Complex Chebyshev Approximation for IIR Digital Filters Based on Eigenvalue Problem

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***Abstract*—This paper presents an efficient method for designing complex infinite impulse response digital filters in the complex Chebyshev sense. The proposed method is based on the formulation of a generalized eigenvalue problem by using the Remez multiple exchange algorithm. Hence, the filter coefficients can be easily obtained by solving the eigenvalue problem to find the absolute minimum eigenvalue, and then the complex Chebyshev approximation is attained through a few iterations starting from a given initial guess. The proposed algorithm is computationally efficient because it not only retains the speed inherent in the Remez exchange algorithm but also simplifies the interpolation step. Some design examples are presented and compared with the conventional methods. It is shown that the results obtained by using the method proposed in this paper are better than those obtained by the conventional methods.**

***Index Terms*—Complex Chebyshev approximation, eigenvalue problem, infinite impulse response (IIR) digital filter, Remez multiple exchange algorithm.**

I. Introduction

It is well known [1]–[4], [6] that the Remez exchange algorithm is an efficient tool for designing finite impulse response (FIR) digital filters with linear phase, where the design problem is a real Chebyshev approximation and the Remez exchange algorithm is based on the alternation theorem. In many applications such as equalization, beamforming, and so on, the design of digital filters with arbitrary magnitude and phase responses is required, which results in a complex Chebyshev approximation problem [1]–[21], [24]. Although the alternation theorem no longer holds in the complex case, the Remez exchange algorithm has also been generalized to design complex FIR filters [11], [17]–[21], [24]. Compared with FIR filters, infinite impulse response (IIR) filters tend to be of much lower order for meeting the same specifications [5], [7]–[9], [12]–[15], [22], [23]. However, IIR filter design is more difficult than FIR design because it is a rational approximation problem. In [22] and [25], the Remez exchange algorithm has been applied to the real Chebyshev approximation for IIR filters, where the interpolation problem has been reduced to a generalized eigenvalue problem. Thus, the solution can be easily obtained by finding the absolute minimum eigenvalue in most cases. In this paper, we wish to generalize the method proposed in [22] to the complex Chebyshev approximation for IIR filters.

Several design methods using nonlinear programming [5], [7], linear programming [8], [13], multiple criterion optimization [9], [12], and differential correction algorithm [14] have been suggested to design IIR digital filters in the complex domain also. However, the major disadvantages thereof are quite computationally expensive and/or poor frequency responses.

In this paper, we consider the complex Chebyshev approximation problem of IIR digital filters and propose an efficient method to attain the specified magnitude and phase responses in the complex Chebyshev sense. The proposed method is based on the formulation of a generalized eigenvalue problem by using the Remez multiple exchange algorithm. Hence, the filter coefficients can be easily obtained by solving the eigenvalue problem to find the absolute minimum eigenvalue, and then the complex Chebyshev approximation is attained through a few iterations starting from a given initial guess. The proposed algorithm is computationally efficient because it not only retains the speed inherent in the Remez exchange algorithm but also simplifies the interpolation step. Finally, some design examples are presented and compared with the conventional methods. It is shown that the results obtained by using the method proposed in this paper are better than the conventional methods.

This paper is organized as follows. Section II states the complex Chebyshev approximation problem of IIR digital filters. Problem formulation based on the generalized eigenvalue problem by using the Remez multiple exchange algorithm is presented in Section III. Section IV discusses how to select a set of initial filter coefficients. A design algorithm is given in detail in Section V, and the stability issue of IIR filters is addressed in Section VI. Section VII presents some numerical examples and a comparison with other existing IIR filter design methods.

II. Problem Statement

Let $H(z)$ be the transfer function of an IIR digital filter with numerator degree N and denominator degree M

$$H(z) = \frac{\sum_{n=0}^{N} a_n z^{-n}}{\sum_{m=0}^{M} b_m z^{-m}} \tag{1}$$

Manuscript received February 2000; revised October 2000. This paper was recommended by Associate Editor Y. C. Lim.

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Publisher Item Identifier S 1057-7130(00)11667-2.

where filter coefficients a_n, b_m are complex in general and $b_0 = 1$. The frequency response of $H(z)$ is generally a complex-valued function of the normalized frequency ω

$$H(e^{j\omega}) = \frac{\sum_{n=0}^{N} a_n e^{-jn\omega}}{\sum_{m=0}^{M} b_m e^{-jm\omega}} = \frac{N(\omega)}{D(\omega)}. \tag{2}$$

The complex Chebyshev approximation problem may be briefly stated as follows. Let $H_d(e^{j\omega})$ be the desired frequency response

$$H_d(e^{j\omega}) = |H_d(e^{j\omega})|e^{j\theta_d(\omega)} \qquad (\omega \in R) \tag{3}$$

where

$\lvert H_d(e^{j\omega})\rvert$	desired magnitude response;
$\theta_d(\omega)$	desired phase response;
$R \subset (-\pi, \pi]$	interest bands (e.g., passband and stopband).

The approximation problem consists in finding the filter coefficients a_n, b_m that will minimize the Chebyshev norm

$$\|E(\omega)\| = \max_{\omega \in R} |E(\omega)| \tag{4}$$

of the weighted error

$$E(\omega) = W(\omega)[H(e^{j\omega}) - H_d(e^{j\omega})] \tag{5}$$

among all possible choices of a_n, b_m. The weighting $W(\omega)$ must be a real, strictly positive and continuous function on R.

In order to guarantee the filter causality and stability, the poles are required to locate inside the unit circle. It is known in [13] that the optimal complex Chebyshev approximation may not exist when the poles are restricted inside the unit circle. In some applications, such as image processing, it is not necessary for the filter to be causal since the signal length is finite [3]. Therefore, the constraint can be relaxed and only the stability remains to be considered. In this case, the poles are required only not to locate on the unit circle. It was pointed out in [13] that there is no guarantee of the uniqueness of the complex Chebyshev approximation problem, and the number of the optimal approximation may be arbitrarily large. The characterization of the optimal rational approximation in the complex Chebyshev sense is available as sufficient conditions for the general approximation without pole restrictions. One sufficient condition is that the weighted error function $E(\omega)$ has at least $N + M + 2$ extremal points [13]. In the following, we will make use of this sufficient condition in the problem formulation without any pole restrictions. The filter stability issue is addressed in Section VI.

III. Formulation Based on Eigenvalue Problem

In this section, we describe the design of IIR digital filters based on the eigenvalue problem by using the Remez multiple exchange algorithm. Our aim is to find a set of filter coefficients a_n, b_m in such a way that the weighted error function in (5) satisfies

$$|E(\omega)| \le \delta_{\max}, \qquad (\omega \in R) \tag{6}$$

where $\delta_{\max}(> 0)$ is the maximum error to be minimized.

To solve the above complex Chebyshev approximation problem, we use the Remez multiple exchange algorithm and formulate the condition for $H(e^{j\omega})$ in the form of a generalized eigenvalue problem. Assume that there are $N+M+2$ extremal frequencies ω_i $(i = 0, 1, \ldots, N+M+1)$ in the bands R. We first formulate $H(e^{j\omega})$ at these frequencies ω_i as

$$E(\omega_i) = W(\omega_i)[H(e^{j\omega_i}) - H_d(e^{j\omega_i})] = \delta e^{j\theta_e(\omega_i)} \tag{7}$$

where δ is the magnitude error and $\theta_e(\omega_i)$ is the phase of $E(\omega)$ at ω_i. Notice that the denominator polynomial $D(\omega)$ of $H(e^{j\omega})$ has to satisfy

$$D(\omega) = \sum_{m=0}^{M} b_m e^{-jm\omega} \neq 0, \qquad (\omega \in R). \tag{8}$$

Substituting (2) into (7), we obtain

$$N(\omega_i) - H_d(e^{j\omega_i})D(\omega_i) = \delta \frac{e^{j\theta_e(\omega_i)}}{W(\omega_i)} D(\omega_i). \tag{9}$$

We then rewrite (9) in the matrix form as

$$\boldsymbol{Pa} = \delta \boldsymbol{Qa} \tag{10}$$

where $\boldsymbol{a} = [a_0, a_1, \ldots, a_N, b_0, b_1, \ldots, b_M]^T$. The elements of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ are given by

$$P_{mn} = \begin{cases} e^{-jn\omega_m}, & (n = 0, 1, \ldots, N) \\ -H_d(e^{j\omega_m})e^{-j(n-N-1)\omega_m}, & (n = N+1, \ldots, N+M+1) \end{cases} \tag{11}$$

$$Q_{mn} = \begin{cases} 0, & (n = 0, 1, \ldots, N) \\ \dfrac{e^{j[\theta_e(\omega_m)-(n-N-1)\omega_m]}}{W(\omega_m)}, & (n = N+1, \ldots, N+M+1). \end{cases} \tag{12}$$

Once $N+M+2$ extremal frequencies ω_i and their phases $\theta_e(\omega_i)$ are given, it is seen from (11) and (12) that the elements of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ are known. Therefore, it should be noted that (10) corresponds to a generalized eigenvalue problem, i.e., δ is an eigenvalue and $\boldsymbol{a}$ is a corresponding eigenvector. In order to minimize δ, we must find the absolute minimum eigenvalue by solving the above eigenvalue problem [22], [25], so that the corresponding eigenvector gives a set of filter coefficients a_n, b_m. Since we are interested in only one eigenvector corresponding to the absolute minimum eigenvalue, this computation can be done efficiently by using the iterative power method without invoking general methods such as the QR technique. By using the obtained filter coefficients, we compute the weighted error function $E(\omega)$ and search for all extremal frequencies Ω_i in the bands R. As a result, it could be found that the obtained $E(\omega)$ may not be equiripple. We then choose those $N+M+$ extremal frequencies having the largest deviations as the sampling frequencies ω_i in the next iteration and calculate the phase of $E(\omega)$ at ω_i to obtain $\theta_e(\omega_i)$. Therefore, the eigenvalue problem of (10) can be again solved to obtain a set of filter coefficients a_n, b_m. The above procedure is iterated until the equiripple response is attained. Notice that a set of initial extremal frequencies ω_i

and their phases $\theta_e(\omega_i)$ are required to start the proposed iteration procedure. The selection of the initial value will directly influence the convergence of the iteration procedure. In the following, we will discuss how to select a set of initial extremal frequencies ω_i and their phases $\theta_e(\omega_i)$.

IV. Selection of Initial Value

In the above-mentioned iteration procedure, an initial step is required to get a first solution with at least $N+M+2$ extremal values. Arbitrarily selecting a set of extremal frequencies ω_i and their phases $\theta_e(\omega_i)$ cannot guarantee the algorithm to converge to the optimal solution. Hence, how to select an initial value is very important. We should give an initial value close enough to the optimal solution to guarantee the convergence of the algorithm. Here, we refer to the method proposed in [11]. Since the aim is to minimize the weighted error function $E(\omega)$ on R, we construct an initial solution by picking $N+M+1$ frequency points $\overline{\omega}_i$ within R and by assuming $E(\omega)$ to be zero at these frequency points

$$E(\overline{\omega}_i) = W(\overline{\omega}_i)[H(e^{j\overline{\omega}_i}) - H_d(e^{j\overline{\omega}_i})] = 0. \quad (13)$$

A possible choice is to pick these frequencies $\overline{\omega}_i$ equally spaced within R. Other distributions may also be preferred to decrease the number of iterations. The denominator polynomial $D(\omega)$ has to satisfy (8). Since $b_0 = 1$, we substitute (2) into (13) and obtain

$$\sum_{n=0}^{N} a_n e^{-jn\overline{\omega}_i} - H_d(e^{j\overline{\omega}_i}) \sum_{m=1}^{M} b_m e^{-jm\overline{\omega}_i} = H_d(e^{j\overline{\omega}_i}) \quad (14)$$

which is a set of linear equations. Hence, there is always a unique solution, and we can get an initial solution by solving the linear equations of (14). By using the obtained filter coefficients, we compute $E(\omega)$ and search for all extremal frequencies Ω_i in R. Since we have assumed $E(\omega)$ to be zero at $N+M+1$ frequency points, there always exist at least $N+M+2$ extremal frequencies Ω_i. We then choose those $N+M+2$ extremal frequencies having the largest deviations as the initial frequency points ω_i and calculate the phase of $E(\omega)$ at ω_i to obtain $\theta_e(\omega_i)$. With the generation of $N+M+2$ extremal frequencies, we can start the next step in the optimization procedure. When IIR filters with real coefficients are designed, we must symmetrically select $\overline{\omega}_i$ between positive and negative frequencies, because the real filters have a complex-conjugate frequency response. Similarly, $N+M+2$ extremal frequencies ω_i should be symmetric also. The design algorithm is shown as follows.

V. Design Algorithm

Procedure {Design Algorithm of IIR Digital Filters}
Begin
1. Read N, M, $H_d(e^{j\omega})$ and $W(\omega)$.
2. Select $N+M+1$ frequency points $\overline{\omega}_i$ within R.
3. Solve (14) to get an initial solution.
4. Compute $E(\omega)$ and search for all extremal values in R, then choose $N+M+2$ extremal frequencies Ω_i having the largest deviations and calculate $\theta_e(\Omega_i)$.

Repeat
5. Set $\omega_i = \Omega_i$ for $i = 0, 1, \ldots, N+M+1$.
6. Compute $\boldsymbol{P}$ and $\boldsymbol{Q}$ by using (11) and (12), then find the absolute minimum eigenvalue of (10) to obtain a set of filter coefficients a_n, b_m.
7. Compute $E(\omega)$ and search for all extremal values in R, then choose $N+M+2$ extremal frequencies Ω_i having the largest deviations and calculate $\theta_e(\Omega_i)$.

Until Satisfy the following condition for a prescribed small constant ϵ:

$$\sum_{i=0}^{N+M+1} |\Omega_i - \omega_i| \le \epsilon.$$

End.

VI. Stability Issue

In the above design algorithm, the obtained filter $H(z)$ may not be stable. The stability must be checked by finding the pole location. To guarantee the filter stability, we have to avoid the poles located on the unit circle, i.e., (8) must be satisfied for all ω. In Section III, we have chosen the absolute minimum eigenvalue to minimize the error δ, which ensures that (8) is satisfied in R. However, (8) may not be satisfied in the "don't care" band. For example, IIR filters with nearly linear phase always have a pair of poles in the transition band near the passband edge. This pair of poles maybe moves toward the unit circle as the desired group delay varies. It has been pointed out in [13] that the stability of $H(z)$ is mainly dependent on the specifications, i.e., the filter degree N, M; the desired frequency response $H_d(e^{j\omega})$; and the weighting $W(\omega)$. Therefore, the specifications should be carefully chosen to guarantee the filter stability. See [13] in detail.

VII. Design Examples

In this section, we present several numerical examples to demonstrate the effectiveness of the proposed method and compare the filter performance with other existing IIR filter design methods. We first design two real-valued IIR low-pass filters with the same specifications as *Example 1* and *Example 2* in [13]. Next, a real-valued bandpass filter in [23] and a complex-valued IIR filter are presented.

Example 1: The filter specification is $N = M = 4$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j5\omega}, & (0 \le |\omega| \le 0.2\pi) \\ 0, & (0.4\pi \le |\omega| \le \pi). \end{cases}$$

The weighting is set to $W(\omega) = 1$ in both passband and stopband. In the following, we show only the positive frequency response since it is a real filter. The initial $\overline{\omega}_i$ is selected as shown in Fig. 1. Note that either $\omega = 0$ or $\omega = \pi$ has to be included in $\overline{\omega}_i$ since $N+M+1$ is odd. We then obtained a first solution and chose a set of initial extremal frequencies ω_i, as shown in Fig. 1. Notice that the magnitude response of $E(\omega)$ in Fig. 1

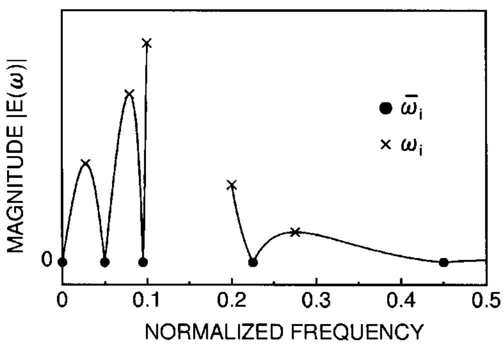

Fig. 1. Initial guess in Example 1.

TABLE I
ERRORS AND ERROR REDUCTIONS IN EXAMPLE 1

No. of iteration	Chebyshev error	Error reduction
0	0.265451	
1	0.045440	484.2%
2	0.030374	49.6%
3	0.025800	17.7%
4	0.023406	10.2%
5	0.023398	0.03%
6	0.023398	

TABLE II
FILTER COEFFICIENTS IN EXAMPLE 1

a_0	−0.030963	b_0	1.000000
a_1	0.065455	b_1	−2.572785
a_2	−0.017905	b_2	2.897621
a_3	0.036301	b_3	−1.608927
a_4	0.028163	b_4	0.365009

has been magnified for illustration and is not the actual one. Starting with these initial extremal frequencies, we obtained an equiripple solution after six iterations. The approximation errors and the error reductions of each iteration are shown in Table I, and the obtained filter coefficients are given in Table II. The magnitude response of $E(\omega)$ is shown in Fig. 2 and the maximum error is $\delta_{\max} = 0.0234$, whereas $\delta_{\max} = 0.0420$ in [13]. The magnitude response and group delay of $H(z)$ are shown in Figs. 3 and 4, respectively. The results in [13] are also shown in the dotted line for comparison. The magnitude error is 0.0233 in passband and 0.0234 (32.6 dB) in stopband, while the error in [13] is 0.0420 and 0.0420 (27.5 dB), respectively. The group delay in passband is between 4.83 and 5.97, and its maximum deviation from the desired group delay is 0.97 in the passband edge. In [13], the group delay is between 4.65 and 6.34, and its maximum deviation is 1.34. The pole-zero location of the obtained filter is shown in Fig. 5, and it is clear that it is causal and stable. To examine the relationship between the specifications and stability, we show the plot of the maximum pole radius versus group delay in Fig. 6. It is seen in Fig. 6 that when the

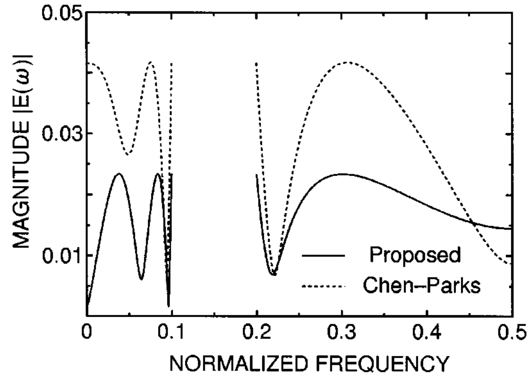

Fig. 2. Magnitude responses of $E(\omega)$ in Example 1.

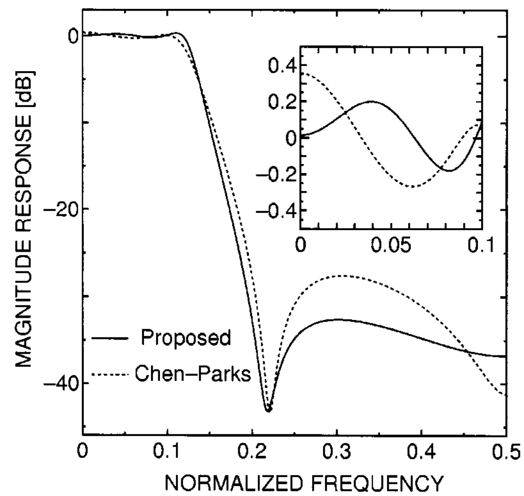

Fig. 3. Magnitude responses of $H(z)$ in Example 1.

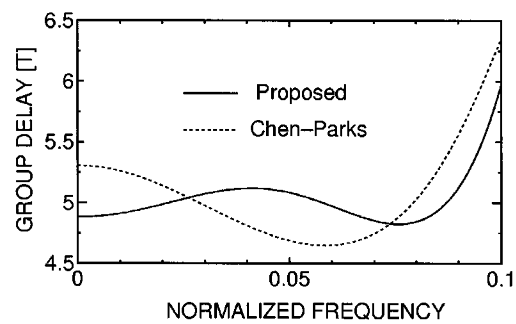

Fig. 4. Group delays of $H(z)$ in Example 1.

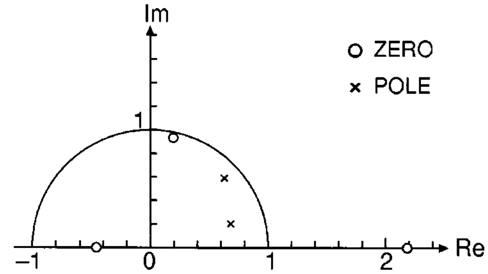

Fig. 5. Pole-zero location of $H(z)$ in Example 1.

group delay is 2.5, the maximum pole radius is equal to one, i.e., this pair of poles locates on the unit circle and the filter is unstable. When the group delay is larger than 2.5, the maximum

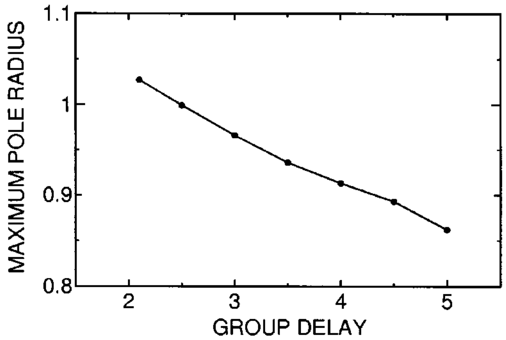

Fig. 6. Maximum pole radius versus group delay.

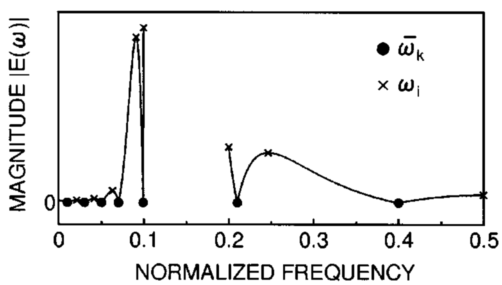

Fig. 7. Initial guess in Example 2.

TABLE III
ERRORS AND ERROR REDUCTIONS IN EXAMPLE 2

No. of iteration	Chebyshev error	Error reduction
0	0.158518	
1	0.007738	1948.6%
2	0.004594	68.4%
3	0.003748	22.6%
4	0.003731	0.5%
5	0.003731	

pole radius is smaller than one; then the filter becomes causal and stable. Therefore, we should specify a larger group delay to guarantee the causality and stability.

Example 2: The filter specification is $N = 6$, $M = 7$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j10\omega}, & (0 \le |\omega| \le 0.2\pi) \\ 0, & (0.4\pi \le |\omega| \le \pi). \end{cases}$$

The weighting is set to $W(\omega) = 1$ in passband and $W(\omega) = 3$ in stopband. The initial $\overline{\omega}_i$ is selected as shown in Fig. 7. Note that both $\omega = 0$ and $\omega = \pi$ are not included in $\overline{\omega}_i$ since $N + M + 1$ is even. We then obtained a first solution and chose a set of initial extremal frequencies ω_i, as shown in Fig. 7. Notice that the magnitude response of $E(\omega)$ in Fig. 7 has been magnified for illustration also. Starting with these initial extremal frequencies, we obtained an equiripple solution after five iterations. The approximation errors and the error reductions of each iteration are shown in Table III, and the filter coefficients are given in Table IV. The magnitude response of $E(\omega)$ is shown in Fig. 8, and the maximum error is $\delta_{\max} = 0.003\,73$, whereas $\delta_{\max} = 0.0155$ in [13]. The magnitude response and

TABLE IV
FILTER COEFFECIENTS IN EXAMPLE 2

a_0	0.002676	b_0	1.000000
a_1	−0.008455	b_1	−4.809253
a_2	0.010671	b_2	10.621628
a_3	−0.011995	b_3	−13.796082
a_4	0.013294	b_4	11.294999
a_5	−0.006682	b_5	−5.799266
a_6	0.007497	b_6	1.722892
		b_7	−0.227885

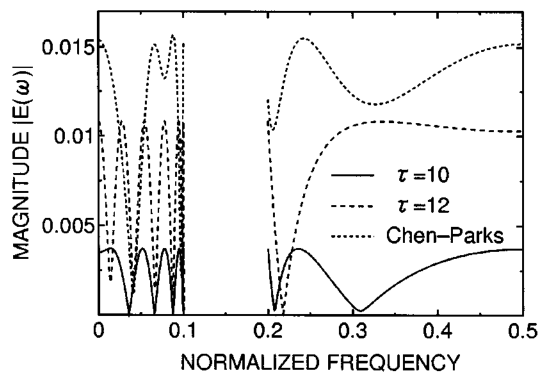

Fig. 8. Magnitude responses of $E(\omega)$ in Example 2.

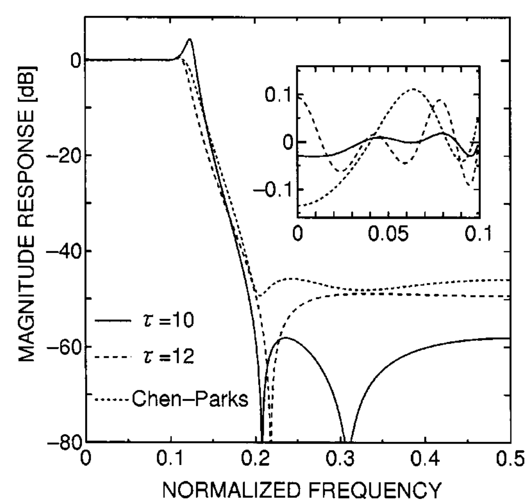

Fig. 9. Magnitude responses of $H(z)$ in Example 2.

group delay of $H(z)$ are shown in Figs. 9 and 10, respectively. The magnitude error is 0.003 50 in passband and 0.001 24 (58.1 dB) in stopband, while the error in [13] is 0.0155 and 0.0052 (45.7 dB), respectively. The maximum deviation from the desired group delay $\tau = 10$ is 0.081, whereas it is 0.80 in [13]. It is seen in Fig. 9 that a magnitude overshoot appears in the transition band. It is because there is a pair of poles in the transition band near the passband edge, as shown in Fig. 11. To avoid this overshoot, we try to vary the desired group delay to $\tau = 12$. The obtained magnitude responses of $E(\omega)$ and $H(z)$ are shown in the dashed line in Figs. 8 and 9, respectively. It is clear that it does not have a magnitude overshoot and is better than that in

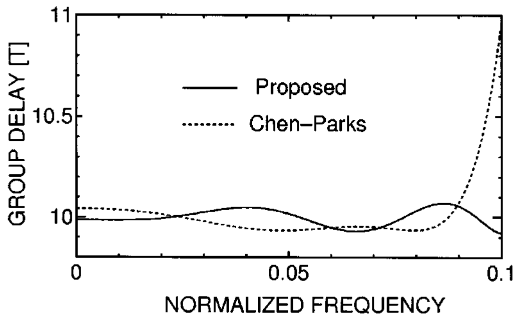

Fig. 10. Group delays of $H(z)$ in Example 2.

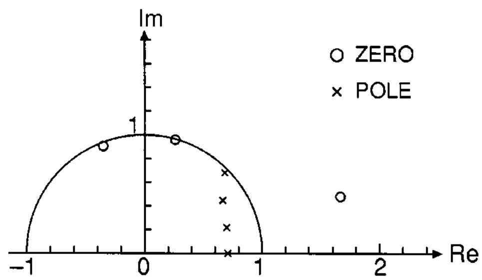

Fig. 11. Pole-zero location of $H(z)$ in Example 2.

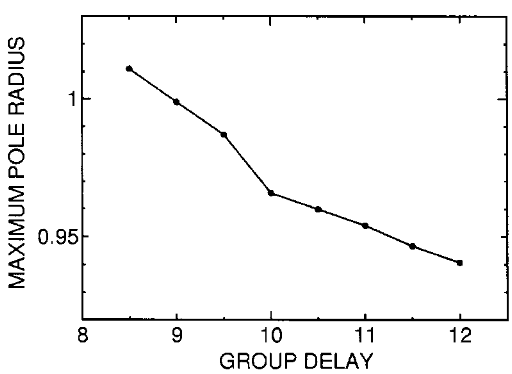

Fig. 12. Maximum pole radius versus group delay.

[13]. The plot of the maximum pole radius versus group delay is shown in Fig. 12. When the group delay is larger than $\tau = 9$, the filters are causal and stable.

Example 3: We consider a real-valued IIR bandpass filter with $N = M = 12$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j20\omega}, & (0.4\pi \leq |\omega| \leq 0.6\pi) \\ 0, & (0 \leq |\omega| \leq 0.28\pi,\ 0.72\pi \leq |\omega| \leq \pi). \end{cases}$$

The weighting is set to $W(\omega) = 1$ in passband and $W(\omega) = 10$ in stopband. The obtained magnitude response of $E(\omega)$ is shown in Fig. 13, and the maximum error is $\delta_{\max} = 0.0115$. The number of iterations is six. The resulting magnitude response and group delay of $H(z)$ are shown in Figs. 14 and 15, respectively. The pole-zero location is shown in Fig. 16, and the resulting filter is causal and stable. A comparison with the results in [15] and [23] is summarized in Table V. In Table V, A_p and

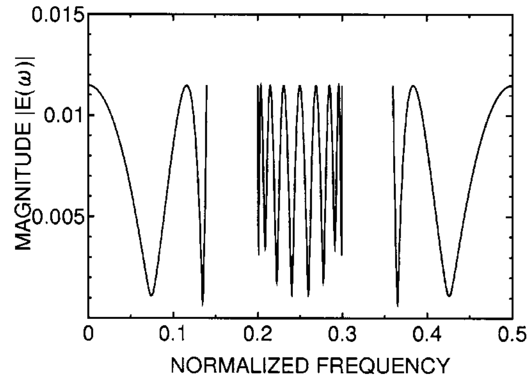

Fig. 13. Magnitude response of $E(\omega)$ in Example 3.

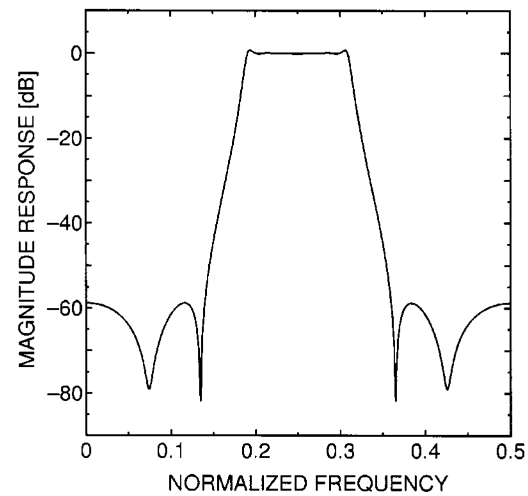

Fig. 14. Magnitude response of $H(z)$ in Example 3.

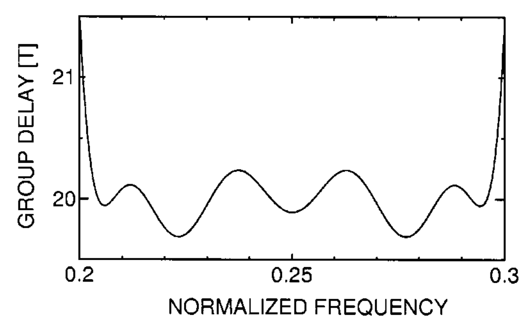

Fig. 15. Group delay of $H(z)$ in Example 3.

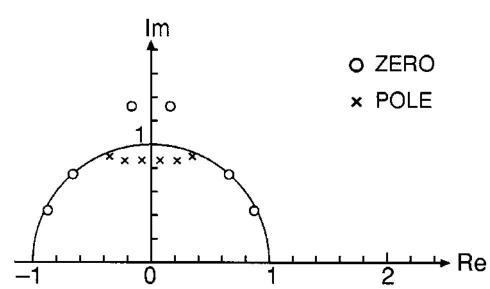

Fig. 16. Pole-zero location of $H(z)$ in Example 3.

TABLE V
A COMPARISON OF RESULTS IN EXAMPLE 3

	$A_p(dB)$	$A_s(dB)$	τ	Q
I.R.Gramian [15]	0.977	42.01	14.0	0.38%
Freq.Weighting [23]	0.984	42.72	14.0	0.55%
This Method	0.110	58.79	20.0	3.78%

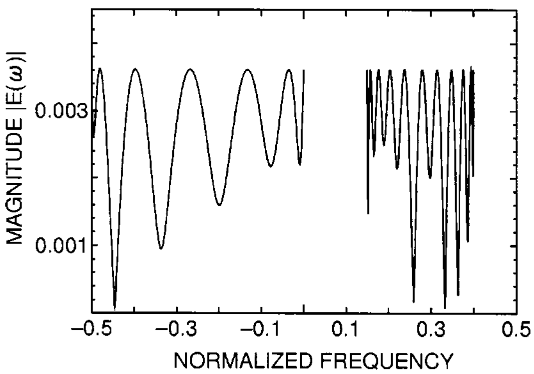

Fig. 17. Magnitude response of $E(\omega)$ in Example 4.

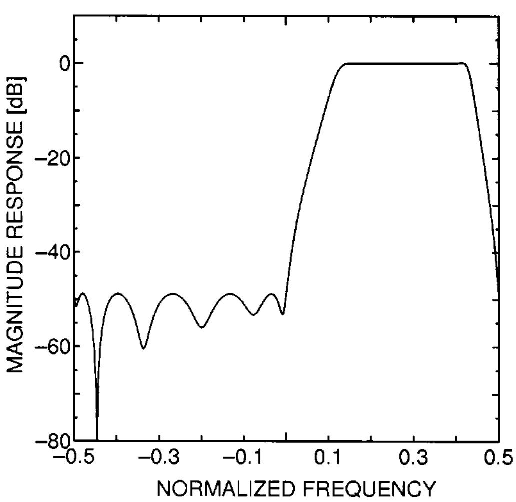

Fig. 18 . Magnitude response of $H(z)$ in Example 4.

A_s are the maximum passband and minimum stopband attenuations in decibels, respectively. τ is the group delay and Q is the group delay quality factor defined by [23]

$$Q = \frac{\tau_{\max} - \tau_{\min}}{\tau_{\max} + \tau_{\min}}. \tag{15}$$

It is seen in Table V that the magnitude response of the obtained filter is better than those in [15] and [23], whereas the group delay is worse. We have also tried to design an IIR filter with the same group delay ($\tau = 14$) as in [15] and [23] but cannot obtain a causal and stable solution.

Example 4: We consider a complex-valued IIR filter with $N = 10, M = 6$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j12\omega}, & (0.3\pi \le \omega \le 0.8\pi) \\ 0, & (-\pi \le \omega \le 0). \end{cases}$$

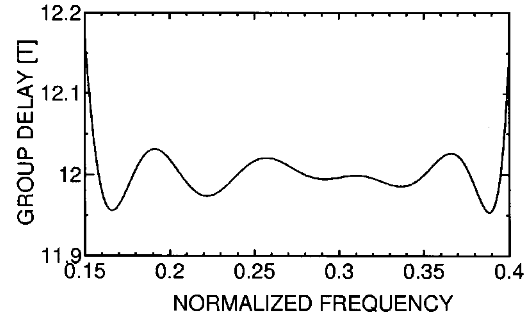

Fig. 19. Group delay of $H(z)$ in Example 4.

The weighting is set to $W(\omega) = 2$ in passband and $W(\omega) = 1$ in stopband. The obtained magnitude response of $E(\omega)$ is shown in Fig. 17, and the maximum error is $\delta_{\max} = 0.003\,62$. The resulting magnitude response and group delay of $H(z)$ are shown in Figs. 18 and 19, respectively. The magnitude error is 0.001 71 in passband and 0.003 62 (48.8 dB) in stopband, respectively, and the maximum deviation from the desired group delay $\tau = 12$ is 0.152. We have also calculated the poles of the resulting filter and found that it is causal and stable.

VIII. CONCLUSIONS

In this paper, we have proposed an efficient method for designing complex IIR digital filters in the complex Chebyshev sense. The proposed method is based on the formulation of a generalized eigenvalue problem by using the Remez multiple exchange algorithm. Hence, the filter coefficients can be easily obtained by solving the eigenvalue problem to find the absolute minimum eigenvalue, and then the complex Chebyshev approximation is attained through a few iterations starting from a given initial guess. The proposed algorithm is computationally efficient because it not only retains the speed inherent in the Remez exchange algorithm but also simplifies the interpolation step. Since the convergence is dependent on the initial solution, the proposed algorithm maybe fail to converge, in particular, in the design of high-order and multiband filters. Finally, it also has been shown through some design examples that the results obtained by using the method proposed in this paper are better than those obtained by the conventional methods. The proposed method can be applied to the design of Hilbert transformers and differentiators.

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