

LETTER

Reducing Stopband Peak Errors of R -Regular 4th-Band Linear Phase FIR Filters by Superimposing

LinnAung PE[†], Nonmember, Toshinori YOSHIKAWA^{††a)}, Yoshinori TAKEI^{††}, Xi ZHANG^{†††},
and Yasunori SUGITA^{††}, Members

SUMMARY R -regular M th band filters are an important class of digital filters and are used in constructing M th-band wavelet filter banks, where the regularity is essential. But this kind of filter has larger stopband peak errors compared with a minimax filter of the same length. In this paper, peak errors in stopband of R -regular 4th-band filters are reduced by means of superimposing two filters with successive regularities. Then the stopband peak errors in the resulting filters are compared with the original ones. The results show that the stopband peak errors are reduced significantly in the synthesized filter that has the same length as the longer one of the two original filters, at the cost of regularity.

key words: linear phase FIR filters, R -regular M th-band filters, superimposing

1. Introduction

Among the digital filters, R -regular M th-band filters play an important role in constructing M th-band wavelet filter banks, where the regularity is essential. FIR M th-band filters have been studied well [1]–[6]. Among these methods, a closed-form solution is given for the maxflat R -regular FIR M th-band filters with exact linear phase in [2]–[4], while the minimax solution can be found in [1] and [5]. But R -regular M th band linear phase FIR filters have larger peak errors than mini-max filters of the same length. Their stopband peak errors should be known as a function of the regularity R to control their stopband behavior or to determine their sufficient degree to achieve a prescribed requirement for the error bound. Also, it is desirable to develop a method that reduces the stopband peak errors while keeping a reasonable regularity for that filter length.

In this paper, we concentrate on Type I linear phase low pass filters in the case where band number M is 4, and propose a method to reduce the stopband peak errors of R -regular 4th-band filters, at the small decrease in regularity. In this method, two filters with different regularities and different signs of the stopband peak errors are superimposed so that the resulting peak error is minimized.

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[†]The author is with Intevac Asia Pte. Ltd., 25 Tagore Lane, #03-10, Singapore, 787602.

^{††}The authors are with the Department of Electrical Engineering, Nagaoka University of Technology, Nagaoka-shi, 940-2188 Japan.

^{†††}The author is with the Department of Information and Communication Engineering, The University of Electro-Communications, Chofu-shi, 182-8585 Japan.

a) E-mail: tyoshi@nagaokaut.ac.jp

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This paper is organized as follows: in Sect. 2, the amplitude response of an R -regular M th-band lowpass filter is presented. Peak errors in stopband of R -regular 4th-band FIR filters are presented in Sect. 3. We also investigate the relations between the regularity R and the sign of magnitude response in stopband with the use of Taylor expansion. In Sect. 4, we develop a method of superimposing the two filters with different regularity and also present the stopband peak errors of the superimposed filters. Finally, we end this paper with conclusions in Sect. 5.

2. Amplitude Response of R -Regular M th-Band Low-pass Filter

The transfer function $H(z)$ of an FIR digital filter with filter length N can be represented with the following equation:

$$H(z) = \sum_{i=0}^{N-1} h[i] z^{-i} \quad (1)$$

where $h[i]$ is the impulse response and its amplitude response for the case of type I linear phase FIR filter can be calculated by using

$$A(\omega) = \sum_{i=0}^{N-1} h[i] e^{-j(i-\frac{N-1}{2})\omega}. \quad (2)$$

A filter $H(z)$ is said to be a maxflat R -regular M th-band low-pass filter, if the amplitude response $A(\omega)$ of Eq. (2) satisfies for the frequency index k and regularity index r the relations

$$\left. \frac{\partial^r A(\omega)}{\partial \omega^r} \right|_{\omega=\frac{2\pi k}{M}} = \begin{cases} 1 & \text{if } k = r = 0; \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

where the range of k and r are $(0 \leq k \leq M)$ and $(0 \leq r < R)$, respectively. The closed formula for the coefficients $h[i]$ that satisfy Eq. (3) has been given in [2]–[4], as shown in

$$h[Mi + n] = \frac{(-1)^i \prod_{\substack{s=0 \\ s \neq i}}^{R-1} \left(Ms + n - \frac{(N-1)}{2} \right)}{M^R i! (R-i-1)!} \quad (4)$$

$(0 \leq i < R, 0 \leq n \leq M-1).$

These filters have the regularity R at every M division frequency point and they are not optimized with respect to stopband errors. However, it will be necessary for us to know the stopband errors as a function of regularity.

3. Peak Errors in Stopband of R -Regular 4th-Band FIR Filters

In this section, we inspect the magnitude responses of R -regular M th-band linear phase FIR filters with the fixed band number of $M = 4$. Then positive peak errors with even regularity and negative peak errors with odd regularity in stopband are observed.

With the specification of Eq. (3), type I maximally flat 4th-band lowpass filters are designed for different regularities. In Fig. 1, magnitude responses of lowpass filters with regularities of $R = 10, 30, 11$ and 31 are presented. From the figure, it can be found that all the magnitude responses have a flat behavior at $\omega/2\pi = 0.0, 0.25$ and 0.5 . On the other hand, near (but not exactly on) the midpoint of 0.25 and 0.5 in the stopband, namely at $\omega = 3\pi/4$, the magnitude responses have peaks, that give the maximum error in the stopband. We shall call them peak errors. Magnitude of positive and negative peak errors are plotted as a function of

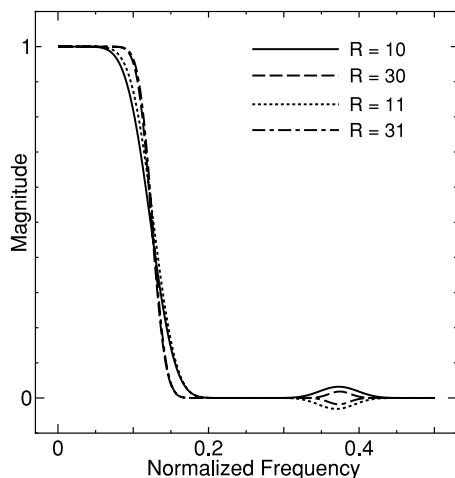


Fig. 1 Magnitude response of type I lowpass filters.

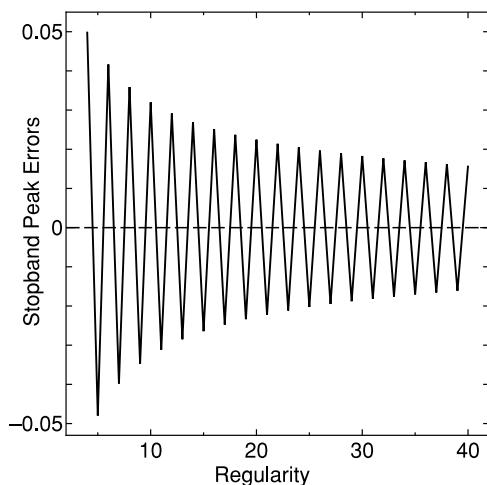


Fig. 2 Stopband peak errors vs. regularities ($M = 4$).

R in Fig. 2. It is found that the absolute value of the stopband peak error decreases as the regularity R increases in both case of regularity R .

There are positive peak errors with even regularities and negative peak errors with odd regularities in the stopband of the filters. Indeed, in terms of regularity R , the magnitude response has a Taylor expansion of the form $A(\omega) = a_R(\omega - 2\pi/4)^R + o((\omega - 2\pi/4)^R)$ where a_R is a real number and $o(x^R)$ denotes a function $f(x)$ that $f(x)/x^R \rightarrow 0$ ($x \rightarrow 0$). Then we have $A(2\pi/4 + \xi)/A(2\pi/4 - \xi) \rightarrow (-1)^R$ ($\xi \rightarrow 0$), i.e., the magnitude response $A(\omega)$ does change its sign between $\omega/2\pi = 0.25$ and 0.5 if and only if the regularity R is odd.

4. Superimposing

As we have shown in Sect. 3, an even regularity results in a positive peak error, while an odd regularity results in a negative peak error. Also it was observed that the position of the peak error is about $\omega = 3\pi/4$, independently of R . By these facts we can expect that superimposing two filters with mutually different parities of regularities cancels negative and positive peak errors and gives rise to a reduced error.

In this section, two filters of successive regularities $R, R+1$ are superimposed and then stopband peak errors in the resulting filter, to which we shall refer as a superimposed filter, will be presented.

We restrict ourselves to the case of band number $M = 4$ because in this case it can be assumed that there is only one positive peak error with even regularity and only one negative peak error with odd regularity in the stopband of the two filters which are going to be superimposed.

4.1 Method of Superimposing Two Filters

When the two filters are superimposed with different parities of regularities, let us call the filter with odd regularity as Filter1 and the filter with even regularity as Filter2. Let $H_1(z)$ and $H_2(z)$ be their transfer functions respectively. We consider only the following cases: the regularity of $H_2(z)$ which is either smaller or larger than $H_1(z)$ by one. The reason why we consider only these case is that other choice makes the resulting regularity after superposition small relative to the resulting filter length. The two filters have different regularities and also different filter lengths. The formula for filter length is given in the following equation as a special case of Table 1 of [2]:

$$N = \begin{cases} 4R + 1 & (R \text{ is odd}); \\ 4R - 1 & (R \text{ is even}). \end{cases} \quad (5)$$

To coincide the center of the shorter filter impulse response with that of the longer impulse, the shorter filter should be delayed with an appropriate delay d which is determined by the length of two filters. With the use of two filter's transfer function, transfer function of new superimposed filter can be calculated by using the following equation:

$$H_3(z) = \begin{cases} \alpha H_1(z) + \beta z^{-d} H_2(z) & \text{(if } H_1(z) \text{ is longer);} \\ \alpha z^{-d} H_1(z) + \beta H_2(z) & \text{(if } H_2(z) \text{ is longer),} \end{cases} \quad (6)$$

where α and β are weights satisfying $\alpha + \beta = 1$, which are determined so that the positive and negative peak errors of the resulting filter are balanced, i.e., have the same magnitude. And the weights are calculated numerically by an application of the bisection method whose initial value is set as the ratio of the magnitudes of the positive and negative peak errors of Filter1 and Filter2, respectively.

4.2 Magnitude Responses of Superimposed Filters

In superimposing $H_1(z)$ with odd regularity and $H_2(z)$ with even regularity, there are two cases. They are

1. $R_1 > R_2$ and
2. $R_1 < R_2$.

As we mentioned above, we only consider the cases $R_2 = R_1 \pm 1$. For the first case filter length difference is 6 and for the second case the difference is only 2.

4.2.1 Pairs of Odd Regularity R_1 and Even Regularity R_2 ($R_1 > R_2$)

In the first case, odd regularity R_1 of Filter1 is greater than even regularity R_2 of Filter2 by one. The length N_1 of Filter1 is longer than Filter2 and this case correspond to the first case of Eq. (6). Namely,

$$H_3(z) = \alpha H_1(z) + (1 - \alpha) z^{-d_1} H_2(z), \quad (7)$$

where

$$d_1 = \frac{N_1 - N_2}{2} \quad (8)$$

$$= \frac{((4R_1 + 1) - (4R_2 - 1))}{2} = 6. \quad (9)$$

In Fig. 3, magnitude responses of the superimposed filter and the two original filters ($R_1 = 11, R_2 = 10$) are presented. The superimposed filter has the regularity $R_2 = \min(R_1, R_2)$, which can be easily concluded by considering the Taylor expansion of $A(\omega)$ at a 4-division point in the stopband and has the filter length $N_3 = \max(N_1, N_2)$.

Figure 4(a) shows the enlarged part of the peak error in stopband of the superimposed filter and (b) shows the zero location of that filter. Although there are only one positive and one negative peak errors in the original two filters, there are two positive peak errors and one negative peak error in the stopband of the superimposed filter. But peak errors in the stopband of the superimposed filter are much smaller than the peak errors in original two filters. The filter lengths, positive and negative peak errors of Filter1, Filter2 and superimposed Filter3 are summarized in Table 1. By comparing the peak errors of Filter1 and Filter3 which are of the same length, we can conclude that our method can reduce the peak errors in stopband of the filter effectively (nearly 95%) at only one decrease of the regularity.

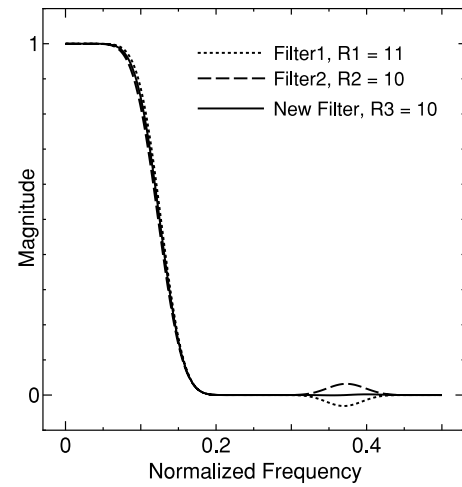


Fig. 3 Magnitude responses of two original filters and the superimposed filter (case 1).

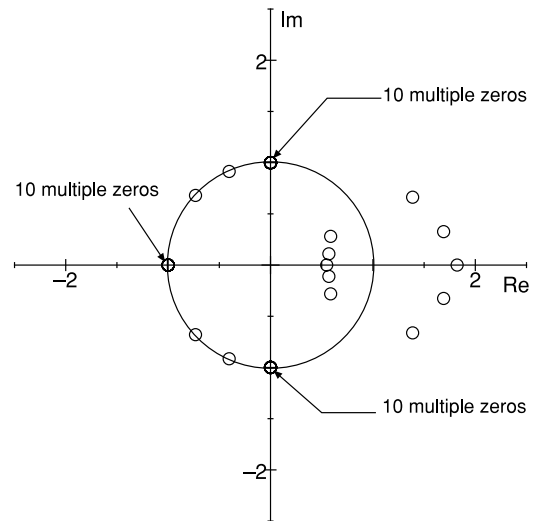
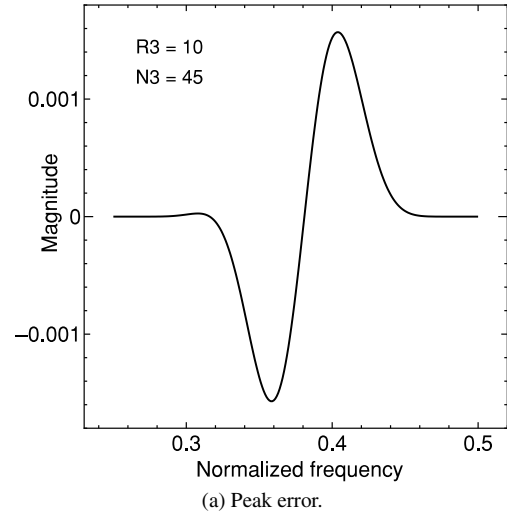
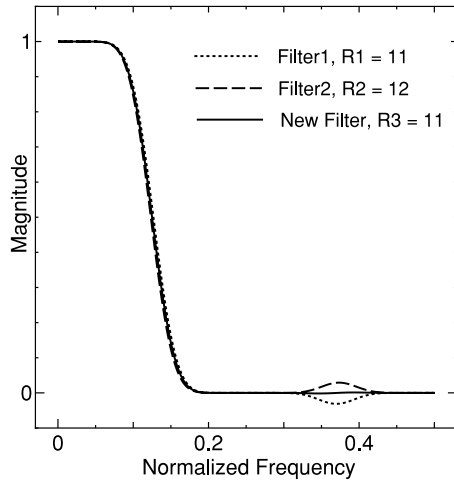


Fig. 4 Peak error in stopband and zero location of the superimposed filter (case 1).

Table 1 Peak errors in new and original filters (case 1).

$M=4$	Filter length N	Positive peak error	Negative peak error
$R_1=11$	$N_1 = 45$	—	-0.0311 (-30.1 [dB])
$R_2=10$	$N_2 = 39$	0.0320 (-29.9 [dB])	—
$R_3=10$	$N_3 = 45$	0.0016 (-55.9 [dB])	-0.0016 (-55.9 [dB])
Errors reduced in %		95.00%	94.86%

**Fig. 5** Magnitude responses of two original filters and the superimposed filter (case 2).

4.2.2 Pairs of Odd Regularity R_1 and Even Regularity R_2 ($R_1 < R_2$)

Filter1 with odd regularity R_1 which is less than even regularity R_2 of Filter2 is used in the second case. In this case,

$$H_3(z) = \alpha z^{-d_2} H_1(z) + (1 - \alpha) H_2(z), \quad (10)$$

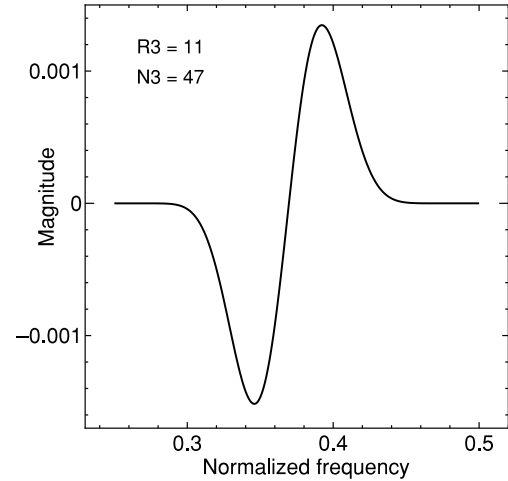
where

$$d_2 = \frac{N_2 - N_1}{2} \quad (11)$$

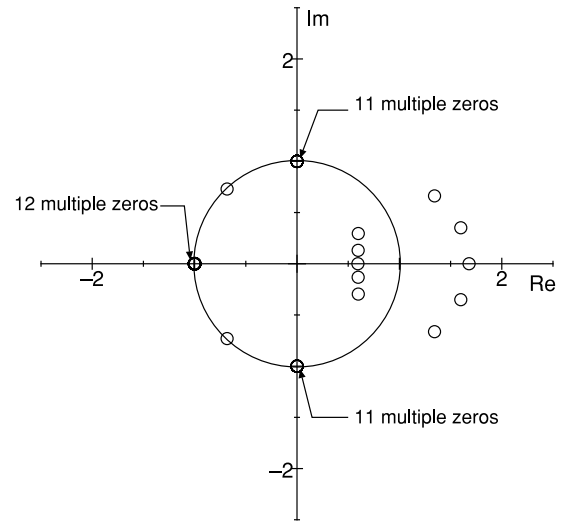
$$= \frac{((4R_2 - 1) - (4R_1 + 1))}{2} = 2. \quad (12)$$

Magnitude responses of Filter1 ($R_1 = 11$), Filter2 ($R_2 = 12$) and superimposed Filter3 are shown in Fig. 5. The enlarged part of the stopband peak errors in Fig. 6(a) shows that there are only one positive and one negative peak errors in stopband of the superimposed filter. With the zero location of the superimposed filter in Fig. 6(b), we also confirm that there is only one zero location in the stopband of the superimposed filter.

Table 2 gives summary of three filters. As was the case in 4.2.1, the superimposition reduces peak errors considerably at the cost of one regularity, compared with Filter1 of the same length.



(a) Peak error.



(b) Zero location.

Fig. 6 Peak error in stopband and zero location of the superimposed filter (case 2).**Table 2** Peak errors in new and original filters (case 2).

$M=4$	Filter length N	Positive peak error	Negative peak error
$R_1=11$	$N_1 = 45$	—	-0.0311 (-30.1 [dB])
$R_2=12$	$N_2 = 47$	0.0291 (-30.7 [dB])	—
$R_3=11$	$N_3 = 47$	0.0014 (-57.1 [dB])	-0.0014 (-57.1 [dB])
Errors reduced in %		95.19%	95.49%

5. Concluding Remarks

Peak errors in stopband of R -regular 4th-band linear phase Type I low pass FIR filters have been reduced by superimposing two filters of successive regularities. We have presented the two cases depending on the pairs of regularities while maintaining the zero-crossing at every four points

from the center in the impulse responses. Compared with the longer filter of the two filters that are superimposed, the resulting filter has the same length as that filter, the regularity smaller by one, and much smaller stopband peak error than those of the two original filters.

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