LOSSY TO LOSSLESS IMAGE CODING USING A COMPLEX ALLPASS FILTER

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ABSTRACT

Since wavelet filters composed of a single complex allpass filter satisfy both of the conditions of orthonormality and symmetry, it is expected to obtain better coding performance than biorthogonal wavelets in image coding. This paper proposes an effective implementation of the orthonormal symmetric wavelets composed of a complex allpass filter for image coding. Firstly, an implementation of irreversible real-to-real wavelets for lossy coding is presented by realizing the complex allpass filter. Then, the reversible integer-to-integer wavelets is realized by utilizing the invertible implementation of complex allpass filter. Finally, the coding performance of the orthonormal symmetric wavelets is evaluated and compared with the D-9/7 and D-5/3 wavelets. It is shown from the experimental results that the allpass-based orthonormal symmetric wavelets can achieve better coding performance than the D-9/7 and D-5/3 wavelets.

Keywords: Orthonormal symmetric wavelet, Lossy to lossless coding, Complex allpass filter, Invertible implementation.

1. INTRODUCTION

Wavelet-based image coding has been extensively studied in [1] $\sim$ [12] and adopted in the international standard JPEG2000 [4], [14]. In the wavelet-based image coding, two-band PR (perfect reconstruction) filter banks play a very important role. The analysis and synthesis filters are required to have exactly linear phase responses (corresponding to symmetric wavelet bases), allowing us to use the symmetric extension method to accurately handle the boundaries of images. The wavelet filters should also be orthonormal to avoid redundancy between the subband images. Unfortunately, there are no nontrivial orthonormal symmetric wavelets with FIR filters, except for the Haar wavelet. To achieve better coding performance, a reasonable regularity is necessary for wavelet bases. Therefore, at least one of the above-mentioned conditions has to be given up to get more regularity than the Haar wavelet. For example, the D-9/7 and D-5/3 wavelets supported by the baseline codec of JPEG2000 are biorthogonal. On the other hand, it is known in [8] that IIR wavelet filters can simultaneously satisfy both of the orthonormality and symmetry. A class of orthonormal symmetric wavelets has been proposed in [13] by using a complex allpass filter.

In this paper, we propose an effective implementation of the orthonormal symmetric wavelets composed of a single complex allpass filter for image compression. Firstly, we present an implementation of irreversible real-to-real wavelets for lossy coding by realizing the complex allpass filter. Then, we give the invertible implementation of complex allpass filter to realize the reversible integer-to-integer wavelets for lossless coding. Finally, we investigate the coding performance of the allpass-based orthonormal symmetric wavelets by using the reference software of JPEG2000 provided in [14], and compare the coding performance with the D-9/7 and D-5/3 wavelets. It is shown from the experimental results that the allpass-based orthonormal symmetric wavelets can achieve better lossy to lossless coding performance than the D-9/7 and D-5/3 wavelets.

2. ORTHONORMAL SYMMETRIC WAVELETS

It is well-known [1] $\sim$ [3] that wavelet bases can be generated by two-band PR filter banks $\{H(z), G(z)\}$, where $H(z)$ is a lowpass filter and $G(z)$ is highpass. The orthonormal filter banks $H(z)$ and $G(z)$ must satisfy

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 2 \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 2 \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0 \end{cases}. \tag{1}$$

If symmetric wavelets are needed, $H(z)$ and $G(z)$ must have exactly linear phase responses also. In [13], a class of orthonormal symmetric wavelets have been proposed by using a single complex allpass filter, i.e.,

$$\begin{cases} H(z) = \dfrac{1}{\sqrt{2}}\{A(z) + \tilde{A}(z)\} \\ G(z) = \dfrac{z^{-1}}{\sqrt{2}j}\{A(z) - \tilde{A}(z)\} \end{cases}, \tag{2}$$

where $A(z)$ is a complex allpass filter of order $2N$ and defined by

$$A(z) = e^{j\eta}\frac{a_0 z^N + ja_1 z^{N-1} + \cdots + ja_1 z^{1-N} + a_0 z^{-N}}{a_0 z^N - ja_1 z^{N-1} + \cdots - ja_1 z^{1-N} + a_0 z^{-N}}, \tag{3}$$

where $\eta = \pm\frac{\pi}{4}$ or $\pm\frac{3\pi}{4}$, a_n are real and $a_0 = 1$. $\tilde{A}(z)$ has a set of coefficients that are complex conjugate with ones of $A(z)$.

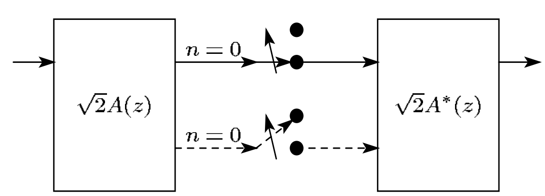

Fig. 1. Wavelet filter bank composed of a complex allpass filter.

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It can be easily verified that $H(z)$ and $G(z)$ in Eq.(2) satisfy the orthonormality condition in Eq.(1). Assume that $\theta(\omega)$ is the phase response of $A(z)$, that is,

$$\theta(\omega) = \eta + 2\varphi(\omega), \tag{4}$$

where if N is even,

$$\varphi(\omega) = \tan^{-1} \frac{\displaystyle\sum_{n=0}^{N/2-1} a_{2n+1} \cos(N-2n-1)\omega}{\displaystyle\frac{a_N}{2} + \sum_{n=0}^{N/2-1} a_{2n} \cos(N-2n)\omega}, \tag{5}$$

and if N is odd,

$$\varphi(\omega) = \tan^{-1} \frac{\displaystyle\frac{a_N}{2} + \sum_{n=1}^{(N-1)/2} a_{2n-1} \cos(N-2n+1)\omega}{\displaystyle\sum_{n=0}^{(N-1)/2} a_{2n} \cos(N-2n)\omega}, \tag{6}$$

then the frequency responses of $H(z)$ and $G(z)$ are given by

$$\begin{cases} H(e^{j\omega}) = \sqrt{2}\cos\theta(\omega) \\ G(e^{j\omega}) = e^{-j\omega}\sqrt{2}\sin\theta(\omega) \end{cases}, \tag{7}$$

which have exactly linear phase responses and satisfy the following power-complementary relation;

$$|H(e^{j\omega})|^2 + |G(e^{j\omega})|^2 = 2. \tag{8}$$

Therefore, the design problem becomes the phase approximation of the complex allpass filter $A(z)$, and has been discussed in [13]. In [13], the closed-form solution for the maximally flat wavelet filters has been given by

$$a_n = \begin{cases} \binom{2N}{n} & (n : \text{even}) \\ -\binom{2N}{n}\tan\frac{\eta}{2} & (n : \text{odd}) \end{cases} \tag{9}$$

Moreover, the design method for the wavelet filters with a given degrees of flatness has been also proposed by using the Remez exchange algorithm. See [13] in detail.

3. IMPLEMENTATION OF IRREVERSIBLE WAVELETS

In this section, we present an effective implementation of the irreversible real-to-real wavelets using a complex allpass filter. Firstly, we assume that input signal is of length M, and it is extended to a periodic signal by employing symmetric extension at the boundaries. It is seen in Fig.1 that the input signal will be filtered by the complex allpass filter $A(z)$. Since the filter coefficients are symmtric, i.e., $a_n = a_{2N-n}$, $A(z)$ can be divided into the causal-stable part $A_S(z)$ and its inverse $A_S(z^{-1})$ as follows;

$$\sqrt{2}A(z) = (1 \pm j)A_S(z)A_S(z^{-1}), \tag{10}$$

where $A_S(z)$ has the poles inside the unit circle only, and its transfer function is given by

$$A_S(z) = \frac{z^{-N} + \displaystyle\sum_{k=1}^{N} j^k \gamma_k z^{k-N}}{1 + \displaystyle\sum_{k=1}^{N} j^{-k} \gamma_k z^{-k}}, \tag{11}$$

where γ_k are real. $A_S(z^{-1})$ can be realized by reversing its input signal, filtering it with the causal-stable filter $A_S(z)$, and then re-reversing the output signal. Therefore, $A(z)$ is realized only by using $A_S(z)$, as shown in Fig.2.

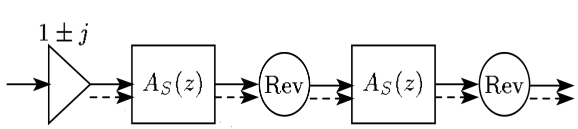

Fig. 2. Cascade connection of causal-stable allpass filters.

For the causal-stable filter $A_S(z)$ with input $p(n)$ and output $q(n)$, its input-output relation is given by

$$q(n) = p(n-N) + \sum_{k=1}^{N} j^k \gamma_k [p(n+k-N) - (-1)^k q(n-k)]. \tag{12}$$

It is clear in Fig.1 that only even-indexed real part and odd-indexed imaginary part of output of $A(z)$ are necessary in analysis filters. Thus, we need only even-indexed real and odd-indexed imaginary parts of output of $A_S(z)$ when N is even, while odd-indexed real and even-indexed imaginary parts when N is odd. In the following, we will consider the case of even N. The case of odd N is similar, then is omitted here. Let $p(n) = p_r(n) + jp_i(n)$ and $q(n) = q_r(n) + jq_i(n)$, where the indices r and i mean the real and imaginary parts, respectively. Therefore, the even-indexed real part $q_r(2n)$ and odd-indexed imaginary part $q_i(2n+1)$ of output of $A_S(z)$ are obtained by

$$\begin{aligned} q_r(2n) &= p_r(2n-N) + \\ &\sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_r(2n+2k-N) - q_r(2n-2k)] \\ &+\gamma_{2k-1}[p_i(2n+2k-N-1) + q_i(2n-2k+1)]\}, \end{aligned} \tag{13}$$

$$\begin{aligned} q_i(2n+1) &= p_i(2n-N+1) + \\ &\sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_i(2n+2k-N+1) - q_i(2n-2k+1)] \\ &-\gamma_{2k-1}[p_r(2n+2k-N) + q_r(2n-2k+2)]\}, \end{aligned} \tag{14}$$

where only N real multipliers are needed per output sample. It is seen in Eqs.(13) and (14) that only the even-indexed real part $p_r(2n)$ and odd-indexed imaginary part $p_i(2n+1)$ of input signal $p(n)$ are necessary for getting $q_r(2n)$ and $q_i(2n+1)$. This means that down-sampling operation is done before the filtering. Therefore, the computational complexity can be reduced. Since the input signal is periodic, some initial values are needed for starting the processing in Eqs.(13) and (14). IIR filters have infinite impulse responses in theory, but the impulse responses of the stable filters will become very small beyond a certain interval in practice. We then make use of this property and approximately calculate the initial values by using the truncated impulse response.

4. REALIZATION OF REVERSIBLE WAVELETS

In this section, we consider the realization of the reversible integer-to-integer wavelets for lossless coding. In most of the cases, $A_S(z)$ has floating point coefficients. Although the input images are matrices of integer values, the filtered output no longer consists of integers. For lossless compression, it is necessary to make an invertible mapping from an integer input to an integer wavelet coefficient. To obtain an integer output, we revise Eqs.(13) and (14) as follows;

$$q_r(2n) = p_r(2n-N) + \lfloor \sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_r(2n+2k-N) - q_r(2n-2k)] + \gamma_{2k-1}[p_i(2n+2k-N-1) + q_i(2n-2k+1)]\} + 0.5\rfloor, \quad (15)$$

$$q_i(2n+1) = p_i(2n-N+1) + \lfloor \sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_i(2n+2k-N+1) - q_i(2n-2k+1)] - \gamma_{2k-1}[p_r(2n+2k-N) + q_r(2n-2k+2)]\} + 0.5\rfloor, \quad (16)$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. Therefore, we can get integer output $q_r(2n), q_i(2n+1)$ for $n = 0, 1, \cdots, M/2-1$. To recover $p(n)$ from $q(n)$, we have from Eqs.(15) and (16)

$$p_r(2n-N) = q_r(2n) - \lfloor \sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_r(2n+2k-N) - q_r(2n-2k)] + \gamma_{2k-1}[p_i(2n+2k-N-1) + q_i(2n-2k+1)]\} + 0.5\rfloor, \quad (17)$$

$$p_i(2n-N+1) = q_i(2n+1) - \lfloor \sum_{k=1}^{N/2} (-1)^k \{\gamma_{2k}[p_i(2n+2k-N+1) - q_i(2n-2k+1)] - \gamma_{2k-1}[p_r(2n+2k-N) + q_r(2n-2k+2)]\} + 0.5\rfloor. \quad (18)$$

This means that if all of $q_r(2n)$ and $q_i(2n+1)$ and some of $p_r(2n)$ and $p_i(2n+1)$, e.g., $\{p_i(M-1), p_r(M-2), \cdots, p_r(M-N)\}$ are known a priori, we can exactly reconstruct $p_r(2n)$ and $p_i(2n+1)$ for $n = (M-N)/2-1, (M-N)/2-2, \cdots, 0$. In lossless coding, $q_r(2n)$ and $q_i(2n+1)$ is transmitted to the decoder without loss. However, it is generally difficult to recover $\{p_i(M-1), p_r(M-2), \cdots, p_r(M-N)\}$ perfectly from only $q_r(2n)$ and $q_i(2n+1)$, since the quantization error has been included in $q(n)$ by using Eqs.(15) and (16). Thus, $\{p_i(M-1), p_r(M-2), \cdots, p_r(M-N)\}$ also need to be transmited as a side information. By using the transmitted side information, we can realize an invertible complex allpass filter with Eqs.(17) and (18). The amount of the side information is relatively small since the complex allpass filters used in image coding are of low order. Moreover, we can calculate the prediction values of $\{p_i(M-1), p_r(M-2), \cdots, p_r(M-N)\}$ from $q_r(2n)$ and $q_i(2n+1)$ by using the method for calculating the initial values, then only the difference between the prediction and actual values need to be transmitted.

Table 1. Lossless coding results: Bit Rate (bpp)

Image	D-5/3	N=1	N=2	N=3	N=4
Barbara	4.695	4.758	4.545	4.503	**4.501**
Boat	4.438	4.527	**4.431**	4.434	4.451
Crowd	**4.234**	4.452	4.268	4.281	4.307
Goldhill	**4.871**	4.942	4.886	4.886	4.894
Lena	4.348	4.440	**4.335**	**4.335**	4.346
Man	**4.730**	4.840	4.740	4.750	4.770
Mandrill	6.149	6.204	6.127	**6.121**	6.122
Pepper	4.653	4.705	**4.649**	4.663	4.679
Woman	**3.345**	3.479	3.382	3.402	3.426
Zelda	4.019	4.090	3.972	**3.965**	3.975
Average	4.548	4.647	**4.534**	**4.534**	4.547

5. IMAGE CODING APPLICATION

In this section, we will investigate the compression performance of the proposed allpass-based orthonormal symmetric wavelets with the maximally flat magnitude responses. The filter order is chosen to $N = 1 \sim 4$. The reference software of JPEG2000 provided in [14] has been used to evaluate the coding performance. Ten images (Barbara, Boat, Crowd, Goldhill, Lena, Man, Mandrill, Pepper, Woman and Zelda) of size 512×512, 8 bpp are used as test images, and the decomposition level of the wavelet transform is set to 6.

5.1. Lossy coding performance

We have examined the lossy coding performance of the irreversible real-to-real wavelets proposed in Section **3**. The distortion is measured by the peak signal to noise ratio (PSNR) between the original and reconstructed images. The lossy coding results for images Barbara and Lena are given in Fig.3 and Fig.4, respectively. It is seen in Fig.3 that when $N > 1$, the orthonormal symmetric wavelets composed of a complex allpass filter have better lossy coding performance than the D-9/7 wavelet for image Barbara. For image Lena, almost same results have been gotten except for $N = 1$, as shown in Fig.4.

5.2. Lossless coding performance

We have investigated the lossless coding performance of the reversible integer-to-integer wavelets proposed in Section **4** for ten test images. The lossless coding results with the comparison with the D-5/3 wavelet are given in Table 1. For each image, the best result has been highlighted. It is seen in Table 1 that the orthonormal symmetric wavelets composed of a complex allpass filter with $N = 2$ and $N = 3$ have better average lossless coding performance than the D-5/3 wavelet. Although there are four images getting the best results for the D-5/3 wavelet, the proposed orthonormal symmetric wavelets with $N = 2$ and $N = 3$ have obtained the best lossless coding performance for three images, respectively.

6. CONCLUSION

In this paper, we have proposed an effective implementation of the orthonormal symmetric wavelets composed of a single com-

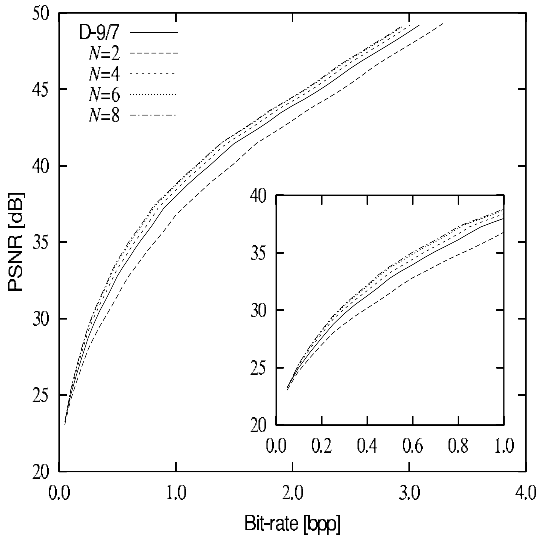

Fig. 3. Lossy coding results of irreversible wavelets for Barbara.

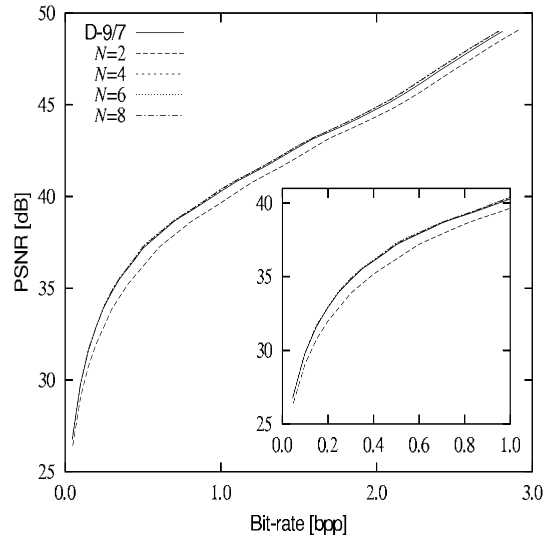

Fig. 4. Lossy coding results of irreversible wavelets for Lena.

plex allpass filter for lossy to lossless image compression. Firstly, we have discussed an implementation of irreversible real-to-real wavelets by realizing the complex allpass filter. Then, we have presented the invertible implementation of complex allpass filter to realize the reversible integer-to-integer wavelets. Finally, we have investigated the compression performance of the proposed allpass-based orthonormal symmetric wavelets by using the reference software of JPEG2000, and compared the lossy coding performance with the D-9/7 wavelet and the lossless coding performance with the D-5/3 wavelet. It have been shown from the experimental results that the proposed allpass-based orthonormal symmetric wavelets can achieve better lossy to lossless coding performance than the conventional D-9/7 and D-5/3 wavelets.

7. ACKNOWLEDGMENT

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