-1} \\ \cos\Theta_0(\omega_0) & \cos\Theta_1(\omega_0) & \cdots & \cos\Theta_N(\omega_0) \\ \cos\Theta_0(\omega_1) & \cos\Theta_1(\omega_1) & \cdots & \cos\Theta_N(\omega_1) \\ \vdots & \vdots & \ddots & \vdots \\ \cos\Theta_0(\omega_{(N-M)}) & \cos\Theta_1(\omega_{(N-M)}) & \cdots & \cos\Theta_N(\omega_{(N-M)}) \end{bmatrix}, \tag{21}$$

$$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ \sin\Theta_0(\omega_0) & \sin\Theta_1(\omega_0) & \cdots & \sin\Theta_N(\omega_0) \\ -\sin\Theta_0(\omega_1) & -\sin\Theta_1(\omega_1) & \cdots & -\sin\Theta_N(\omega_1) \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{N-M}\sin\Theta_0(\omega_{(N-M)}) & (-1)^{N-M}\sin\Theta_1(\omega_{(N-M)}) & \cdots & (-1)^{N-M}\sin\Theta_N(\omega_{(N-M)}) \end{bmatrix}. \tag{22}$$

It should be noted that Eq. (20) is a generalized eigenvalue problem, i.e., δ is an eigenvalue, and $\boldsymbol{A}$ is a corresponding eigenvector. It is known in [15,16] that to obtain a solution that satisfies Eq. (19), we only need to find the eigenvector corresponding to the positive minimum eigenvalue. In this design problem, we have found that the positive minimum eigenvalue is equal to the absolute minimum one, and then this computation can be done efficiently by using the iterative power method. Therefore, we can easily get a set of filter coefficients by solving the above eigenvalue problem to compute the absolute minimum eigenvalue. To achieve an equiripple magnitude response, we make use of an iteration procedure to obtain the optimal solution. Once the optimal filter coefficients a_n are obtained, we compute poles of $U(z)$ and then assign the poles inside the unit circle to $A_2(z)$ as its poles and the poles outside the unit circle to $A_1(z)$ as its zeros. Hence, we can get two causal stable allpass filters $A_1(z)$ and $A_2(z)$. The design algorithm is shown as follows.

Procedure {Design Algorithm of IIR Filters Using Two Real Allpass Filters}

Begin

1. Read specifications N, K, and cutoff frequency ω_s.
2. Select initial extremal frequencies Ω_i $(i = 0,1,\dots,N-M)$ equally spaced in the stopband $[\omega_s,\pi]$.

Repeat

3. Set $\omega_i = \Omega_i$ $(i = 0,1,\dots,N-M)$.
4. Compute $\boldsymbol{P}$, $\boldsymbol{Q}$ by using Eqs. (21) and (22), then find the absolute minimum eigenvalue of Eq. (20) to get a set of filter coefficients a_n that satisfies Eq. (19).

5. Search the peak frequencies of $|H(e^{j\omega})|$ within the stopband, and store these frequencies into the corresponding Ω_i.

Until Satisfy the following condition for the prescribed small constant ε:

$$\left\{\sum_{i=0}^{N-M} |\Omega_i - \omega_i| \leqslant \varepsilon\right\}$$

6. Compute poles of $U(z)$ and assign them to $A_1(z)$ and $A_2(z)$ to obtain a causal stable $H(z)$.

End

4. Design of IIR filters using a complex allpass filter

In this section, we describe design of IIR filters $H(z)$ using a complex allpass filter. From Eq. (3), we have

$$H(z) = \tfrac{1}{2}\{A(z) + \hat{A}(z)\} = \tfrac{1}{2}A(z)\{1 + U(z)\}, \tag{23}$$

where $U(z)$ is a complex allpass filter, and is given from Eq. (5) by

$$U(z) = \frac{\hat{A}(z)}{A(z)} = \frac{\eta^* \sum_{n=0}^{N_1} b_{2n}\{z^{2n} + z^{-2n}\} - \mathrm{j}\sum_{n=0}^{N_2} b_{2n+1}\{z^{2n+1} + z^{-2n-1}\}}{\eta \sum_{n=0}^{N_1} b_{2n}\{z^{2n} + z^{-2n}\} + \mathrm{j}\sum_{n=0}^{N_2} b_{2n+1}\{z^{2n+1} + z^{-2n-1}\}}, \tag{24}$$

where b_n are real and $b_0 = 1/2$. Then the phase response $\theta(\omega)$ of $U(z)$ is

$$\theta(\omega) = 2\tan^{-1}\frac{\mp \sum_{n=0}^{N} (\pm 1)^n b_n \cos n\omega}{\sum_{n=0}^{N} (\mp 1)^n b_n \cos n\omega}, \tag{25}$$

and the magnitude response of $H(z)$ is

$$|H(e^{j\omega})| = \cos\frac{\theta(\omega)}{2} = \frac{\sum_{n=0}^{N} (\mp 1)^n b_n \cos n\omega}{\sqrt{\{\sum_{n=0}^{N} (\pm 1)^n b_n \cos n\omega\}^2 + \{\sum_{n=0}^{N} (\mp 1)^n b_n \cos n\omega\}^2}}. \tag{26}$$

Similarly, to meet the flatness condition in Eq. (7), the numerator polynomial of Eq. (26) must satisfy

$$\left.\frac{\partial^k\{\sum_{n=0}^{N} (\mp 1)^n b_n \cos n\omega\}}{\partial\omega^k}\right|_{\omega=\pi} = 0 \quad (k = 0,1,\ldots,K-1), \tag{27}$$

where $K = 2M$ is even. Thus, we can get

$$\sum_{n=0}^{N} (\pm 1)^n b_n n^{2m} = 0 \quad (m = 0,1,\ldots,M-1). \tag{28}$$

When $M = N$, the maximally flat filter $H(z)$ can be obtained by solving the above linear equations due to $b_0 = 1/2$. When $0 \leqslant M < N$, we select $(N - M + 1)$ extremal frequencies ω_i in the stopband $[\omega_s,\pi]$ as follows:

$$\omega_s = \omega_0 < \omega_1 < \cdots < \omega_{(N-M)} \leqslant \pi. \tag{29}$$

Note that when $M > 0$, $\omega_{(N-M)} < \pi$, and when $M = 0$, $\omega_{(N-M)} = \pi$, thus $H(z)$ becomes the elliptic filter. We then use the Remez exchange algorithm and formulate $|H(e^{j\omega})|$ as

$$|H(e^{j\omega_i})| = \cos\frac{\theta(\omega_i)}{2} = (-1)^i\delta_m. \tag{30}$$

From Eqs. (26) and (30), we have

$$\frac{\sum_{n=0}^{N} (\mp 1)^n b_n \cos n\omega_i}{\mp \sum_{n=0}^{N} (\pm 1)^n b_n \cos n\omega_i} = (-1)^i\delta, \tag{31}$$

where the denominator polynomial must satisfy

$$\sum_{n=0}^{N} (\pm 1)^n b_n \cos n\omega \neq 0 \quad (\omega_s \leqslant \omega \leqslant \pi). \tag{32}$$

We rewrite Eqs. (28) and (31) in the matrix form as

$$\boldsymbol{PB} = \delta\boldsymbol{QB}, \tag{33}$$

where $\boldsymbol{B} = [b_0, b_1, \dots, b_N]^{\mathrm{T}}$, and the matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are

$$\boldsymbol{P} = \begin{bmatrix} 1 & \pm 1 & \cdots & (\pm 1)^N \\ 0 & \pm 1 & \cdots & (\pm 1)^N N^2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \pm 1 & \cdots & (\pm)^N N^{2(M-1)} \\ 1 & \mp \cos\omega_0 & \cdots & (\mp 1)^N \cos(N\omega_0) \\ 1 & \mp \cos\omega_1 & \cdots & (\mp 1)^N \cos(N\omega_1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mp \cos\omega_{(N-M)} & \cdots & (\mp 1)^N \cos(N\omega_{(N-M)}) \end{bmatrix}, \tag{34}$$

$$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ \mp 1 & -\cos\omega_0 & \cdots & \mp(\pm 1)^N \cos(N\omega_0) \\ \pm 1 & \cos\omega_1 & \cdots & \pm(\pm 1)^N \cos(N\omega_1) \\ \vdots & \vdots & \ddots & \vdots \\ \mp(-1)^{N-M} & (-1)^{N-M+1}\cos\omega_{(N-M)} & \cdots & (\mp 1)^{N+1}(-1)^M \cos(N\omega_{(N-M)}) \end{bmatrix}. \tag{35}$$

Therefore, we can get a set of filter coefficients by solving the above eigenvalue problem to compute the absolute minimum eigenvalue, and obtain the optimal solution with an equiripple magnitude response through a few iterations. When the optimal filter coefficients b_n are known, we compute poles of $U(z)$ and assign those poles inside the unit circle to $\hat{A}(z)$ to get a causal stable $H(z)$. The design algorithm is shown as follows.

Procedure {Design Algorithm of IIR Filters Using a Complex Allpass Filter}

Begin

1. Read specifications N, K, and cutoff frequency ω_s.
2. Select initial extremal frequencies Ω_i ($i = 0,1,\dots,N-M$) equally spaced in the stopband $[\omega_s,\pi]$.

Repeat

3. Set $\omega_i = \Omega_i$ ($i = 0,1,\dots,N-M$).
4. Compute $\boldsymbol{P}$, $\boldsymbol{Q}$ by using Eqs. (34) and (35), then find the absolute minimum eigenvalue of Eq. (33) to get a set of filter coefficients b_n that satisfies Eq. (32).
5. Search the peak frequencies of $|H(e^{j\omega})|$ within the stopband, and store these frequencies into the corresponding Ω_i.

Until Satisfy the following condition for the prescribed small constant ε:

$$\left\{ \sum_{i=0}^{N-M} |\Omega_i - \omega_i| \leqslant \varepsilon \right\}$$

6. Compute poles of $U(z)$ and assign the poles in the unit circle to $\hat{A}(z)$ to get a causal stable $H(z)$.

End

5. Design examples

In this section, we present some design examples to demonstrate the effectiveness of the proposed method.

Example 1. We consider design of an IIR filter bank using two real allpass filters with $N = 4$, $\omega_p = 0.4\pi$ and $\omega_s = 0.6\pi$. The order of $H(z)$ is $2N + 1 = 9$, and the order of $A_1(z)$ and $A_2(z)$ is 2. By setting $K = 5$, i.e., $M = 2$, we have first designed $H(z)$ by using the proposed method. The obtained magnitude responses are shown in Fig. 3 in the solid line, and the scaling and wavelet functions generated according to [14] are shown in Figs. 4 and 5, respectively. For comparison purposes, we have designed $H(z)$ with $K = 9$ (i.e., $M = 4$) and $K = 1$ (i.e., $M = 0$) also. Their magnitude responses are shown in Fig. 3 in the dotted line and in the dashed line, respectively, and the scaling and wavelet functions are shown in Figs. 4 and 5 also. When $K = 9$, it can be seen that $H(z)$ is the maximally flat filter that is the same as the one in [10], and when $K = 1$, it is the elliptic filter that is the same as the one in [5]. Note that $H(z)$ with $K = 5$ cannot be designed by using the methods proposed in [5,10]. It is clear from Fig. 3 that the magnitude error increases with increasing M. To increase M implies that the resulting wavelet functions are more regular. It can be seen in Figs. 4 and 5 that the scaling and wavelet functions decline more rapidly as M increases.

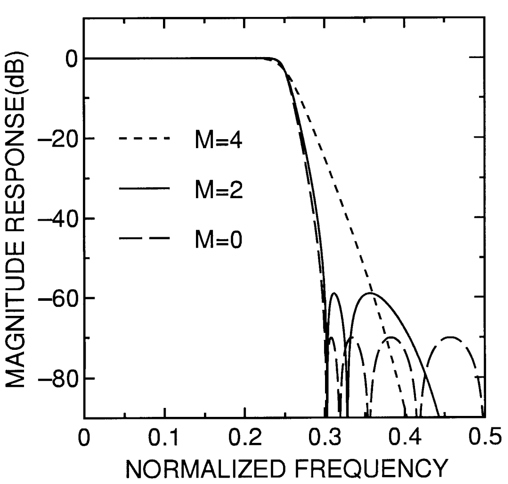

Fig. 3. Magnitude responses of $H(z)$ in Example 1.

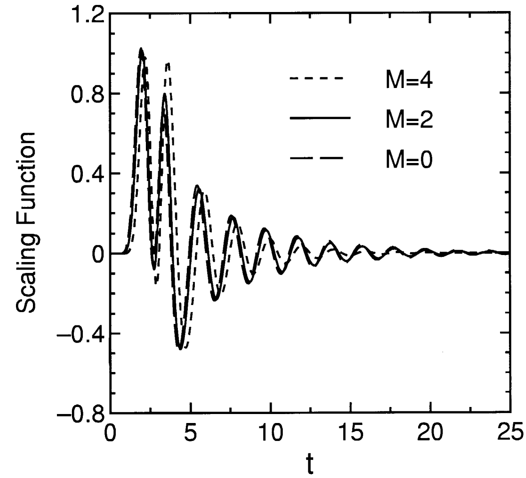

Fig. 4. Scaling functions of Example 1.

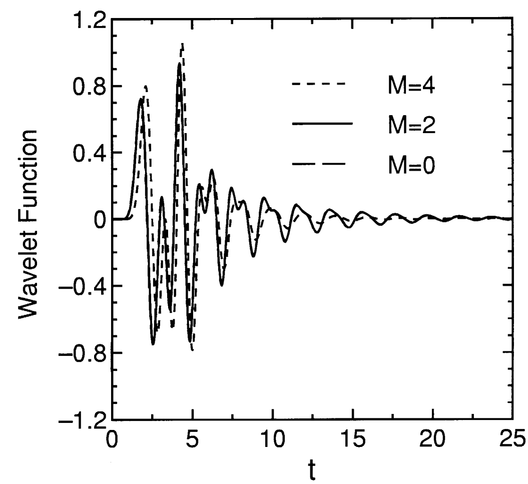

Fig. 5. Wavelet functions of Example 1.

Example 2. We consider the design of an IIR filter bank using a complex allpass filter with $N = 4$, $\omega_p = 0.4\pi$ and $\omega_s = 0.6\pi$. The order of $H(z)$ is $2N = 8$. First, we have set $K = 4$ (i.e., $M = 2$) and designed $H(z)$ by using the proposed method. The obtained magnitude responses are shown in Fig. 6 by a solid line, and the generated scaling and

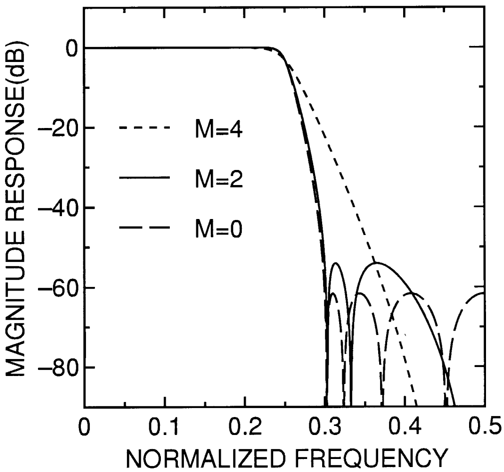

Fig. 6. Magnitude responses of $H(z)$ in Example 2.

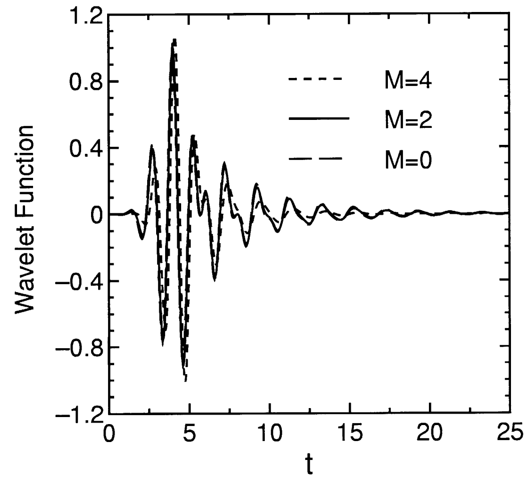

Fig. 8. Wavelet functions of Example 2.

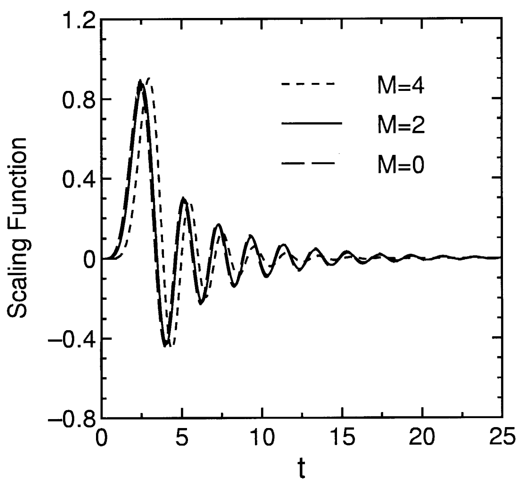

Fig. 7. Scaling functions of Example 2.

wavelet functions are shown in Figs. 7 and 8, respectively. $H(z)$ with $K = 8$ (i.e., $M = 4$) and $K = 0$ (i.e., $M = 0$) have also been designed. Their magnitude responses are also shown in Fig. 6, and the scaling and wavelet functions are shown in Figs. 7 and 8, by a dotted line and in the dashed line, respectively. It is seen in Fig. 6 that $H(z)$ is the maximally flat filter when $K = 8$, and the elliptic filter when $K = 0$. It is noted that $H(z)$ does not have any zero located at $z = -1$ when $K = 0$.

6. Conclusions

In this paper, a new method has been proposed for designing two-band orthonormal IIR wavelet filter banks using two real allpass filters or a complex allpass filter. From the regularity of wavelets, the design problem of IIR filters that have the best possible frequency selectivity for a given flatness condition has been discussed. The proposed design method is based on the formulation of a generalized eigenvalue problem by using the Remez exchange algorithm and considering the given flatness condition. Therefore, a set of filter coefficients can be easily obtained by solving the eigenvalue problem to compute the absolute minimum eigenvalue, and the optimal solution with an equiripple magnitude response can be attained through a few iterations. The main advantages are that the proposed design method is computationally efficient since the efficient Remez exchange algorithm is employed, and the flatness condition can be arbitrarily specified.

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