LOSSLESS IMAGE COMPRESSION USING 2D ALLPASS FILTERS

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ABSTRACT

In this paper, a reversible integer-to-integer wavelet transform based on non-separable 2D allpass filters is proposed for lossless image coding. The number of rounding operations included in the reversible wavelet transform is reduced by using non-separable 2D allpass filters, thus it is expected to get better coding performance. The lossless coding performance of the proposed reversible wavelet transform is evaluated and compared with the conventional separable wavelet transforms. It is shown from the experimental results that the proposed non-separable 2D reversible wavelet tranform can achieve better lossless coding performance than the conventional separable wavelet transforms, including the D-5/3 wavelet tranform in the JPEG 2000.

Keywords: Reversible wavelet transform, Lossless compression, Orthonormal symmetric wavelet, Allpass filter.

1. INTRODUCTION

Wavelet-based image coding has been extensively studied in [1]–[16] and adopted in the international standard JPEG 2000 [4], [16]. In the wavelet-based image coding, two-band wavelet filter banks play a very important role, which are required to be orthonormal and symmetric. For example, the D-9/7 and D-5/3 wavelet transforms in the JPEG 2000 are symmetric and near orthogonal (biorthogonal). It is known in [6] and [12] that both the orthonormality and symmetry of wavelets can be simultaneously satisfied by using allpass filters. It has been shown in [13] and [14] that this class of allpass-based orthonormal symmetric wavelet filters have better lossy and lossless coding performance than the D-9/7 and D-5/3 wavelet transforms.

In the image coding, 1D wavelet transform is typically applied to images horizontally and vertically to realize 2D transform. This is separable 2D wavelet transform. It has been shown in [15] that by using non-separable 2D wavelet filter banks, the number of rounding operations included in the reversible wavelet transform and the error due to the rounding operations can be reduced, then the coding performance can be improved also.

In this paper, we extend the allpass-based 1D orthonormal symmetric wavelet transform to non-separable 2D case, and propose a reversible integer-to-integer wavelet transform based on non-separable 2D allpass filters. We show that the number of rounding operations can be reduced by using non-separable 2D allpass filters. Also, we investigate the lossless coding performance of the proposed reversible wavelet transform by using the reference software of JPEG 2000 provided in [16], and compare the coding performance with the conventional separable wavelet transforms. It is shown from the experimental results that the proposed non-separable reversible wavelet tranform can achieve better lossless coding performance than the conventional separable wavelet transforms, including the D-5/3 wavelet transform in the JPEG 2000.

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2. ORTHONORMAL SYMMETRIC WAVELETS

It is well-known [1]–[3] that wavelet bases can be generated by two-band filter banks $\{H(z), G(z)\}$, where $H(z)$ is a lowpass filter and $G(z)$ is highpass. The orthonormal filter banks $H(z)$ and $G(z)$ must satisfy

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 2 \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 2 \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0 \end{cases} . \quad (1)$$

If symmetric wavelets are needed, $H(z)$ and $G(z)$ must have exactly linear phase responses also. In [6] and [12], a class of orthonormal symmetric wavelet filters have been proposed by using allpass filters, i.e.,

$$\begin{cases} H(z) = \frac{1}{\sqrt{2}}\{z^{-2K-1}A(z^{-2}) + A(z^2)\} \\ G(z) = \frac{1}{\sqrt{2}}\{z^{-2K-1}A(z^{-2}) - A(z^2)\} \end{cases} , \quad (2)$$

where K is integer, and $A(z)$ is an allpass filter of order N and defined by

$$A(z) = z^{-N} \frac{\sum_{n=0}^{N} a_n z^n}{\sum_{n=0}^{N} a_n z^{-n}}, \quad (3)$$

where a_n is real and $a_0 = 1$. It can be easily verified that $H(z)$ and $G(z)$ in Eq.(2) satisfy the orthonormality condition in Eq.(1). Note that since $A(z)$ and $A(z^{-1})$ are used in Eq.(2), $H(z)$ and $G(z)$ are not causal, which is not a problem in image processing.

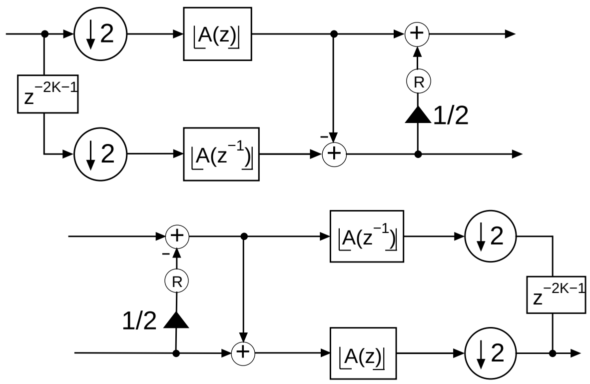

Fig. 1. Allpass-based 1D reversible wavelet filters.

Assume that $\theta(\omega)$ is the phase response of $A(z)$, that is,

$$\theta(\omega) = -N\omega + 2\tan^{-1}\frac{\sum_{n=0}^{N} a_n \sin n\omega}{\sum_{n=0}^{N} a_n \cos n\omega}, \quad (4)$$

then the frequency responses of $H(z)$ and $G(z)$ are

$$\begin{cases} H(e^{j\omega}) = e^{-j(K+\frac{1}{2})\omega}\sqrt{2}\cos\{\theta(2\omega) + (K+\frac{1}{2})\omega\} \\ G(e^{j\omega}) = -je^{-j(K+\frac{1}{2})\omega}\sqrt{2}\sin\{\theta(2\omega) + (K+\frac{1}{2})\omega\} \end{cases}, \quad (5)$$

which have exactly linear phase responses. Therefore, the design problem can be reduced to the phase approximation of the allpass filter $A(z)$, and has been discussed in [12]. It has also been pointed out in [12] that if $K = 4k+1$ or $4k+2$ when N is even and if $K = 4k$ or $4k+3$ when N is odd, then the magnitude response has an undesired zero and bump nearby $\omega = \pi/2$. Therefore, to get a pair of reasonable lowpass and highpass filters, we should choose $K = 4k$ or $4k+3$ when N is even, and $K = 4k+1$ or $4k+2$ when N is odd, where $-(2N+1) \leq K \leq 2N$. See [12] in detail.

3. 1D REVERSIBLE WAVELET TRANSFORM

In this section, we describe the realization of 1D reversible integer-to-integer wavelet transform by using allpass filters. To avoid the dynamic range growth of the transform coefficients in successive lowpass decomposition, the maximum magnitudes of lowpass and highpass filters are required to be 1 and 2 in the passbands, respectively, which are the same as the wavelet transforms supported by JPEG2000. Therefore, we revise Eq.(2) as follows;

$$\begin{bmatrix} \tilde{H}(z) \\ \tilde{G}(z) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} A(z^2) & 0 \\ 0 & A(z^{-2}) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-2K-1} \end{bmatrix}, \quad (6)$$

where $\tilde{H}(z) = \frac{1}{\sqrt{2}}H(z)$ and $\tilde{G}(z) = \sqrt{2}G(z)$. Note that $\tilde{H}(z)$ and $\tilde{G}(z)$ have the maximum passband gains of 1 and 2, respectively. It is known that

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad (7)$$

which corresponds to the well-known lifting scheme proposed in [8] and [9]. Therefore, we can obtain output of integer values by adding a rounding operation after the multiplier with the coefficient $1/2$ as shown in Fig.1. The inverse transform is straightforward.

Next, we consider the invertible realization of the allpass filter $A(z)$. In most of the cases, $A(z)$ has floating point coefficients. Although the input images are matrices of integer values, the output filtered with $A(z)$ no longer consists of integers. In lossless coding, it is necessary to make an invertible mapping from an integer input to an integer wavelet coefficient. For a causal stable $A(z)$ with input $x(n)$ and output $y(n)$, we insert a rounding operation in the filtering process and give its input-output relationship as follows;

$$y(n) = x(n-N) + \lfloor \sum_{i=1}^{N} a_i[x(n+i-N) - y(n-i)] + 0.5 \rfloor \quad (n = 0, 1, \cdots, L-1), \quad (8)$$

where L is the length of $x(n)$, and $\lfloor x \rfloor$ denotes the largest integer not greater than x. It should be noted in Eq.(8) that some initial values $y(-1), \cdots, y(-N)$ are needed, and can be calculated by using the method proposed in [13], while $x(-1), \cdots, x(-N)$ is obtained by using the symmetric extension. Therefore, we can get an integer output $y(n)$ by using Eq.(8). To recover $x(n)$ from $y(n)$, we have from Eq.(8)

$$x(n-N) = y(n) - \lfloor \sum_{i=1}^{N} a_i[x(n+i-N) - y(n-i)] + 0.5 \rfloor \quad (n = L-1, L-2, \cdots, N), \quad (9)$$

which is an invertible realization of $A(z^{-1})$, and must be processed in reverse order of n, since $A(z^{-1})$ is anti-causal stable. It is seen in Eq.(9) that if all of $y(n)$ and some of $x(n)$, for example, $x(L-1), \cdots, x(L-N)$ are known a priori, we can exactly reconstruct $x(n)$ by using Eq.(9). In lossless coding, $y(n)$ is transmitted to the decoder without any error. Thus, some of $x(n)$ also need to be transmitted as a side information. In stead of $x(n)$, we firstly calculate some prediction values of $x(n)$ from $y(n)$ by using the method calculating the initial values in [13], and then transmit the difference between the prediction and actual values of some $x(n)$, for example, $x(L-1), \cdots, x(L-N)$. Compared with the transmission of $x(L-1), \cdots, x(L-N)$, the amount of the side information of its difference is relatively small. Therefore, by making use of the transmitted side information, we can realize the invertible allpass filter with Eqs.(8) and (9). See [14] in detail.

4. 2D REVERSIBLE WAVELET TRANSFORM

In this section, we propose a non-separable 2D reversible integer-to-integer wavelet transform. The conventional 2D wavelet transform is generally realized by applying 1D transform horizontally and vertically to images, as shown in Fig.2, where $\mathbf{M_1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{M_2} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. The transfer functions of four 2D filters $H_{LL}(z_1, z_2), H_{LH}(z_1, z_2), H_{HL}(z_1, z_2), H_{HH}(z_1, z_2)$ in Fig.2 are given by

$$\begin{cases} H_{LL}(z_1, z_2) = \tilde{H}(z_1)\tilde{H}(z_2) \\ H_{LH}(z_1, z_2) = \tilde{H}(z_1)\tilde{G}(z_2) \\ H_{HL}(z_1, z_2) = \tilde{G}(z_1)\tilde{H}(z_2) \\ H_{HH}(z_1, z_2) = \tilde{G}(z_1)\tilde{G}(z_2) \end{cases}, \quad (10)$$

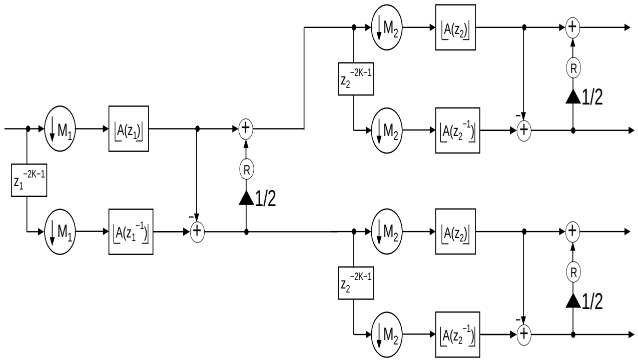

Fig. 2. Separable 2D reversible wavelet filters.

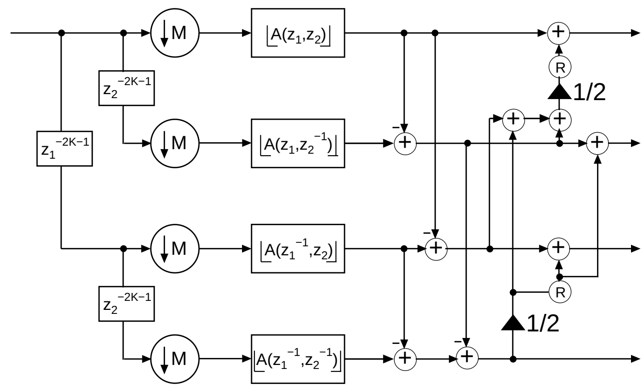

Fig. 3. Non-separable 2D reversible wavelet filters.

which are separable 2D wavelet filters. It is clear in Fig.2 that the separable 2D wavelet filters based on 1D allpass filters have a total of 9 rounding operations, including ones inserted in 1D allpass filters. It has been proposed in [15] that the number of rounding operations can be reduced by using non-separable 2D wavelet filters, thus the coding performance improvement is confirmed by applying the D-5/3 wavelet transform. In the following, we describe a realization of non-separable 2D wavelet filters by using non-separable 2D allpass filters.

By using the polyphase representation, we have

$$\begin{bmatrix} H_{LL}(z_1,z_2) \\ H_{LH}(z_1,z_2) \\ H_{HL}(z_1,z_2) \\ H_{HH}(z_1,z_2) \end{bmatrix} = \mathbf{C}\ \mathbf{A} \begin{bmatrix} 1 \\ z_2^{-2K-1} \\ z_1^{-2K-1} \\ z_1^{-2K-1} z_2^{-2K-1} \end{bmatrix}, \tag{11}$$

where

$$\mathbf{C} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} A(z_1^2,z_2^2) & 0 & 0 & 0 \\ 0 & A(z_1^2,z_2^{-2}) & 0 & 0 \\ 0 & 0 & A(z_1^{-2},z_2^2) & 0 \\ 0 & 0 & 0 & A(z_1^{-2},z_2^{-2}) \end{bmatrix}$$

and $A(z_1,z_2)$ is defined by[1]

$$A(z_1,z_2) = A(z_1)A(z_2) = z_1^{-N} z_2^{-N} \frac{\sum_{m,n=0}^{N} a_{mn} z_1^m z_2^n}{\sum_{m,n=0}^{N} a_{mn} z_1^{-m} z_2^{-n}}, \tag{12}$$

where $a_{mn} = a_m a_n$ and $a_{00} = a_0^2 = 1$. It is known in [8] and [9] that the matrix $\mathbf{C}$ can be decomposed into the lifting scheme. There exist many kinds of decompositions. To minimize the number of rounding operations, we propose the following decomposition;

$$\mathbf{C} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{13}$$

Thus, $\mathbf{C}$ can be realized by using the lifting structure and is shown in Fig.3, where $\mathbf{M} = \mathbf{M_1 M_2} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

Next, we consider the direct implementation of 2D allpass filter $A(z_1,z_2)$, in stead of the cascade structure of 1D allpass filters. For the causal stable $A(z_1,z_2)$ with input $x(n_1,n_2)$ and output $y(n_1,n_2)$, we insert a rounding operation in the 2D filtering process, similarly to the case of 1D allpass filters. Its input-output relationship is given as follows;

$$y(n_1,n_2) = x(n_1-N,n_2-N) + \lfloor \sum_{\substack{i,j=0 \\ i\neq 0 \cup j\neq 0}}^{N} a_{ij}[x(n_1+i-N,n_2+j-N) - y(n_1-i,n_2-j)] + 0.5 \rfloor$$

$$(n_1 = 0,1,\cdots,L_1-1,\ n_2 = 0,1,\cdots,L_2-1), \tag{14}$$

where the size of image is $L_1 \times L_2$. Therefore, an integer output $y(n_1,n_2)$ can be obtained by Eq.(14). It is clear that only one rounding operation is needed in Eq.(14), whereas if the cascade structure of 1D filters is used, then the number of rounding operations is two. Its inverse operation, that is, the invertible realization of $A(z_1^{-1},z_2^{-1})$ is given by

$$x(n_1-N,n_2-N) = y(n_1,n_2) - \lfloor \sum_{\substack{i,j=0 \\ i\neq 0 \cup j\neq 0}}^{N} a_{ij}[x(n_1+i-N,n_2+j-N) - y(n_1-i,n_2-j)] + 0.5 \rfloor$$

$$(n_1 = L_1-1, L_1-2,\cdots,N,\ n_2 = L_2-1, L_2-2,\cdots,N). \tag{15}$$

Therefore, a non-separable 2D reversible wavelet transform is realized, as shown in Fig.3. It is clear in Fig.3 that the number of rounding operations needed in the proposed non-separable 2D reversible wavelet filters is 6, while the separable 2D wavelet filters in Fig.2 have 9 rounding operations, thus it is reduced by $1/3$.

[1] It is possible for $A(z_1,z_2)$ to have a more general form and different degrees of z_1 and z_2.

Table 1. Lossless coding results: Bit Rate (bpp)

Image	F_{1s}	F_{1n}	F_{2s}	F_{2n}	D-5/3
Barbara	4.588	4.581	4.501	**4.486**	4.691
Boat	4.436	4.423	4.418	**4.403**	4.434
Crowd	4.242	4.229	4.207	**4.188**	4.229
Goldhill	4.888	4.881	4.877	4.870	**4.868**
Lena	4.343	4.336	4.321	**4.312**	4.344
Man	4.739	4.731	4.725	**4.715**	4.726
Mandrill	6.142	6.135	6.122	**6.118**	6.145
Pepper	4.654	4.647	4.654	**4.639**	4.649
Woman	3.362	3.336	3.337	**3.305**	3.342
Zelda	3.990	3.974	3.959	**3.943**	4.015
Average	4.538	4.527	4.512	**4.498**	4.544

5. LOSSLESS CODING PERFORMANCE

In this section, we investigate the lossless coding performance of the proposed non-separable 2D reversible wavelet transform, and compare it with the conventional separable 2D wavelet transform and the D-5/3 wavelet in the JPEG 2000. The reference software of JPEG2000 provided in [16] has been used to evaluate the lossless coding performance. Ten images (Barbara, Boat, Crowd, Goldhill, Lena, Man, Mandrill, Pepper, Woman and Zelda) of size 512×512, 8 bpp have been used as test images, and the decomposition level of the wavelet transform is set to 4. The lossless coding results with the comparison with the D-5/3 wavelet are given in Table 1. For each image, the best result has been highlighted. In Table 1, F_{Ns} and F_{Nn} denote the separable and non-separable 2D reversible wavelet filters with the allpass filter $A(z)$ of order N, respectively. Two allpass filters with $\{N = 1, K = 1\}$ and $\{N = 2, K = 3\}$ proposed in [13] and [14] have been used, which are causal stable. It is seen in Table 1 that the proposed non-separable 2D reversible wavelet filters F_{Nn} have better lossless coding performance than the separable filters F_{Ns}, and an improvement of about 0.01 bpp is obtained, where $N = 1$ or 2. Moreover, the proposed wavelet filter F_{2n} has the best lossless coding performance, although there is one image (Goldhill) getting the best result for the D-5/3 wavelet.

6. CONCLUSION

In this paper, we have proposed a non-separable 2D reversible integer-to-integer wavelet transform by using non-separable 2D allpass filters. By extending the allpass-based 1D orthonormal symmetric wavelet transform to non-separable 2D case, the number of rounding operations included in the reversible wavelet transform and the error due to the rounding operations can be reduced. Also, we have investigated the lossless coding performance of the proposed non-separable reversible wavelet filters, and compared it with the conventional separable wavelet transforms. It is shown that the proposed non-separable 2D reversible wavelet filters have better lossless coding performance than the conventional separable transforms and the D-5/3 wavelet in the JPEG 2000. Furthermore, a theoretical analysis will be done in the near future.

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