DESIGN OF TWO CHANNEL IIR LINEAR PHASE PR FILTER BANKS

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ABSTRACT

This paper proposes a novel method for designing two channel biorthogonal perfect reconstruction (PR) filter banks with exact linear phase using IIR filters. Since the structurally PR implementation is adopted, the proposed filter banks are guaranteed to be PR even when all filter coefficients are quantized. From the viewpoint of wavelets, design of IIR linear phase filter banks with an additional flatness condition is considered. The proposed design method is based on the formulation of a generalized eigenvalue problem by using Remez exchange algorithm. Therefore, the filter coefficients can be obtained by solving the eigenvalue problem, and the optimal solution with an equiriple magnitude response is easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness condition can be arbitrarily specified.

1. INTRODUCTION

Two channel PR filter banks have been used in different applications of signal processing. The theory and design of FIR PR filter banks have been well established in recent years [1]. PR filter banks include two cases: orthonormal and biorthogonal. For orthonormal case, FIR PR filter banks, except Haar function, cannot possess exact linear phase that is desired in image signal processing. Thus, biorthogonal PR filter banks are proposed to obtain exact linear phase. Design of biorthogonal FIR linear phase PR filter banks has been also discussed in [1]. However, compared with IIR filters, FIR filters generally require higher-order filters for meeting the same specifications. In this paper, we will consider design of IIR linear phase PR filter banks. Although causal IIR filters can possess approximately linear phase only, we can obtain exact linear phase by using noncausal IIR filters, which are permissible in image signal processing. In the most designs, the PR property cannot be preserved generally when the filter coefficients are quantized. In [4], an efficient structurally PR implementation is proposed, where for IIR case, causal allpass filters are used.

In this paper, we propose a new method for designing two channel biorthogonal linear phase PR filter banks using noncausal IIR filters. We adopt the structurally PR implementation proposed in [4], thus the obtained IIR filter banks still satisfy the PR condition even when all filter coefficients are quantized. It is well-known [2], [3] that wavelet bases can be generated from PR filter banks. From the regularity of wavelets, a flatness constraint is required to impose on the PR filter banks. In this paper. from the viewpoint of wavelets, we consider design of IIR linear phase PR filter banks with an additional flatness condition. By using Remez exchange algorithm, we formulate the design problem in the form of a generalized eigenvalue problem [5], [6]. Therefore, we can get a set of filter coefficients by solving the eigenvalue problem. Then, the optimal solution with an equiripple response is easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness condition can be arbitrarily specified.

2. STRUCTURALLY PR FILTER BANKS

In two channel filter banks, assume that $H_0(z), H_1(z)$ are analysis filters, and $G_0(z), G_1(z)$ are synthesis filters. It is well-known that the PR condition is

$$\begin{cases} G_0(z) = H_1(-z) \\ G_1(z) = -H_0(-z) \\ H_0(z)H_1(-z) - H_1(z)H_0(-z) = z^{-2K-1} \end{cases} , \quad (1)$$

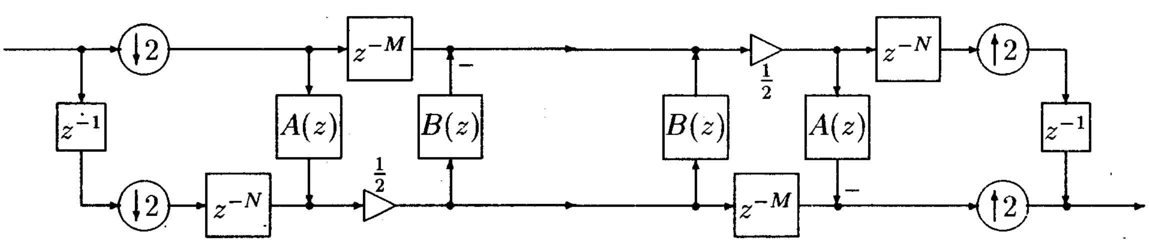

Fig.1 Structurally perfect reconstruction filter bank.

where K is integer. In [4], the analysis filters $H_0(z)$ and $H_1(z)$ are composed by

$$\left\{\begin{array}{ll} H_0(z) & = \frac{1}{2}\{z^{-2N-1} + A(z^2)\} \\ H_1(z) & = z^{-2M} - B(z^2)H_0(z) \\ & = z^{-2M} - \frac{B(z^2)}{2}\{z^{-2N-1} + A(z^2)\} \end{array}\right. , \quad (2)$$

where N and M are integers. Thus the PR condition of Eq.(1) can be satisfied, and the structurally PR implementation is shown in Fig.1. In [4], both $A(z)$ and $B(z)$ are allpass filters for IIR case. In this paper, we use general IIR filters to obtain exact linear phase, i.e.,

$$\left\{\begin{array}{l} A(z) = \sum_{i=0}^{L_1} a_i z^{-i} / \sum_{i=0}^{L_2} b_i z^{-i} \\ B(z) = \sum_{i=0}^{L_3} c_i z^{-i} / \sum_{i=0}^{L_4} d_i z^{-i} \end{array}\right. , \quad (3)$$

where L_1, L_2, L_3, L_4 are integers, and a_i, b_i, c_i, d_i are real coefficients, $b_0 = d_0 = 1$.

3. DESIGN OF IIR LINEAR PHASE PR FILTER BANKS

3.1. Desired Magnitude Responses

From Eq.(2), we have

$$H_0(z) = \frac{z^{-2N-1}}{2}\{1+\frac{A(z^2)}{z^{-2N-1}}\} = \frac{z^{-2N-1}}{2}\{1+\hat{A}(z^2)\}, \quad (4)$$

where $\hat{A}(z) = z^{N+\frac{1}{2}}A(z)$. To obtain an exact linear phase, the filter coefficients of $A(z)$ must be symmetric, that is, $a_i = a_{L_1-i}$ and $b_i = b_{L_2-i}$. L_1 and L_2 must satisfy $L_1 = 2I_1+1, L_2 = 2I_2$ and $L_1 - L_2 = 2N+1$, where I_1 and I_2 are integers. Then $\hat{A}(z)$ becomes zero phase, and its frequency response is given by

$$\hat{A}(e^{j\omega}) = \frac{\sum_{i=0}^{I_1} a_i \cos(I_1 - i + \frac{1}{2})\omega}{\frac{b_{I_2}}{2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega}. \quad (5)$$

Therefore $H_0(z)$ has exact linear phase, and its magnitude response is

$$|H_0(e^{j\omega})| = \frac{1}{2}\{1 + \hat{A}(e^{j2\omega})\}. \quad (6)$$

To make $H_0(z)$ lowpass, the desired magnitude response of $\hat{A}(z^2)$ is

$$\left\{\begin{array}{ll} \hat{A}_d(e^{j2\omega}) = 1 & (0 \le \omega \le \omega_p) \\ \hat{A}_d(e^{j2\omega}) = -1 & (\omega_s \le \omega \le \pi) \end{array}\right. , \quad (7)$$

where ω_p and ω_s are the passband and stopband edge frequencies respectively, and $\omega_p + \omega_s = \pi$. We can see from Eq.(5) that

$$\hat{A}(e^{j(2\pi-\omega)}) = -\hat{A}(e^{j\omega}), \quad (8)$$

thus the desired magnitude response is reduced to

$$\hat{A}_d(e^{j\omega}) = 1 \qquad (0 \le \omega \le 2\omega_p). \quad (9)$$

From Eq.(2), we have

$$H_1(z) = z^{-2M}\{1-\frac{H_0(z)B(z^2)}{z^{-2M}}\} = z^{-2M}\{1-\frac{1+\hat{A}(z^2)}{2}\hat{B}(z^2)\}, \quad (10)$$

where $\hat{B}(z) = z^{M-N-\frac{1}{2}}B(z)$. Similarly, to force $\hat{B}(z)$ to be zero phase, $c_i = c_{L_3-i}, d_i = d_{L_4-i}$ and $L_3 = 2I_3 + 1, L_4 = 2I_4, L_3 - L_4 = 2(M - N) - 1$ must be satisfied, where I_3 and I_4 are integers. Hence the frequency response of $\hat{B}(z)$ is given by

$$\hat{B}(e^{j\omega}) = \frac{\sum_{i=0}^{I_3} c_i \cos(I_3 - i + \frac{1}{2})\omega}{\frac{d_{I_4}}{2} + \sum_{i=0}^{I_4-1} d_i \cos(I_4 - i)\omega}, \quad (11)$$

and then $H_1(z)$ has exact linear phase and its magnitude response is

$$|H_1(e^{j\omega})| = 1 - |H_0(e^{j\omega})|\hat{B}(e^{j2\omega}). \quad (12)$$

Since $|H_0(e^{j\omega})| = 0$ in $[\omega_s, \pi]$, it is clear that $|H_1(e^{j\omega})| = 1$. In $[0, \omega_p]$, due to $|H_0(e^{j\omega})| = 1$, ideally, to make $|H_1(e^{j\omega})| = 0$, the desired magnitude response of $B(z)$ must be

$$\hat{B}_d(e^{j\omega}) = 1 \qquad (0 \le \omega \le 2\omega_p). \quad (13)$$

Therefore, the design problem becomes approximation of $A(z)$ and $B(z)$ to Eqs.(9) and (13).

3.2. Design of Maximally Flat Filters

We consider design of $H_0(z)$, i.e., $A(z)$. First, we define an error function $E_a(\omega)$ between the desired and actual magnitude responses of $A(z)$ as

$$E_a(\omega) = 1 - \hat{A}(e^{j\omega}). \quad (14)$$

The design purpose is to find a set of filter coefficients a_i and b_i to minimize $E_a(\omega)$ in $[0, 2\omega_p]$. It is well-known [1]~[3] that wavelet bases can be generated from PR filter banks, then synthesis of wavelet bases is reduced to design of PR filter banks. From the regularity of wavelets, PR filter banks are required to satisfy certain flatness condition, that is,

$$\left.\frac{\partial^k |H_0(e^{j\omega})|}{\partial \omega^k}\right|_{\omega=\pi} = 0 \qquad (k = 0, 1, \cdots, 2J_1 - 1), \quad (15)$$

where J_1 is integer and $0 \le J_1 \le I_1 + I_2 + 1$. When $J_1 = I_1 + I_2 + 1$, $H_0(z)$ is the maximally flat filter and the corresponding wavelet function has maximum regularity. From Eqs.(6), (8) and (14), Eq.(15) is equivalent to

$$\left.\frac{\partial^k E_a(\omega)}{\partial \omega^k}\right|_{\omega=0} = 0 \qquad (k = 0, 1, \cdots, 2J_1 - 1). \quad (16)$$

Substituting Eqs.(5) and (14) into Eq.(16), we get

$$\left\{\begin{array}{l} \frac{b_{I_2}}{2} + \sum_{i=0}^{I_2-1} b_i - \sum_{i=0}^{I_1} a_i = 0 \\ \sum_{i=0}^{I_2-1} b_i (I_2 - i)^{2k} - \sum_{i=0}^{I_1} a_i (I_1 - i + \frac{1}{2})^{2k} = 0 \\ \qquad\qquad (k = 1, 2, \cdots, J_1 - 1) \end{array}\right. . \quad (17)$$

When $J_1 = I_1 + I_2 + 1$, we can obtain the filter coefficients of the maximally flat filters by solving $(I_1 + I_2 + 1)$ linear equations in Eq.(17).

3.3. Design of Filters with Given Flatness

It is well-known that the maximally flat filters are poorly selective. Frequency selectivity is also thought of as a useful property for many applications. However, frequency selectivity and regularity somewhat contradict each other. For this reason, we consider design of IIR filters that have the best possible frequency selectivity for a given flatness. Assume that the flatness condition of Eq.(15) is required where $J_1 \leq I_1 + I_2$. Our aim is to achieve an equiripple response under the given flatness by using the remaining degree of freedom. First, we select $(I_1 + I_2 - J_1 + 2)$ extremal frequencies ω_i $(2\omega_p = \omega_0 > \omega_1 > \cdots > \omega_{(I_1+I_2-J_1+1)} \geq 0)$ in $[0, 2\omega_p]$. Then we use Remez exchange algorithm and formulate $E_a(\omega)$ as

$$E_a(\omega_i) = 1 - \hat{A}(e^{j\omega_i}) = (-1)^i \delta \ , \tag{18}$$

where δ (> 0) is magnitude error, and the denominator polynomial of $E_a(\omega)$ must satisfy

$$\frac{b_{I_2}}{2} + \sum_{i=0}^{I_2-1} b_i \cos(I_2 - i)\omega \neq 0 \qquad (\text{for all } \omega). \tag{19}$$

Substituting Eq.(5) into Eq.(18), we rewrite Eqs.(17) and (18) in the matrix form as

$$P\,A = \delta\, Q\, A \ , \tag{20}$$

where $A = [a_0, a_1, \cdots, a_{I_1}, b_0, b_1, \cdots, b_{I_2}]^T$. The elements of the matrices P are, when $i = 0, 1, \cdots, J_1 - 1$,

$$P_{ij} = \begin{cases} -(I_1 - j + \frac{1}{2})^{2i} & (j = 0, 1, \cdots, I_1) \\ (I_1 + I_2 + 1 - j)^{2i} & (j = I_1 + 1, \cdots, I_1 + I_2) \end{cases}, \tag{21}$$

when $i = J_1, J_1 + 1, \cdots, I_1 + I_2 + 1$,

$$P_{ij} = \begin{cases} -\cos(I_1 - j + \frac{1}{2})\omega_{(i-J_1)} & (j = 0, 1, \cdots, I_1) \\ \cos(I_1 + I_2 + 1 - j)\omega_{(i-J_1)} & (j = I_1 + 1, \cdots, I_1 + I_2) \end{cases} \tag{22}$$

and when $j = I_1 + I_2 + 1$,

$$P_{ij} = \begin{cases} 0 & (i = 1, 2, \cdots, J_1 - 1) \\ \frac{1}{2} & (\text{else}) \end{cases}. \tag{23}$$

The elements of the matrices Q are, when $i = J_1, J_1 + 1, \cdots, I_1 + I_2 + 1$ and $j = I_1 + 1, I_1 + 2, \cdots, I_1 + I_2$,

$$Q_{ij} = (-1)^{i-J_1} \cos(I_1 + I_2 + 1 - j)\omega_{(i-J_1)}, \tag{24}$$

and when $i = J_1, J_1 + 1, \cdots, I_1 + I_2 + 1$ and $j = I_1 + I_2 + 1$,

$$Q_{ij} = (-1)^{i-J_1}\frac{1}{2}, \tag{25}$$

else $Q_{ij} = 0$. It should be noted that Eq.(20) is a generalized eigenvalue problem, i.e., δ is an eigenvalue, and A is a corresponding eigenvector. It is known in [5] that to obtain a solution that satisfies Eq.(19), we only need to find an eigenvector corresponding to the positive minimum eigenvalue. Therefore, a set of filter coefficients can be easily obtained. To achieve an equiripple response, we apply an iteration procedure to get the optimal solution. The design algorithm is shown as follows.

3.4. Design Algorithm

Procedure {Design Algorithm of IIR Linear Phase Filters}

Begin

1. Read L_1, L_2, J_1, and ω_p.
2. Select initial extremal frequencies Ω_i $(i = 0, 1, \cdots, I_1 + I_2 - J_1 + 1)$ equally spaced in $[0, 2\omega_p]$.

Repeat

3. Set $\omega_i = \Omega_i$ $(i = 0, 1, \cdots, I_1 + I_2 - J_1 + 1)$.
4. Compute P, Q and find the positive minimum eigenvalue of Eq.(20) to obtain a_i and b_i that satisfies Eq.(19).
5. Search the peak frequencies of $E_a(\omega)$ within $[0, 2\omega_p]$, and store these frequencies into the corresponding Ω_i.

Until Satisfy the following condition for the prescribed small constant ϵ :

$$\left\{ \sum_{i=0}^{I_1+I_2-J_1+1} |\Omega_i - \omega_i| \leq \epsilon \right\}$$

End.

3.5. Design of $H_1(z)$

We consider design of $H_1(z)$, i.e., $B(z)$. $B(z)$ can be similarly designed by using the design algorithm proposed in Section 3.4. It is seen from Eq.(12) that the magnitude response of $H_1(z)$ is dependent on both $A(z)$ and $B(z)$. Even if both $A(z)$ and $B(z)$ have equiripple magnitude responses, we cannot guarantee that the magnitude response of $H_1(z)$ must be equiripple. To achieve an equiripple magnitude response of $H_1(z)$, we define an error function $E_b(\omega)$ as

$$E_b(\omega) = 1 - |H_0(e^{j\omega/2})|\hat{B}(e^{j\omega}), \tag{26}$$

and use Remez exchange algorithm to formulate $E_b(\omega)$ as

$$E_b(\omega_i) = 1 - |H_0(e^{j\omega_i/2})|\hat{B}(e^{j\omega_i}) = (-1)^i\delta, \tag{27}$$

where $|H_1(e^{j\omega/2})|$ can be considered to be a weighting function. Hence, $H_1(z)$ will have an equiripple magnitude response in stopband. Similarly, $H_1(z)$ is also required to satisfy the given flatness condition, that is,

$$\left.\frac{\partial^k |H_1(e^{j\omega})|}{\partial \omega^k}\right|_{\omega=0} = 0 \qquad (k = 0, 1, \cdots, 2J_2 - 1), \tag{28}$$

where J_2 is integer. Since the flatness of $H_1(z)$ is decided by the flatness-lower one between $A(z)$ and $B(z)$, then $J_2 \leq J_1$. It is seen from Eq.(12) that the flatness condition of Eq.(28) is equivalent to

$$\begin{cases} \hat{B}(e^{j0}) = 1 \\ \left.\dfrac{\partial^k \hat{B}(e^{j\omega})}{\partial \omega^k}\right|_{\omega=0} = 0 \qquad (k = 1, 2, \cdots, 2J_2 - 1) \end{cases}, \tag{29}$$

then we can get the similar linear equations as Eq.(17). Therefore, we can formulate the design problem of $B(z)$ in the form of eigenvalue problem as shown in Section 3.3. The design algorithm is the same as that shown in Section 3.4.

4. DESIGN EXAMPLE

We consider design of an IIR linear phase PR filter bank with a given flatness. The design specification is $J_1 = J_2 = 5$, $\omega_p = 0.45\pi$ and $\omega_s = 0.55\pi$. The order of $A(z)$ and $B(z)$ are $L_1 = 7$, $L_2 = 6$ and $L_3 = 9$, $L_4 = 6$. The obtained magnitude responses of $A(z)$ and $B(z)$ are shown in Fig.2, and ones of $H_0(z)$ and $H_1(z)$ are shown in Fig.3 in the solid line. The stopband attenuations of $H_0(z)$ and $H_1(z)$ are $56.7dB$ and $68.0dB$, respectively. It is clear that the equiripple responses with given flatness have been obtained. We have also designed another PR filter bank with $J_1 = 7$ and $J_2 = 5$. The magnitude responses of $A(z)$ and $B(z)$ are shown in Fig.2, and ones of $H_0(z)$ and $H_1(z)$ are shown in Fig.3 in the dotted line also. It is seen that $H_0(z)$ is the maximally flat filter.

5. CONCLUSION

In this paper, we have proposed a new method for designing two channel IIR linear phase PR filter banks. Since we have adopted the structurally PR implementation proposed in [4], the PR condition is still satisfied even when all filter coefficients are quantized. From the viewpoint of wavelets, we have shown design of IIR linear phase PR filter banks with an additional flatness condition. By using Remez exchange algorithm, we have formulated the design problem in the form of a generalized eigenvalue problem. Therefore, by solving the eigenvalue problem to compute the positive minimum eigenvalue, a set of filter coefficients can be getten as the corresponding eigenvector. The optimal solution with an equiripple magnitude response is easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness condition can be arbitrarily specified.

6. REFERENCES

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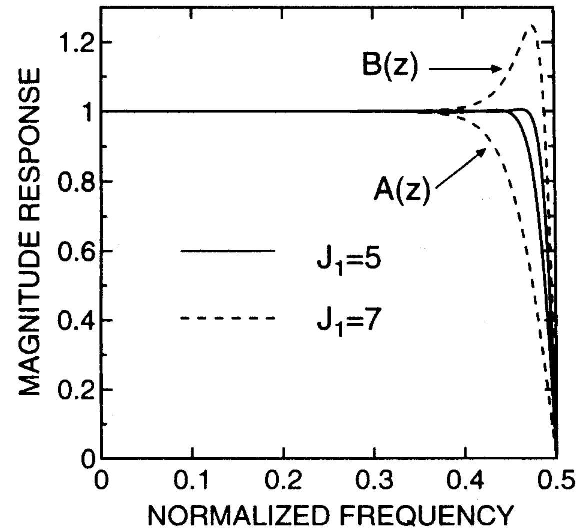
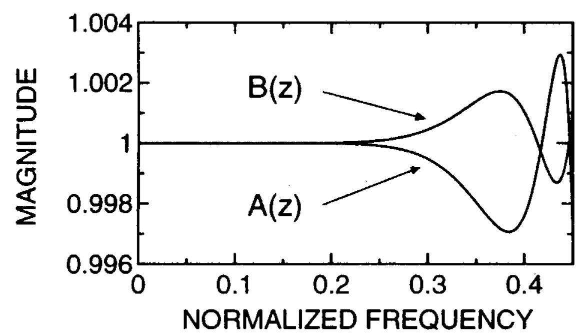


(a)



(b)

Fig.2 Magnitude responses of $A(z)$ and $B(z)$.

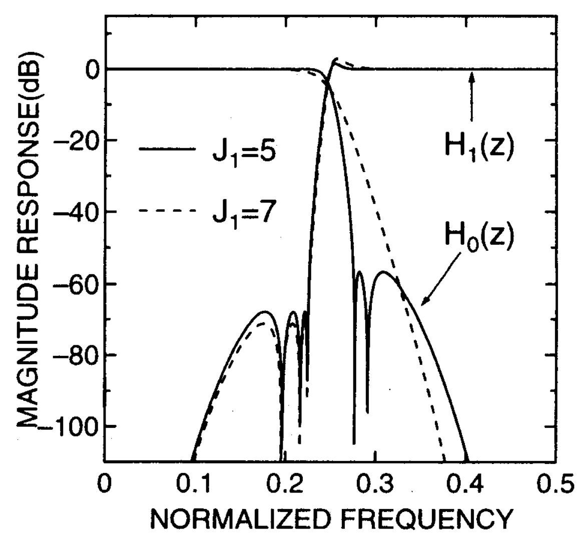


Fig.3 Magnitude responses of $H_0(z)$ and $H_1(z)$.