LOSSY TO LOSSLESS IMAGE COMPRESSION USING ALLPASS FILTERS

Xi Zhang†

Dept. of Information & Communication Eng.
University of Electro-Communications
Chofu-shi, Tokyo 182-8585 Japan

K. Kawai, T. Yoshikawa, Y. Takei

Dept. of Electrical Engineering
Nagaoka University of Technology
Nagaoka, Niigata 940-2188 Japan

ABSTRACT

In this paper, an effective implementation of the allpass-based orthonormal symmetric wavelets is proposed for image compression. Since the orthonormal symmetric wavelets are used, it can be expected to get better compression performance than biorthogonal wavelets. Firstly, the implementation of irreversible real-to-real wavelets is presented and its decomposition process is shown by using allpass filters. Then, the realization of the reversible integer-to-integer wavelets is given by utilizing the invertible implementation of allpass filters. Finally, the coding performance of the orthonormal symmetric wavelets is evaluated and compared with the D-9/7 and D-5/3 wavelets. It is shown from the experimental results that the allpass-based orthonormal symmetric wavelets can achieve better compression performance than the D-9/7 and D-5/3 wavelets.

Keywords: Orthonormal symmetric wavelet, Lossy to lossless coding, Allpass filter, Invertible implementation.

1. INTRODUCTION

Wavelet-based image coding has been extensively studied in [1]–[13] and adopted in the international standard JPEG2000 [4], [14]. In the wavelet-based image coding, two-band PR (perfect reconstruction) filter banks play a very important role. The analysis and synthesis filters are required to have exactly linear phase responses (corresponding to symmetric wavelet bases), allowing us to use the symmetric extension method to accurately handle the boundaries of images. The wavelet filter banks should also be orthonormal to avoid redundancy between the subband images. Unfortunately, there are no nontrivial orthonormal symmetric wavelets with FIR filters, except for the Haar wavelet. To achieve better compression performance, a reasonable regularity is necessary for wavelet bases. Therefore, at least one of the above-mentioned conditions has to be given up to get more regularity than the Haar wavelet. For example, the D-9/7 and D-5/3 wavelets supported by the baseline codec of JPEG2000 are biorthogonal. On the other hand, it is known in [6] that IIR wavelet filters can simultaneously satisfy both of the orthonormality and symmetry. A class of IIR orthonormal symmetric wavelets has been proposed in [12] by using allpass filters.

In this paper, we apply the allpass-based orthonormal symmetric wavelets to image compression, and propose an effective implementation of the wavelet filter banks. Firstly, we present the implementation of irreversible real-to-real wavelets and show its decomposition process by using allpass filters. Then, we make use of the invertible implementation of allpass filters to realize the reversible integer-to-integer wavelets. Finally, we investigate the coding performance of the allpass-based orthonormal symmetric wavelets by using the reference software of JPEG2000 provided in [14], and compare the performance with the D-9/7 and D-5/3 wavelets. It is shown from the experimental results that the allpass-based orthonormal symmetric wavelets can achieve better compression performance than the D-9/7 and D-5/3 wavelets.

2. ORTHONORMAL SYMMETRIC WAVELETS

It is well-known [1]–[3] that wavelet bases can be generated by two-band PR filter banks $\{H(z), G(z)\}$, where $H(z)$ is a lowpass filter and $G(z)$ is highpass. The orthonormal filter banks $H(z)$ and $G(z)$ must satisfy

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 2 \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 2 \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0 \end{cases} . \tag{1}$$

If symmetric wavelets are needed, $H(z)$ and $G(z)$ must have exactly linear phase responses also. In [6] and [12], a class of orthonormal symmetric wavelets have been proposed by using allpass filters, i.e.,

$$\begin{cases} H(z) = \dfrac{1}{\sqrt{2}}\{z^{-2K-1}A(z^{-2}) + A(z^2)\} \\ G(z) = \dfrac{1}{\sqrt{2}}\{z^{-2K-1}A(z^{-2}) - A(z^2)\} \end{cases} , \tag{2}$$

where K is integer, and $A(z)$ is an allpass filter of order N and defined by

$$A(z) = z^{-N} \frac{\sum_{n=0}^{N} a_n z^n}{\sum_{n=0}^{N} a_n z^{-n}}, \tag{3}$$

where a_n is real and $a_0 = 1$. It can be easily verified that $H(z)$ and $G(z)$ in Eq.(2) satisfy the orthonormality condition in Eq.(1). Assume that $\theta(\omega)$ is the phase response of $A(z)$, that is,

$$\theta(\omega) = -N\omega + 2\tan^{-1} \frac{\sum_{n=0}^{N} a_n \sin n\omega}{\sum_{n=0}^{N} a_n \cos n\omega}, \tag{4}$$

† E-mail: xiz@ice.uec.ac.jp

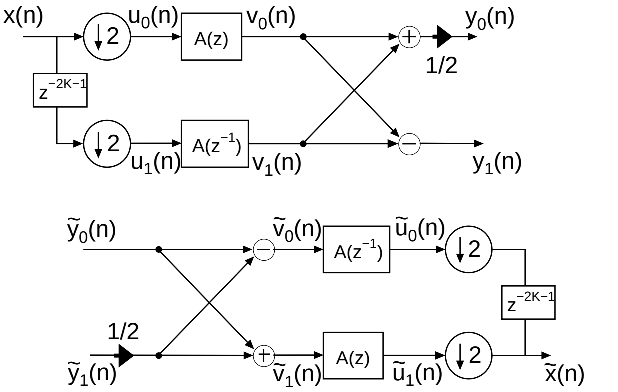

Fig. 1. Allpass-based wavelet filter bank.

then the frequency responses of $H(z)$ and $G(z)$ are

$$\begin{cases} H(e^{j\omega}) = e^{-j(K+\frac{1}{2})\omega}\sqrt{2}\cos\{\theta(2\omega) + (K+\frac{1}{2})\omega\} \\ G(e^{j\omega}) = -je^{-j(K+\frac{1}{2})\omega}\sqrt{2}\sin\{\theta(2\omega) + (K+\frac{1}{2})\omega\} \end{cases}, \tag{5}$$

which have exactly linear phase responses and satisfy the following power-complementary relation;

$$|H(e^{j\omega})|^2 + |G(e^{j\omega})|^2 = 2. \tag{6}$$

Therefore, the design problem becomes the phase approximation of the allpass filter $A(z)$, and has been discussed in [12]. It has also been pointed out in [12] that if $K = 4k+1$ or $4k+2$ when N is even and if $K = 4k$ or $4k+3$ when N is odd, then the magnitude response has an undesired zero and bump nearby $\omega = \pi/2$. To get a pair of reasonable lowpass and highpass filters, we should choose $K = 4k$ or $4k+3$ when N is even, and $K = 4k+1$ or $4k+2$ when N is odd, where $-(2N+1) \le K \le 2N$. See [12] in detail.

3. IMPLEMENTATION OF IRREVERSIBLE WAVELETS

In this section, we present an effective implementation of the irreversible real-to-real wavelets using allpass filters. Firstly, we assume that input signal is of length M, and $x(n)$ is a periodic signal obtained by employing symmetric extension at the boundaries of input signal, whose z transform is $X(z)$. In the following, the capital letter denotes the z transform of the signal. The allpass-based wavelet filter banks in Eq.(2) with a little modification can be realized by using the polyphase structure shown in Fig.1. This modification ensures that the maximum magnitudes of $H(z)$ and $G(z)$ are 1 and 2 in the passbands respectively, which are the same as the wavelet transforms supported by the baseline codec of JPEG2000, to avoid the dynamic range growth of the transform coefficients in successive lowpass decomposition [4], [14]. It is seen that $x(n)$ will be filtered by two allpass filters $A(z)$ and $A(z^{-1})$ after decimation. In the following, we will demonstrate the decomposition process with an example of $M = 8$ and $K = 0$. $x(n)$ is obtained by doubling the boundary points of input signal and its period is $2M$, as shown in Fig.2. Firstly, $x(n)$ is decimated to get $u_0(n)$ and $u_1(n)$. It is seen in Fig.2 that $u_0(n)$ and $u_1(n)$ are periodic with period M and satisfy the following symmetric relation;

$$u_0(n) = u_1(M-1-n), \tag{7}$$

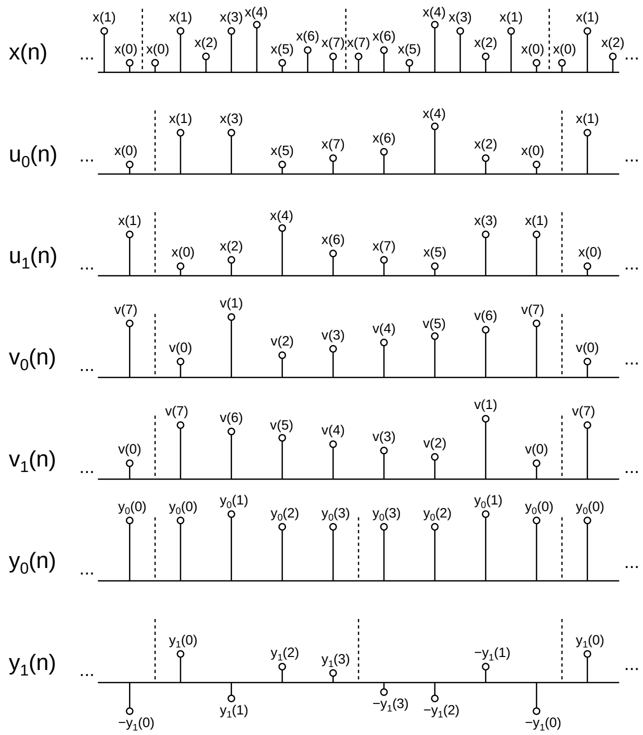

Fig. 2. Decomposition process.

that is,

$$U_0(z) = z^{-M+1}U_1(z^{-1}). \tag{8}$$

It should be noted that when $K \ne 0$, the symmetric relation still holds, although the symmetric point is different. $u_0(n)$ and $u_1(n)$ are then filtered by $A(z)$ and $A(z^{-1})$ to get $v_0(n)$ and $v_1(n)$ respectively;

$$\begin{cases} V_0(z) = U_0(z)A(z) \\ V_1(z) = U_1(z)A(z^{-1}) \end{cases}. \tag{9}$$

It follows from Eqs.(8) and (9) that $v_0(n)$ and $v_1(n)$ satisfy the symmetric relation also;

$$V_0(z) = z^{-M+1}V_1(z^{-1}), \tag{10}$$

that is,

$$v_0(n) = v_1(M-1-n). \tag{11}$$

Thus, the subband signals $y_0(n)$ and $y_1(n)$ can be obtained by

$$\begin{cases} y_0(n) = \frac{1}{2}[v_0(n) + v_1(n)] = \frac{1}{2}[v_0(n) + v_0(M-1-n)] \\ y_1(n) = v_1(n) - v_0(n) = v_0(M-1-n) - v_0(n) \end{cases}, \tag{12}$$

where $y_0(n)$ and $y_1(n)$ are symmetric and antisymmetric respectively. Therefore, only M samples of $v_0(n)$ are needed to get $M/2$ samples of $y_0(n)$ and $y_1(n)$. To sum up, the decomposition process is composed of arranging the input signal in odd- and even-index order to get $u_0(n)$ as shown in Fig.2, filtering it with $A(z)$ to output $v_0(n)$, and then adding or subtracting $v_0(n)$ to obtain $y_0(n)$ and $y_1(n)$ as shown in Eq.(12). The reconstruction can be done in the reversed order, and it is omitted here.

In general, $A(z)$ has its poles inside and outside the unit circle, and then can be divided into the causal part $A_c(z)$ and anti-causal part $A_a(z)$, that is,

$$A(z) = A_c(z)A_a(z), \tag{13}$$

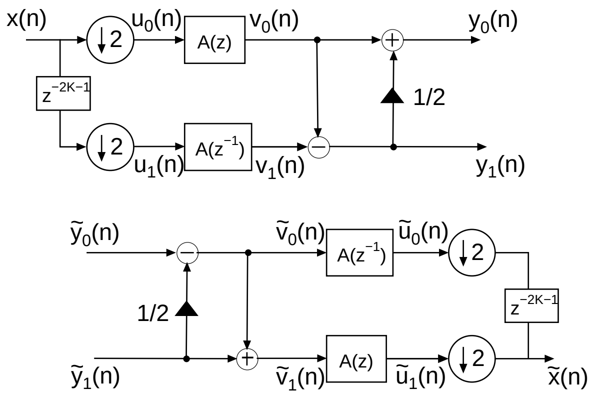

Fig. 3. Reversible realization.

where $A_c(z)$ and $A_a(z)$ have the poles only inside and outside the unit circle respectively. For the causal part $A_c(z)$ of order N_1 with input $x(n)$ and output $y(n)$, since $a_0 = 1$, its input-output relation is given by

$$y(n) = x(n - N_1) + \sum_{i=1}^{N_1} a_i[x(n + i - N_1) - y(n - i)], \quad (14)$$

where only N_1 multipliers are needed. Since the input signal is periodic, some initial values are needed for starting the processing. We use the method proposed in [13] to calculate the initial values. The anti-causal part $A_a(z)$ is realized by reversing the input signal, filtering it with $A_a(z^{-1})$ that has only the poles inside the unit circle, and then re-reversing the output signal.

4. REALIZATION OF REVERSIBLE WAVELETS

In this section, we consider the realization of the reversible integer-to-integer wavelets. In most of the cases, $A(z)$ has floating point coefficients. Since the input images are matrices of integer values, the filtered output no longer consists of integers. For lossless compression, it is necessary to make an invertible mapping from an integer input to an integer wavelet coefficient. To obtain an integer output, we revise Eq.(14) as follows;

$$y(n) = x(n - N_1) + \lfloor \sum_{i=1}^{N_1} a_i[x(n + i - N_1) - y(n - i)] + 0.5 \rfloor, \quad (15)$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. Therefore, we can get an integer output $y(n)$ for $n = 0, 1, \cdots, M-1$ by using the initial values $\{y(-1), y(-2), \cdots, y(-N_1)\}$ obtained by the method in [13]. To recover $x(n)$ from $y(n)$, we have from Eq.(15)

$$x(n - N_1) = y(n) - \lfloor \sum_{i=1}^{N_1} a_i[x(n + i - N_1) - y(n - i)] + 0.5 \rfloor. \quad (16)$$

This means that if all of $y(n)$ and some of $x(n)$, e.g., $\{x(M - 1), x(M - 2), \cdots, x(M - N_1)\}$ are known a priori, we can exactly reconstruct $x(n)$ for $n = M - N_1 - 1, M - N_1 - 2, \cdots, 0$. In lossless coding, $y(n)$ is transmitted to the decoder without rounding error. However, it is generally difficult to recover $\{x(M - 1), x(M - 2), \cdots, x(M - N_1)\}$ only from $y(n)$. Thus, $\{x(M - 1), x(M - 2), \cdots, x(M - N_1)\}$ are also needed to transmit as a side information. By using the transmitted side information, we can realize an invertible allpass filter with Eqs.(15) and (16). The amount of the side information is relatively small since allpass filters used in image coding are of low order. Moreover, we can calculate the prediction values of $\{x(M-1), x(M-2), \cdots, x(M-N_1)\}$ from $y(n)$ by using the method for calculating the initial value proposed in [13], then only the difference between the prediction and actual values needs to be transmitted. After the invertible allpass filtering, we can get an integer output $y_1(n)$ from Eq.(12), but $y_0(n)$ is no longer integer since divided by two. We rewrite Eq.(12) as

$$y_0(n) = \frac{1}{2}[v_0(n) + v_1(n)] = v_0(n) + \frac{y_1(n)}{2}. \quad (17)$$

Therefore, the allpass-based wavelet filter bank shown in Fig.1 is modified to get the structure shown in Fig.3, which corresponds to the well-known lifting scheme proposed in [8], [9]. By quantizing output of the multiplier $1/2$, we obtain an integer output $y_0(n)$;

$$y_0(n) = v_0(n) + \lfloor \frac{y_1(n)}{2} \rfloor. \quad (18)$$

The inverse operation is straightforward as shown in Fig.3.

Table 1. Lossless coding results: Bit Rate (bpp)

	D-5/3	N1-K1	N2-K3	N3-K1
Barbara	4.694	4.587	4.532	**4.511**
Boat	4.438	4.438	**4.423**	4.464
Crowd	4.234	4.254	**4.218**	4.320
Lena	4.348	4.349	**4.330**	4.358
Mandrill	6.149	6.130	6.124	**6.125**
Pepper	**4.653**	4.661	**4.653**	4.690
Woman	**3.345**	3.366	3.347	3.453
Zelda	4.019	3.992	**3.966**	3.994
Average	4.485	4.473	**4.449**	4.489

5. IMAGE CODING APPLICATION

In this section, we investigate the compression performance of three allpass-based orthonormal symmetric wavelets with $\{N = 1, K = 1\}$, $\{N = 2, K = 3\}$ and $\{N = 3, K = 1\}$. Two wavelet filter banks with $\{N = 1, K = 1\}$ and $\{N = 2, K = 3\}$ have all poles inside the unit circle, thus need not be divided into the causal and anti-causal parts. The reference software of JPEG2000 provided in [14] has been used to evaluate the coding performance. Eight images (Barbara, Boat, Crowd, Lena, Mandrill, Pepper, Woman and Zelda) of size 512×512, 8 bpp are used as test images, and the decomposition level of the wavelet transform is set to 6.

5.1. Lossy coding performance

We examine the lossy coding performance of three irreversible real-to-real wavelets. The distortion is measured by the peak signal to noise ratio (PSNR) between the original and reconstructed images. The lossy coding results for images Barbara and Lena are given in Fig.4 and Fig.5, respectively. It is seen in Fig.4 that the allpass-based orthonormal symmetric wavelets have better lossy coding performance than the D-9/7 wavelet for image Barbara, while almost same results are gotten for image Lena, as shown in Fig.5.

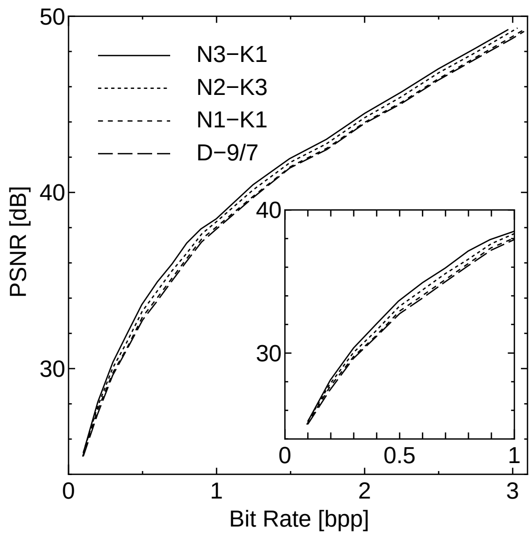

Fig. 4. Lossy coding results of irreversible wavelets for Barbara.

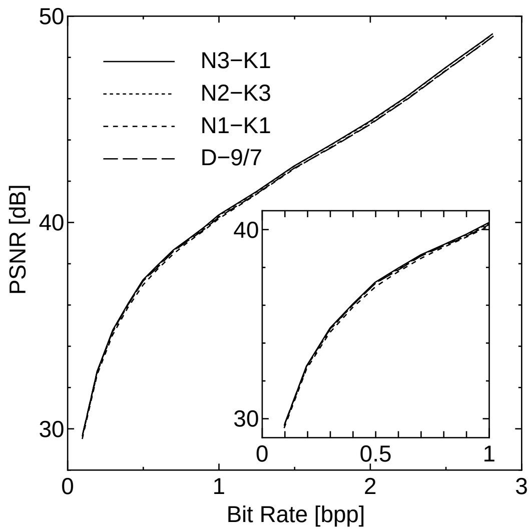

Fig. 5. Lossy coding results of irreversible wavelets for Lena.

5.2. Lossless coding performance

we investigate the lossless coding performance of three reversible integer-to-integer wavelets. The lossless coding results with the comparison with the D-5/3 wavelet are given in Table 1. For each image, the best result has been highlighted. It is seen in Table 1 that the allpass-based orthonormal symmetric wavelets have better average lossless coding performance than the D-5/3 wavelet, and the wavelet of $\{N = 2, K = 3\}$ is the best, although there are two images getting the best results for the D-5/3 wavelet.

6. CONCLUSION

In this paper, we have proposed an effective implementation of the allpass-based orthonormal symmetric wavelets for lossy to lossless image compression. Firstly, we have discussed the implementation of irreversible real-to-real wavelets and shown its decomposition process by using allpass filters. Then, we have presented the invertible implementation of allpass filters to realize the reversible integer-to-integer wavelets. Finally, we have investigated the compression performance of the allpass-based orthonormal symmetric wavelets by using the reference software of JPEG2000, and compared the performance with the D-9/7 and D-5/3 wavelets. It have been shown from the experimental results that the allpass-based orthonormal symmetric wavelets can achieve better compression performance than the D-9/7 and D-5/3 wavelets.

7. REFERENCES

[1] I. Daubechies, "Ten Lectures on Wavelets," SIAM, Philadelphia, 1992.

[2] M. Vetterli and J.Kovacevic, "Wavelets and Subband Coding," Prentice Hall PRT, New Jersey, 1995.

[3] A. N. Akansu and M. J. T. Smith, "Subband and Wavelet Transforms: Design and Applications," Kluwer Academic Publishers, Boston, 1996.

[4] D. S. Taubman and M. W. Marcellin, "JPEG2000: Image Compression Fundamentals, Standards and Practice," Kluwer Academic Publishers, Boston, 2002.

[5] M. Antonini, M. Barlaud, P. Mathieu and I. Daubechies, "Image coding using wavelet transform," IEEE Trans. Image Processing, Vol.1, No.2, pp.205–220, 1992-04.

[6] C. Herley and M. Vetterli, "Wavelets and recursive filter banks," IEEE Trans. Signal Processing, Vol.41, No.8, pp.2536–2556, 1993-08.

[7] A. Said and W. A. Pearlman, "An image multiresolution representation for lossless and lossy compression," IEEE Trans. Image Processing, Vol.5, No.9, pp.1303–1310, 1996-09.

[8] W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," Appl. Comput. Harmon. Anal., Vol.3, No.2, pp.186–200, 1996.

[9] W. Sweldens, "The lifting scheme: A construction of second generation wavelets," SIAM J.Math. Anal., Vol.29, No.2, pp.511–546, 1997.

[10] A. R. Calderbank, I. Daubechies, W. Sweldens and B. L. Yeo, "Wavelet transforms that map integers to integers," Appl. Comput. Harmon. Anal., Vol.5, No.3, pp.332–369, 1998.

[11] M. D. Adams and F. Kossentini, "Reversible integer-to-integer wavelet transforms for image compression: Performance evaluation and analysis," IEEE Trans. Image Processing, Vol.9, No.6, pp.1010–1024, 2000-06.

[12] X. Zhang, T. Muguruma and T. Yoshikawa, "Design of orthonormal symmetric wavelet filters using real allpass filters," Signal Processing, Vol.80, No.8, pp.1551–1559, 2000-08.

[13] S. Kamimura, X. Zhang and T. Yoshikawa, "Wavelet-based image coding using allpass filters," Electronics and Communications in Japan, Parts 3, Vol.85, No.2, pp.13–21, 2002-02.

[14] ISO/IEC 15444: Information technology — JPEG 2000 image coding system, 2000.