Design of zero phase IIR Nyquist filters

X. Zhang and T. Yoshikawa

Indexing terms: Nyquist criterion, Digital filters

A new method for designing zero phase IIR Nyquist filters with zero intersymbol interference is presented. The proposed method is based on the eigenvalue problem by using the Remez exchange algorithm in the stopband, and the optimal filter coefficients can be easily obtained by computing the absolute minimum eigenvector and applying an iteration procedure.

Introduction: Nyquist filters play an important role in designing data transmission systems and filter banks. Nyquist filters are required to band-limit data spectrum and to minimise intersymbol interference simultaneously. FIR Nyquist filters that have been exhaustively studied in [1 – 3] have an exact linear phase, but generally need high-order filters to meet stringent magnitude specifications. IIR Nyquist filters [4] can obtain better magnitude responses than FIR filters. However, the proposed design procedure is time-consuming.

In this Letter, we present a new method for designing zero phase IIR Nyquist filters with zero intersymbol interference. First, we show that zero phase IIR Nyquist filters with zero intersymbol interference have a property where the magnitude in the passband is mainly decided by a stopband error. Therefore, the design problem will become the minimisation of magnitude error in the stopband. By applying the Remez exchange algorithm in the stopband, we formulate the design problem in the form of an eigenvalue problem [5]. Then the optimal filter coefficients with an equiripple stopband response can then be easily obtained after solving the eigenvalue problem and using an iteration procedure.

Property of IIR Nyquist filters: To minimise intersymbol interference, the Nyquist filter $H(z)$ is required to have an exact zero-cross impulse response except for one point, i.e.

$$\begin{cases} h(K) = \frac{1}{M} & (k = 0) \\ h(K + kM) = 0 & (k = \pm 1, \pm 2, ...) \end{cases} \qquad (1)$$

where K, M are integers. $H(z)$ is also required to be lowpass with passband and stopband cutoff frequencies $\omega_p = (1-\rho)\pi/M$, $\omega_s = (1+\rho)\pi/M$, where ρ is a rolloff rate. It is known in [4] that the transfer function of IIR Nyquist filters that satisfy eqn. 1 can be expressed in the form

$$H(z) = \frac{z^{-K}}{M} + \frac{\sum_{\substack{i=0 \\ i \neq K+kM}}^{N_n} a_i z^{-i}}{\sum_{i=0}^{N_d} b_i z^{iM}} \qquad (2)$$

where the filter coefficients a_i, b_i are real, and N_n, N_d are integers. To obtain zero phase, we set $K = 0$ and the filter coefficients to be symmetric, i.e.

$$H(z) = \frac{1}{M} + \frac{\sum_{\substack{i=1 \\ i \neq kM}}^{N_n} c_i (z^i + z^{-i})}{\sum_{i=0}^{N_d} d_i (z^{iM} + z^{-iM})} \qquad (3)$$

Note that eqn. 3 is noncausal, and must be decomposed into causal and anticausal parts to be implemented. The anticausal part can be realised by using time reversal for finite length inputs, or using the time reversed section technique [6] for infinite length inputs. The magnitude response of $H(z)$ is given by

$$H(e^{j\omega}) = \frac{1}{M} + \frac{\sum_{\substack{i=1 \\ i \neq kM}}^{N_n} c_i \cos(i\omega)}{\sum_{i=0}^{N_d} d_i \cos(iM\omega)} \qquad (4)$$

It is well-known that the cosine function satisfies

$$\begin{cases} \sum_{m=0}^{M-1} \cos\left(i\left(\omega + \frac{2m\pi}{M}\right)\right) = 0 & (i \neq kM) \\ \cos\left(i\left(\omega + \frac{2m\pi}{M}\right)\right) = \cos(i\omega) & (i = kM) \end{cases} \qquad (5)$$

Therefore, we obtain

$$\sum_{m=0}^{M-1} H\left(e^{j(\omega + (\frac{2m\pi}{M}))}\right) \equiv 1 \qquad (6)$$

which means that the sum of the magnitudes at the frequency points $\omega + 2m\pi/M$ ($m = 0, 1, ..., M-1$) keep unity regardless of what the values of the coefficients c_i and d_i are. Since $H(e^{j(2\pi-\omega)})$ $H(e^{j\omega})$, eqn. 7 can be rewritten as

$$H(e^{j\omega_0}) = 1 - \sum_{k=1}^{M-1} H(e^{j\omega_k}) \qquad (0 \leq \omega_0 \leq \omega_p) \qquad (7)$$

where $\omega_k = \lfloor \{k+1\}/2 \rfloor 2\pi/M + (-1)^k \omega_0$, and $\lfloor x \rfloor$ denotes the integer part of x. It is clear that if the magnitude response is 0 in the stopband, then the passband response must be 1. Therefore, the passband ripple is mainly decided by the stopband ripple. Let δ_s be the maximum error in the stopband, the maximum error in the passband is in the worst case $\delta_p = (M-1)\delta_s$. Usually, δ_p is much smaller than this upper limit in practical designs. Since δ_p is guaranteed to be relatively small for a small value of δ_s, the filter design can concentrate on approximating the stopband response. It can also be explained according to the pole-zero locations. $H(z)$ has $2(N_n + N_d - \lfloor N_n/M \rfloor)$ independent zeros which must be located on the unit circle to provide the desired stopband response. The poles and the remaining zeros off the unit circle are used for satisfying the time-domain condition so that the passband response is naturally formed. In the following, we will directly apply the Remez exchange algorithm in the stopband to design zero phase IIR Nyquist filters with zero intersymbol interference.

Design of IIR Nyquist filters: It is known that the time-domain condition of eqn. 1 has been satisfied by using the transfer function of eqn. 3. Then the time-domain optimisation is not required, and we only need to optimise the magnitude response of eqn. 4 in the stopband. Since $2(N_n + N_d - \lfloor Ln/M \rfloor)$ independent zeros of $H(z)$ have to locate on the unit circle to minimise the stopband error, we can select $(N_n + N_d - \lfloor Ln/M \rfloor + 1)$ extremal frequencies $\bar{\omega}$ in the stopband. By using the Remez exchange algorithm, we formulate the magnitude response of eqn. 4 as

$$W(\bar{\omega}_i) H(e^{j\bar{\omega}_i}) = (-1)^{i-1} \delta \qquad (8)$$

where $W(\omega)$ is a weighting function, and δ is the error to be minimised. Substituting eqn. 4 into eqn. 8 we can rewrite eqn. 8 in the matrix form as

$$\mathbf{PC} = \delta \mathbf{QC} \qquad (9)$$

where $\mathbf{C} = [c_1, c_2, ..., cM_{-1}, c_{M+1}, ..., c_{N_n}, d_0, d_1, ..., d_{N_d}]^T$, and the elements of the matrices $\mathbf{P}$, $\mathbf{Q}$ are given by

$$P_{ij} = \begin{cases} \cos\left(\left(j + \left\lfloor \frac{j-1}{M-1} \right\rfloor\right) \bar{\omega}_i\right) & j = 1, 2, ..., N_n - \left\lfloor \frac{N_n}{M} \right\rfloor \\ \frac{\cos(mM\bar{\omega}_i)}{M} & j + \left\lfloor \frac{N_n}{M} \right\rfloor - N_n - 1 = m \\ & = 0, 1, ..., N_d \end{cases} \qquad (10)$$

$$Q_{ij} = \begin{cases} 0 & j = 1, 2, ..., N_n - \left\lfloor \frac{N_n}{M} \right\rfloor \\ \frac{(-1)^{i-1}}{W(\bar{\omega}_i)} \cos(mM\bar{\omega}_i) & j + \left\lfloor \frac{N_n}{M} \right\rfloor - N_n - 1 = m \\ & = 0, 1, ..., N_d \end{cases} \qquad (11)$$

It should be noted that eqn. 9 is a generalised eigenvalue problem, i.e., δ is an eigenvalue and $\mathbf{C}$ is a corresponding eigenvector. Therefore, we can obtain a solution by solving the eigenvalue problem of eqn. 9. To minimise the stopband error, we compute the absolute minimum eigenvalue, then its corresponding eigenvector gives a set of filter coefficients. Since the extremal frequencies initially selected may not be the current peak frequencies, we must apply an iteration procedure to obtain the optimal solution with an equiripple stopband response. The design algorithm is shown as follows.

Design algorithm:
procedure {design algorithm of IIR Nyquist filters}
begin

(i) Read N_n, N_d, M, ρ, and weighting function $W(\omega)$

(ii) Select initial external frequencies $\bar{\Omega}_i$ (for $i = 1, 2, ..., N_n+N_d - \lfloor N_n/M \rfloor+1$) equally spaced in stopband

repeat

(ii) Set $\bar{\omega}_i = \bar{\Omega}_i$ (for $i = 1, 2, ..., N_n+N_d-\lfloor N_n/M \rfloor+1$).

(iv) Compute **P**, **Q** by using eqns. 10 and 11, and find the absolute minimum eigenvalue to obtain a set of filter coefficients c_i and d_i

(v) Search the peak frequencies $\bar{\Omega}_i$ of $H(e^{j\omega})$ in the stopband

until Satisfy the following condition for the prescribed small constant ϵ:

$$\left\{ |\bar{\Omega}_i - \bar{\omega}_i| \le \epsilon \quad \left(\text{for } i = 1, 2, ..., N_n + N_d - \left\lfloor \frac{N_n}{M} \right\rfloor + 1\right) \right\}$$

end.

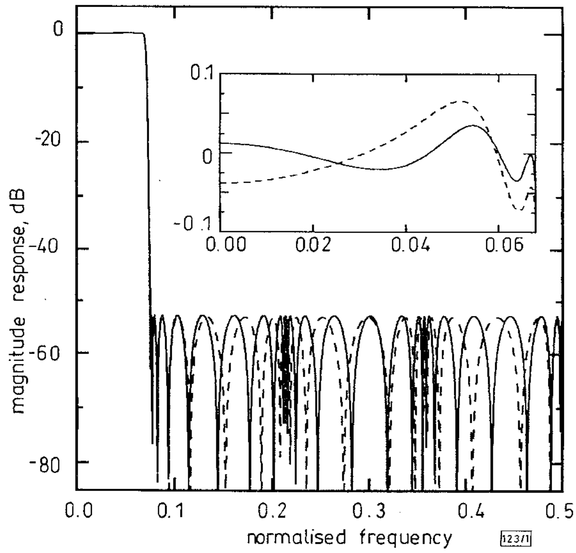

Fig. 1 *Magnitude responses of zero phase IIR Nyquist filters*

——— $N_d = 2$
– – – – $N_d = 3$

Design example: The aim is to design a zero phase IIR Nyquist filter with $N_n = 24$, $N_d = 2$, $M = 7$ and $\rho = 0.05$. The weighting function is set to $W(\omega) = 1$ in the stopband. The magnitude response of the filter designed by using the proposed method is shown in Fig. 1 with the solid line. The passband and stopband attenuations are 0.0363 and 52.67dB, respectively. We also designed a Nyquist filter with $N_n = 20$ and $N_d = 3$, and the passband and stopband attenuations are 0.0722 and 52.96dB, respectively.

Conclusion: A new method for designing zero phase IIR Nyquist filters with zero intersymbol interference has been proposed. It has been shown that the passband ripple of IIR Nyquist filters with zero intersymbol interference is mainly decided by stopband ripple. Therefore, the design problem can be formulated in the form of an eigenvalue problem by using the Remez exchange algorithm only in the stopband, and the optimal filter coefficients with an equiripple stopband response can be easily obtained by computing the absolute minimum eigenvalue and applying an iteration procedure.

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X. Zhang and T. Yoshikawa (*Department of Electrical Engineering, Nagaoka University of Technology, 1603-1, Kamitomiokacho, Nagaoka-shi, Niigata 940-21, Japan*)

References

1 LIANG, J.K., DEFIGUEIREDO, R.J.P., and LU, F.C.: 'Design of optimal Nyquist, partial response, Nth-band, and nonuniform tap spacing FIR digital filters using linear programming techniques', *IEEE Trans.*, 1985, **CAS-32**, pp. 386–392

2 VAIDYANATHAN, P.P., and NGUYEN, T.Q.: 'Eigenfilters: a new approach to least-squares FIR filter design and applications including Nyquist filters', *IEEE Trans.*, 1987, **CAS-34**, pp. 11–23

3 SARAMAKI, T., and NEUVO, Y.: 'A class of FIR Nyquist (Nth-band) filters with zero intersymbol interference', *IEEE Trans.*, 1987, **CAS-34**, pp. 1182–1190

4 NAKAYAMA, K., and MIZUKAMI, T.: 'A new IIR Nyquist filter with zero intersymbol interference and its frequency response approximation', *IEEE Trans.*, 1982, **CAS-29**, pp. 23–34

5 ZHANG, X., and IWAKURA, H.: 'Novel method for designing digital allpass filters based on eigenvalue problem', *Electron. Lett.*, 1993, **29**, pp. 1279–1281

6 POWELL, S.R., and CHAU, P.M.: 'A technique for realizing linear phase IIR filters', *IEEE Trans. Sig. Process.*, 1991, **39**, pp. 2425–2435

Fast *M*-band orthogonal wavelet transform algorithm when base length equals 2*M*

Sang-keun Kwon, Young-seok Oh and Dong Shin Seo

Indexing terms: Wavelet transforms, Digital filters

The fast implementation algorithm of M-band orthogonal wavelet filter bank (OWFB) with length L equal to $2M$ is proposed by using the lossless matrix factorisation method. The computational complexity of the proposed algorithm becomes lower than that of the direct filtering method (DFM): ($M^2 + M + 1$) multiplications and $M^2 + 2M - 1$) additions against $2M^2$ and ($2M^2 - M$), respectively.

Introduction: Before the wavelet transform (WT) was formally introduced, the M-band QMF (quadrature mirror filter) bank had already been used in sub-band coding (SBC). The main differences between the QMF bank and OWFB are that the lowpass filter (LPF) of the OWFB has regularity and normalisation conditions [1]. So by including these conditions into the LPF of the OWFB, it is possible to design the M-band OWFB using a lossless system used to design the M-band QMF bank [3, 4]. Based on this concept, the design methods of an M-band OWFB were completely generalised with M having any positive integer [3, 4].

In this Letter, we propose a fast algorithm which can decompose the signal into M-band signals by OWFB when the length L of the OWFB is equal to $2M$. Since M-band OWFB design is based on a lossless matrix, we implement the fast algorithm by factorising the lossless matrix proposed by Vaidyanathan [2].

The proposed algorithm has lower computational complexity than that of the DFM: ($M^2 + M + 1$) multiplications and ($M^2 + 2M - 1$) additions against $2M^2$ and ($2M^2 - M$), respectively. The OWFB can be implemented with lower computational complexity since unitary matrices such as DCT, WUT etc.. which have a fast algorithm, can be used to design the OWFB for $M = 2^n$.

Design of generalised M-band OWFB: Consider the maximally decimated M-band QMF bank system. Let $\underline{E}(z)$ be the M-component polyphase matrix of the decomposing filter bank $\underline{H}(z) = [H_0(z), H_1(z), ..., H_{M-1}(z)]$. If $\underline{E}(z)$ satisfies eqn. 1, the QMF bank system has perfect reconstruction (PR) [2] and we call it the paraunitary matrix:

$$\tilde{\underline{E}}(z)\underline{E}(z) = \underline{I} \tag{1}$$

where $\underline{I}$ is the identity matrix and $\tilde{\underline{E}}(z)$ is the transpose-conjugate of $\underline{E}(z)$.

Since the filter coefficients length L of $H_k(z)$ is $2M$, $\underline{E}(z)$ can be decomposed as

$$\underline{E}(z) = \underline{A}_0 + \underline{A}_1 z^{-1} \tag{2}$$

where

$$\underline{A}_i = \begin{bmatrix} h_0(Mi) & h_0(Mi+1) & \cdots & h_0(Mi+M-1) \\ h_1(Mi) & h_1(Mi+1) & \cdots & h_1(Mi+M-1) \\ \vdots & \vdots & \cdots & \vdots \\ h_{M-1}(Mi) & h_{M-1}(Mi+1) & \cdots & h_{M-1}(Mi+M-1) \end{bmatrix}$$