Design of IIR Orthogonal Wavelet Filter Banks Using Lifting Scheme

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***Abstract*—The lifting scheme is well known to be an efficient tool for constructing second generation wavelets and is often used to design a class of biorthogonal wavelet filter banks. For its efficiency, the lifting implementation has been adopted in the international standard JPEG2000. It is known that the orthogonality of wavelets is an important property for many applications. This paper presents how to implement a class of infinite-impulse-response (IIR) orthogonal wavelet filter banks by using the lifting scheme with two lifting steps. It is shown that a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. Then, the design of the proposed IIR orthogonal wavelet filter banks is discussed. The designed IIR orthogonal wavelet filter banks have approximately linear phase responses. Finally, the proposed IIR orthogonal wavelet filter banks are applied to the image compression, and then the coding performance of the proposed IIR filter banks is evaluated and compared with the conventional wavelet transforms.**

***Index Terms*—Allpass filter, approximate linear phase, infinite-impulse-response (IIR) filter, image coding, lifting scheme, orthogonal wavelet filter bank.**

I. Introduction

The discrete wavelet transform (DWT) has been applied extensively to digital signal and image processing [1]–[24]. It is well known that the wavelet bases can be generated by two-band perfect reconstruction filter banks. Both of the orthogonality and symmetry of wavelets are desirable properties for many applications. The symmetry requires all filters in the filter banks to possess exactly linear phase. It is known in [2] that finite-impulse-response (FIR) filters (corresponding to the compactly supported wavelets) can easily realize the linear phase response. However, it is widely appreciated [1]–[4] that the only FIR solution that produces a orthogonal symmetric wavelet basis is the Haar wavelet, which is not continuous. To get more regularity than the Haar wavelet, one of the constraints will be relaxed. Therefore, various classes of orthogonal wavelet filter banks with approximately linear phase responses and biorthogonal wavelet filter banks with exactly linear phase responses have been proposed by using FIR [1]–[4], [11] and IIR filters [7], [10]–[13], [19]. On the other hand, it is shown in [10] that IIR filter banks can produce the orthogonal symmetric wavelet bases. A class of IIR orthogonal symmetric wavelet filter banks has been proposed by using allpass filters in [10], [18], [22], [23].

The lifting scheme proposed by Sweldens in [14] and [15] is an efficient tool for constructing second generation wavelets, and has advantages such as faster implementation, fully in-place calculation, reversible integer-to-integer transforms, and so on. It has been proved in [16] and [17] that every wavelet transform with FIR filters can be decomposed into a finite number of lifting steps. However, it is not always possible for IIR wavelet filter banks to be decomposed into a finite number of lifting steps. For example, it is difficult to realize the IIR orthogonal symmetric wavelet filter banks proposed in [10], [18], [22], [23] by a finite number of lifting steps. In general, the lifting scheme is often used to construct a class of biorthogonal wavelet filter banks. Although the existing orthogonal FIR filter banks can be realized by lifting scheme, the number of the lifting step is required to be more than two, except for the Haar wavelet. In lossy to lossless image coding application, more lifting steps mean that more rounding errors are involved, resulting in degradation of the coding performance [21]. In this paper, we will consider the design of IIR orthogonal wavelet filter banks with two lifting steps. It should be noted that the Haar wavelet is the only FIR orthogonal wavelet filter bank with two lifting steps.

This paper presents how to implement IIR orthogonal wavelet filter banks by using the lifting scheme with two lifting steps. First, we derive the transfer functions for subfilters in the lifting steps from the orthogonality condition of wavelets. It is shown that a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. Then, we discuss the design problem of the proposed IIR orthogonal wavelet filter banks with the flat or equiripple frequency responses, and show that the resultant IIR orthogonal wavelet filter banks have approximately linear phase responses. Finally, we apply the proposed IIR orthogonal wavelet filter banks to the image compression, and investigate the coding performance of the proposed IIR filter banks by using the reference software of JPEG2000 provided in [24]. The coding results are compared with the wavelet transforms supported by the baseline codec of JPEG2000. It can be seen from the experimental results that the proposed IIR orthogonal wavelet filter banks can achieve a better coding performance than the conventional wavelet transforms.

This paper is organized as follows. The lifting scheme is briefly reviewed in Section II. In Section III, the transfer functions of subfilters in the lifting steps are derived from the condition of the orthogonality. In Section IV, the design of the proposed IIR orthogonal wavelet filter banks is discussed, and some design examples are shown. In Section V, the evaluation

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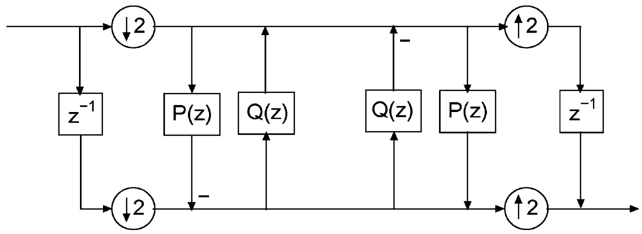

Fig. 1. Lifting scheme.

and comparison of the coding performance with the conventional wavelet transforms are presented. Finally, Section VI contains a conclusion.

II. Lifting Scheme

The lifting scheme is an efficient tool for constructing second generation wavelets, and has many advantages such as faster implementation, fully in-place calculation, and so on [14], [15]. The similar structure has been proposed in 1992 by Breukers and van den Enden [8], and has been extended for the filter bank design in [11]. It has been proven in [16] and [17] that every wavelet transform with FIR filters can be decomposed into a finite number of lifting steps, thus, this allows the construction of an integer version of the transform. Such integer wavelet transforms are invertible, and then are attractive in lossless coding applications. Due to these properties, the lifting implementation has been adopted in the international standard JPEG2000 [5], [24]. Conventionally, the lifting scheme is often used to construct a class of biorthogonal wavelet filter banks [14], [15]. It has been shown in [16] that the orthogonal wavelet filter banks can also be realized by lifting scheme, however, more than two lifting steps are needed, except for the Haar wavelet that has only two lifting steps. In lossy to lossless image coding application, since more lifting steps mean that more rounding errors are involved, wavelet filter banks with fewer lifting steps will tend to perform better, resulting in less degradation in the coding performance [21]. In this paper, we restrict ourselves to the lifting scheme with two lifting steps shown in Fig. 1. In Fig. 1, subfilter $P(z)$ is a prediction operator and $Q(z)$ is an update operator. It is clear that a reversible integer-to-integer transform can be easily realized by rounding the outputs of subfilters $P(z)$ and $Q(z)$ in the lifting scheme, since the inverse wavelet transform is immediately derived [14], [15]. Therefore, the lifting scheme is attractive in lossless coding applications, where the original image can be completely restored after decoding the compressed image.

Let $H_0(z)$ and $H_1(z)$ be a pair of lowpass and highpass filters in the analysis bank. Their transfer functions are given by

$$H_0(z) = 1 + Q(z^2)H_1(z) \tag{1}$$
$$H_1(z) = z^{-1} - P(z^2). \tag{2}$$

Therefore, the design problem of the filter bank $H_0(z)$ and $H_1(z)$ becomes how to determine two transfer functions $P(z)$ and $Q(z)$ to meet the given design specification, such as regularity, vanishing moments, frequency localization, and so on. In the design of $P(z)$ and $Q(z)$, FIR filters are often used so far to get a class of biorthogonal wavelet filter banks with exact linear phase [11]–[17], while IIR filters have been also used to design a class of biorthogonal wavelet filter banks with the causal stability [11] or with exact linear phase [19]. It is still open to design the IIR orthogonal wavelet filter banks by using the lifting scheme. In the following, we will describe how to obtain a class of orthogonal wavelet filter banks by using IIR allpass filters in the lifting scheme.

III. IIR Orthogonal Wavelet Filter Banks

It is well-known that the orthogonality of wavelets is an important property for many applications of digital signal and image processing. The orthogonality conditions of the filter banks are given by

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = c_1 \tag{3}$$
$$H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) = c_2 \tag{4}$$
$$H_0(z)H_1(z^{-1}) + H_0(-z)H_1(-z^{-1}) = 0 \tag{5}$$

where c_1 and c_2 are constants, and $c_1 = c_2 = 2$ in the case of the orthonormal wavelet filter banks.

Now, we will derive the conditions imposed on the subfilters $P(z)$ and $Q(z)$ from the conditions of the orthogonality in (3), (4), and (5). First, by substituting $H_1(z)$ in (2) into (4), we have

$$H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) = 2 + 2P(z^2)P(z^{-2}) = c_2 \tag{6}$$

that is

$$P(z)P(z^{-1}) = \frac{c_2}{2} - 1. \tag{7}$$

For $H_1(z)$ to be a highpass filter, we choose $c_2 = 4$ to ensure $P(z)$ with unit gain. Thus, we get from (7)[1]

$$\left|P(e^{j\omega})\right|^2 = 1 \tag{8}$$

which means that $P(z)$ should possess a constant magnitude response at all frequencies. Therefore, $P(z)$ must be an allpass filter and is defined by

$$P(z) = z^M A(z) = z^{M-N} \frac{\sum_{n=0}^{N} a_n z^n}{\sum_{n=0}^{N} a_n z^{-n}} \tag{9}$$

where M and N are the degree of the delay section and the allpass filter $A(z)$ respectively, a_n are real-valued filter coefficients, and $a_0 = 1$. Therefore, the highpass filter $H_1(z)$ becomes

$$H_1(z) = z^{-1} - z^{2M} A(z^2). \tag{10}$$

Next, we substitute $H_1(z)$ in (10) and $H_0(z)$ in (1) into (5) and get

$$H_0(z)H_1(z^{-1}) + H_0(-z)H_1(-z^{-1}) = 4Q(z^2) - 2P(z^{-2}) = 0 \tag{11}$$

that is

$$Q(z) = \frac{1}{2}P(z^{-1}) = \frac{1}{2}z^{-M}A(z^{-1}). \tag{12}$$

[1]Note that we consider the filter banks with real-valued coefficients only.

Thus, $H_0(z)$ becomes

$$H_0(z) = 1 + \frac{1}{2} z^{-2M} A(z^{-2}) \left\{ z^{-1} - z^{2M} A(z^2) \right\} = \frac{1}{2} \left\{ 1 + z^{-2M-1} A(z^{-2}) \right\}. \quad (13)$$

Finally, we will check whether or not $H_0(z)$ in (13) meets the condition in (3). By substituting $H_0(z)$ in (13) into (3), we have

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 1. \quad (14)$$

If we choose $c_1 = 1$, then the orthogonality conditions in (3), (4) and (5) are satisfied. Therefore, a class of orthogonal wavelet filter banks is realized by using the allpass filter $A(z)$ in the lifting step.

If we choose $N = 0$ in (9), then $P(z)$ becomes

$$P(z) = z^M \quad (15)$$

which is FIR filter, specifically, a pure delay. Therefore, we have

$$H_0(z) = \frac{1}{2}\{1 + z^{-2M-1}\} \quad (16)$$

$$H_1(z) = z^{-1} - z^{2M}. \quad (17)$$

To get a pair of reasonable lowpass and highpass filters, it is clear that $M = 0$ or $M = -1$ has to be chosen. The obtained orthogonal wavelet filter banks are correspondent to the (unnormalized) Haar wavelets in [16]. Therefore, the Haar wavelet is the only FIR orthogonal wavelet filter bank with two lifting steps.

IV. Design of IIR Orthogonal Wavelet Filter Banks

In this section, we will discuss the design of the proposed IIR orthogonal wavelet filter banks. For convenience, we rewrite $H_0(z)$ and $H_1(z)$ as

$$H_0(z) = \frac{1}{2} \left\{ 1 + z^{-2M-1} A(z^{-2}) \right\} \quad (18)$$

$$H_1(z) = z^{-1} - z^{2M} A(z^2). \quad (19)$$

It can be seen that $H_0(z)$ and $H_1(z)$ are composed of the delay section and allpass filter $A(z)$ [20]. Therefore, the design problem of the proposed IIR orthogonal wavelet filter banks can be reduced to the phase approximation of the allpass filter $A(z)$.

Assume that $\theta(\omega)$ is the phase response of $A(z)$. The magnitude and phase responses of $H_0(z)$ and $H_1(z)$ are given by

$$\left|H_0(e^{j\omega})\right| = \left|\cos\left\{\left(M + \frac{1}{2}\right)\omega + \frac{\theta(2\omega)}{2}\right\}\right| \quad (20)$$

$$\left|H_1(e^{j\omega})\right| = 2\left|\sin\left\{\left(M + \frac{1}{2}\right)\omega + \frac{\theta(2\omega)}{2}\right\}\right| \quad (21)$$

$$\theta_0(\omega) = -\left(M + \frac{1}{2}\right)\omega - \frac{\theta(2\omega)}{2} \quad (22)$$

$$\theta_1(\omega) = \left(M - \frac{1}{2}\right)\omega + \frac{\theta(2\omega)}{2} - \frac{\pi}{2} \quad (23)$$

where $\theta_0(\omega)$ and $\theta_1(\omega)$ are the phase responses of $H_0(z)$ and $H_1(z)$, respectively. Note that the maximum magnitudes of $H_0(z)$ and $H_1(z)$ in the passband are 1 and 2, respectively, which are the same as the wavelet transforms supported by the baseline codec of JPEG2000, to avoid the dynamic range growth of the transform coefficients in successive lowpass decomposition [5], [24].

For $H_0(z)$ and $H_1(z)$ to be a pair of lowpass and highpass filters, the phase response $\theta(\omega)$ of $A(z)$ must satisfy

$$\left(M + \frac{1}{2}\right)\omega + \frac{\theta(2\omega)}{2} = \begin{cases} 0 & (0 \le \omega \le \omega_p) \\ \pm\frac{\pi}{2} & (\omega_s \le \omega \le \pi) \end{cases} \quad (24)$$

where ω_p and ω_s are the edge frequencies of $H_0(z)$ in the passband and stopband, respectively, and $\omega_p + \omega_s = \pi$. Since the phase response $\theta(\omega)$ is periodic with period 2π and antisymmetric between the positive and negative frequency region, it is straightforward from (24) that the desired phase response of $A(z)$ is

$$\theta_d(\omega) = -\left(M + \frac{1}{2}\right)\omega \quad (0 \le \omega \le 2\omega_p). \quad (25)$$

Once the phase response $\theta(\omega)$ of $A(z)$ is approximated to the desire phase response $\theta_d(\omega)$ in (25), it is clear from (22) and (23) that $H_0(z)$ and $H_1(z)$ have approximate linear phase responses.

It is known in [1]–[4] that the flat-frequency responses of the wavelet filter banks are generally required for the regularity of wavelets. For $H_0(z)$ and $H_1(z)$ to have the flat-frequency responses, the allpass filter $A(z)$ must have a flat phase response [18], [22], that is

$$\left.\frac{\partial^i \theta(\omega)}{\partial \omega^i}\right|_{\omega=0} = \begin{cases} -\left(M + \frac{1}{2}\right) & (i = 1) \\ 0 & (i = 2, 3, \cdots, 2K) \end{cases} \quad (26)$$

where K is integer, and $0 < K \le N$. For the maximally flat design with $K = N$, it has been shown in [18] that the filter coefficients a_n of $A(z)$ can be analytically determined. Thus, the closed-form solution is given by

$$a_n = \binom{N}{n} \prod_{i=1}^{n} \frac{N - M - i + \frac{1}{2}}{M + i + \frac{1}{2}}. \quad (27)$$

On the other hand, the optimal design with an equiripple frequency response is also needed in many applications of signal processing [2], [11], [19]. The optimization method for designing the equiripple phase response of allpass filters has been proposed in [20] by using the well-known Remez exchange algorithm, where the obtained equiripple solution is optimal in the minimax (Chebyshev) sense. Furthermore, the allpass filters with a specified degree of flatness $(K < N)$ can also be designed by the approach proposed in [22]. In the following, we will design some examples and examine the property of the proposed IIR orthogonal wavelet filter banks.

Example 1: We consider the design of the maximally flat (MF) filter banks with $N = 2$, and have designed $A(z)$ with various M. It has been found that if $M > N$ or $M < -(N+1)$, we cannot obtain a pair of reasonable lowpass and highpass filters. We then examine only the MF filters with $-(N+1) \le M \le N$ below. The obtained phase responses of $A(z)$ are shown in Fig. 2. It can be seen in Fig. 2 that the phase responses of $A(z)$ with $M = k$ and $M = -(k+1)$ for $k = 0, 1, \cdots, N$ are mutually symmetric, and we have found that their poles each

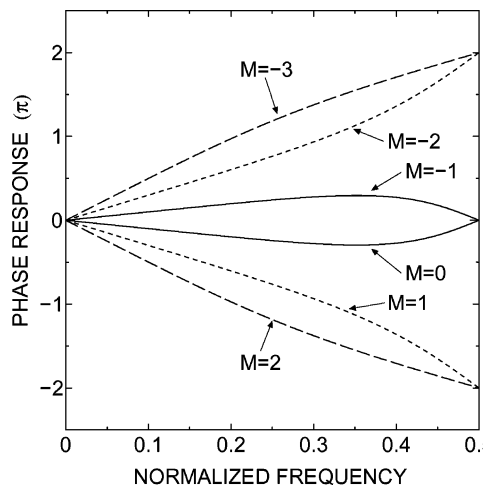

Fig. 2. Phase responses of $A(z)$ in Example 1.

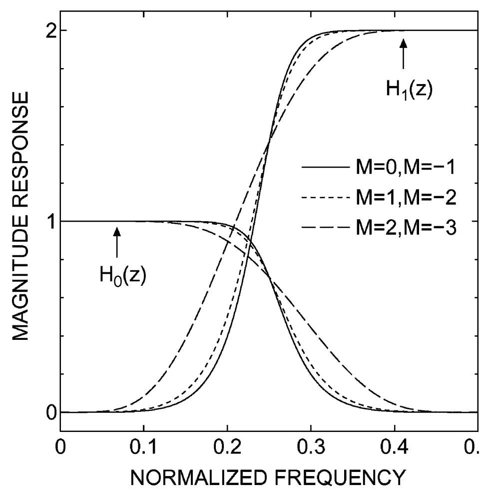

Fig. 3. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 1.

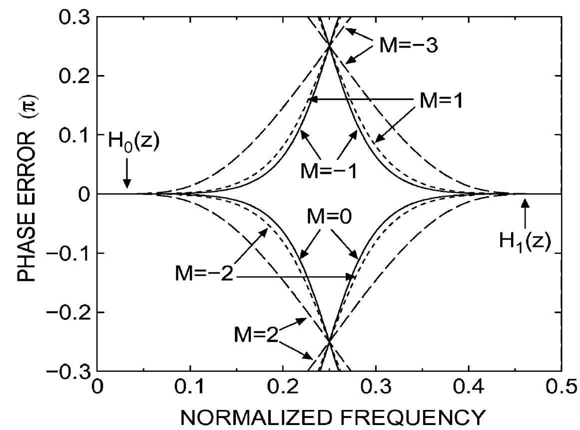

Fig. 4. Phase errors of $H_0(z)$ and $H_1(z)$ in Example 1.

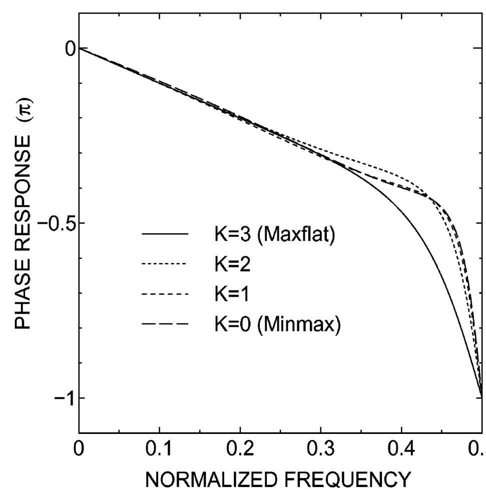

Fig. 5. Phase responses of $A(z)$ in Example 2.

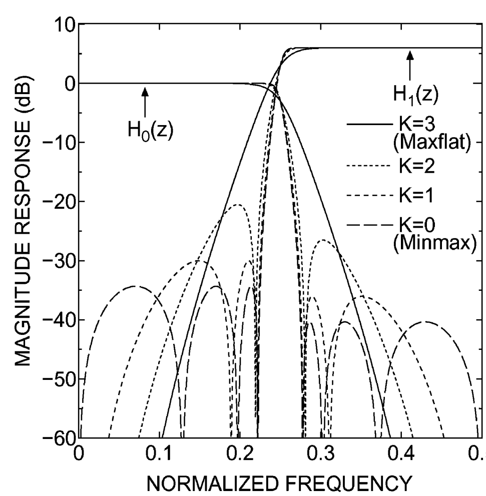

Fig. 6. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 2.

other satisfy the mirror-image relation with respect to the unit circle. The magnitude responses of $H_0(z)$ and $H_1(z)$ are shown in Fig. 3, while their phase errors are shown in Fig. 4. It is clear in Figs. 3 and 4 that the filter banks $H_0(z)$, $H_1(z)$ with $M = k$ and $M = -(k+1)$ have the same magnitude responses and symmetric phase errors. Furthermore, it is seen in Fig. 3 that the magnitude responses of the filter bank with $M = 0 (M = -1)$ are more flat than that with other M. It is also found that when $M = N$ and $M = N - 1$, $A(z)$ has its poles inside the unit circle and becomes causal stable.

Example 2: We consider the design of IIR filter banks with $N = 3$ and $M = 0$. First, we have designed $A(z)$ with the maximally flat phase response $(K = 3)$, and shown its phase response in the solid line in Fig. 5. Next, we set $\omega_p = 0.45\pi$ and use the design method of allpass filters proposed in [20] to get $A(z)$ with the equiripple phase response $(K = 0)$. The obtained phase response is also shown in the dashed line in Fig. 5, and it is optimal in the minimax (Chebyshev) sense. Other two allpass filters with $K = 1$ and $K = 2$ have also been designed by using the approach proposed in [22], and their phase responses are shown in Fig. 5. The magnitude responses of these filter banks are shown in Fig. 6, and the phase errors are shown in Fig. 7. It can be seen in Figs. 6 and 7 that the magnitude and phase errors decrease as K decreases.

V. Image Coding Application

In this section, we apply the proposed IIR orthogonal wavelet filter banks only with the maximally flat-frequency responses to

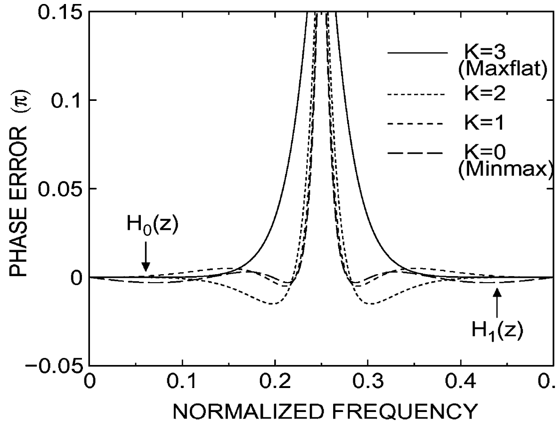

Fig. 7. Phase errors of $H_0(z)$ and $H_1(z)$ in Example 2.

the wavelet-based image coding and investigate its coding performance, because the maximally flat filter banks produce the most numbers of vanishing moments of the wavelets [1], which potentially influence the coding performance [21]. We have used the reference software of JPEG2000 provided in [24] to evaluate the coding performance of the proposed IIR filter banks. Eight images (Barbara, Boat, Crowd, Goldhill, Lena, Mandrill, Woman, and Zelda) of size 512×512, 8 bit per pixel (bpp) have been used as test images. The decomposition level of the wavelet transform is set to 6. Since the proposed IIR filters have only approximate linear phase responses, a simple extension technique where the signals are extended by repetition of their first and last sample values is employed to handle filtering at the signal boundaries, instead of the symmetric extension. Allpass filters $A(z)$ that include the poles inside and outside the unit circle are divided into the causal part $A_c(z)$, which has the poles inside the unit circle only, and anticausal part $A_a(z)$, which has the poles outside the unit circle only, that is, $A(z) = A_c(z)A_a(z)$. Then, the anticausal part $A_a(z)$ is realized by reversing the input signal, filtering it with $A_a(z^{-1})$ that has the poles inside the unit circle only, and then rereversing the output signal.

A. Lossless Coding Performance

First, we investigate the lossless coding performance of the proposed IIR orthogonal wavelet filter banks, and then compare the performance with the reversible integer-to-integer wavelet transform D-5/3 supported by the baseline codec of JPEG2000 [24].

1) Influence of N and M: We examine the influence of the parameters N and M on the lossless coding performance of the IIR filter banks, since the maximally flat filter banks $(K = N)$ are used here. It is seen in *Example 1* that the filter banks with $M = k$ and $M = -(k+1)$ for $k = 0, 1, \cdots, N$ have the same magnitude responses. Therefore, we will investigate the lossless coding performance of the filter banks only with $0 \leq M \leq N$. The lossless coding results of the filter banks with $N = 1$, $N = 2$ and $N = 3$ are given in Table I, Table II, and Table III, respectively. For each image, the best result has been highlighted. It is seen in Table I that the filter bank with $M = 0$ has a lower bit rate than that with $M = 1$ when $N = 1$. If $N = 2$, there are seven images getting the best lossless coding performance when $M = 1$, while two images when $M = 0$. Thus, the lowest average bit rate is obtained when $M = 1$. When $N = 3$, the filter bank with $M = 2$ has the lowest average bit rate, although one with $M = 0$ got the best lossless coding performance for three images. Therefore, $M = N - 1$ is chosen in general, from the viewpoint of the average bit rate. It is because the filter bank with $M = N - 1$ has a more flat-frequency response than that of $M = N$ as shown in *Example 1* and has all poles inside the unit circle, thus needs not be divided into the causal and anticausal parts. The filter banks with $0 \leq M \leq N - 2$ must be divided into the causal and anticausal parts to implement, since some poles

TABLE I
LOSSLESS CODING RESULTS: BIT RATE (bpp)

[N=1]	M=0	M=1
Barbara	**4.567**	4.735
Boat	**4.440**	4.510
Crowd	**4.241**	4.371
Goldhill	**4.892**	4.925
Lena	**4.337**	4.414
Mandrill	**6.140**	6.193
Woman	**3.346**	3.415
Zelda	**3.974**	4.058
Average	**4.492**	4.578

TABLE II
LOSSLESS CODING RESULTS: BIT RATE (bpp)

[N=2]	M=0	M=1	M=2
Barbara	**4.503**	4.512	4.627
Boat	4.447	**4.430**	4.461
Crowd	4.247	**4.231**	4.279
Goldhill	4.892	**4.885**	4.901
Lena	4.333	**4.327**	4.362
Mandrill	**6.128**	**6.128**	6.154
Woman	3.331	**3.330**	3.366
Zelda	3.955	**3.954**	3.998
Average	4.479	**4.475**	4.519

TABLE III
LOSSLESS CODING RESULTS: BIT RATE (bpp)

[N=3]	M=0	M=1	M=2	M=3
Barbara	**4.486**	4.496	4.494	4.574
Boat	4.444	4.457	**4.434**	4.446
Crowd	4.255	4.263	**4.237**	4.251
Goldhill	4.886	4.895	**4.885**	4.892
Lena	4.333	4.337	**4.326**	4.345
Mandrill	**6.127**	6.130	**6.127**	6.139
Woman	3.327	3.332	**3.325**	3.348
Zelda	**3.946**	3.949	3.949	3.978
Average	4.475	4.482	**4.472**	4.497

TABLE IV
COMPARISON OF LOSSLESS CODING RESULTS: BIT RATE (bpp)

	D-5/3	N1-M0	N2-M1	N3-M2
Barbara	4.697	4.567	4.512	**4.494**
Boat	4.440	4.440	**4.430**	4.434
Crowd	4.236	4.241	**4.231**	4.237
Goldhill	**4.873**	4.892	4.885	4.885
Lena	4.350	4.337	4.327	**4.326**
Mandrill	6.151	6.140	6.128	**6.127**
Woman	3.347	3.346	3.330	**3.325**
Zelda	4.021	3.974	3.954	**3.949**
Average	4.514	4.492	4.475	**4.472**

are located outside the unit circle. Then an extra extension is required after the causal filtering, which may influence the lossless coding performance, although these filter banks have slightly more flat-frequency responses than that of $M = N - 1$. It can be seen also in Table IV that the filter bank of $N = 3$ has the best average lossless coding performance. This is because the regularity of wavelets (the flatness of the filter banks) increases with an increasing order N of allpass filters. When N is further increased, we can only get a little improvement. However, the computational complexity required in the implementation of the proposed wavelet transforms becomes higher.

2) Comparison With the Conventional Wavelet D-5/3: We compare the lossless coding performance of the proposed IIR orthogonal wavelet filter banks with the conventional wavelet filter banks. The reversible integer-to-integer wavelet transform D-5/3 supported by the baseline codec of JPEG2000 is chosen as a comparison object. The comparison results of lossless coding performance are shown in Table IV. It is seen from the experimental results in Table IV that the proposed IIR orthogonal wavelet filter banks with approximate linear phase responses have better lossless coding performance than the conventional wavelet transform D-5/3, although there is one image Goldhill getting the best result for D-5/3.

B. Lossy Coding Performance

We further investigate the lossy coding performance of the proposed IIR filter banks, and compare its lossy coding performance with the wavelet transforms supported by the baseline codec of JPEG2000. In the baseline codec of JPEG2000, two types of wavelet transforms have been employed: the irreversible real-to-real wavelet transform D-9/3 and the reversible integer-to-integer wavelet transform D-5/3.

1) Irreversible Real-to-Real Wavelet Transform: We examine the lossy coding performance of irreversible real-to-real wavelet transform. In the proposed IIR orthogonal wavelet filter banks, the rounding operation is not used in the lifting steps, then the transform is irreversible (real-to-real). We have investigated three filter banks with $N = 1$, $N = 2$, and $N = 3$, where $M = N - 1$ is set for all filter banks. The lossy coding results for images Barbara and Lena are given in Fig. 8 and

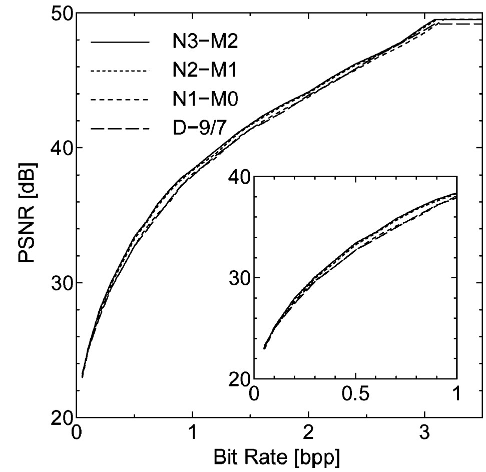

Fig. 8. Lossy coding performance of irreversible wavelet transform for image Barbara.

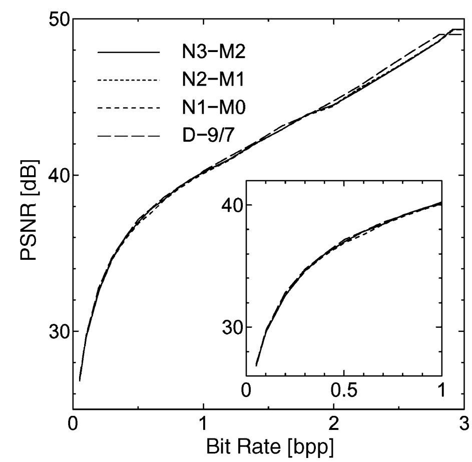

Fig. 9. Lossy coding performance of irreversible wavelet transform for image Lena.

Fig. 9, respectively. It is seen in Fig. 8 that the proposed IIR orthogonal wavelet filter banks have better lossy coding performance than D-9/7 for image Barbara, while almost same results are obtained for image Lena, as shown in Fig. 9. For example, at 0.5 bpp for image Barbara, the proposed filter bank with $N = 3$ has the peak signal-to-noise ratio (PSNR) of 33.415 dB, while D-9/7 is 32.753 dB. To measure the subjective visual quality, the original and reconstructed images of Lena with D-9/7 and with the filter banks of $N = 2$ and $N = 3$ are shown in Figs. 10–13, respectively.

2) Reversible Integer-to-Integer Wavelet Transform: We examine the lossy coding performance of reversible integer-to-integer wavelet transform. To obtain the reversible integer-to-integer wavelet transform, the rounding operation must be used in the lifting steps to round the output of the subfilters $P(z)$ and $Q(z)$ in Fig. 1. We have also investigated three filter banks with $N = 1$, $N = 2$, and $N = 3$ $(M = N - 1)$. The lossy coding results for images Barbara and Lena are given in Fig. 14 and Fig. 15, respectively. It is seen in Fig. 14 that the proposed IIR orthogonal wavelet filter banks have better lossy coding performance than D-5/3 for image Barbara. For image Lena, almost same results are obtained at low bit rate, but D-5/3 is better at high bit rate, as shown in Fig. 15.

Fig. 10. Original image Lena (512 × 512, 8 bpp).

Fig. 11. Reconstructed image with the wavelet D-9/7 at 0.503 bpp (PSNR: 37.173 dB).

Fig. 12. Reconstructed image with the filter bank of N2-M1 at 0.501 bpp (PSNR: 36.975 dB).

Fig. 13. Reconstructed image with the filter bank of N3-M2 at 0.501 bpp (PSNR: 36.980 dB).

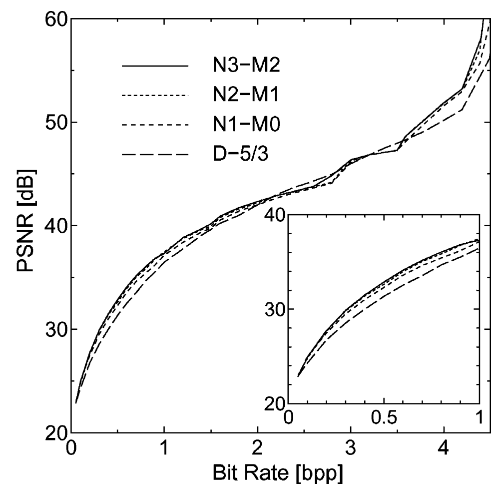

Fig. 14. Lossy coding performance of reversible wavelet transform for image Barbara.

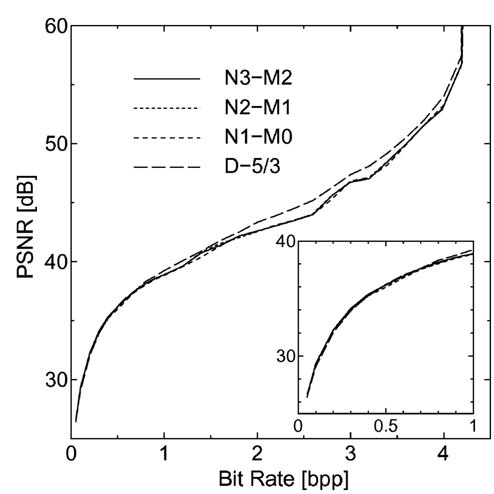

Fig. 15. Lossy coding performance of reversible wavelet transform for image Lena.

C. Comparison of Computational Complexity

It is known that allpass filters $A(z)$ with real-valued coefficients can be decomposed into the cascade connection of first-order and second-order allpass filters. The first-order allpass filters require one multiplier and two adders respectively, while the second-order allpass filters require two multipliers and four adders [2]. Therefore, allpass filters $A(z)$ of order N require only N multipliers and $2N$ adders for implementation. The analysis bank in the proposed wavelet filter banks then requires $2N$ multipliers and $4N + 2$ adders. The comparison of computational complexity with the conventional wavelets D-5/3 and D-9/7 is given in Table V. It is seen that for the proposed wavelet filter banks, the number of multipliers N_M is not

TABLE V
COMPARISON OF COMPUTATIONAL COMPLEXITY

	N=1	N=2	N=3	D-5/3	D-9/7
N_M	2	4	6	2	6
N_A	6	10	14	4	8

TABLE VI
FILTER COEFFICIENTS OF ALLPASS FILTERS

	N1-M0	N2-M1	N3-M2
a_0	1.00000000	1.00000000	1.00000000
a_1	0.33333333	0.40000000	0.42857143
a_2		-0.02857143	-0.04761905
a_3			0.00432900

more than D-9/7 and not less than D-5/3, while the number of adders N_A is more than D-9/7 beside the filter bank of $N = 1$.

For reference, the filter coefficients of the maximally flat allpass filters with $N = 1$, $N = 2$ and $N = 3$ $(M = N - 1)$ are given in Table VI, respectively.

VI. CONCLUSION

In this paper, we have presented how to design the IIR orthogonal wavelet filter banks by using the lifting scheme. It has been shown that a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. Thus, the obtained IIR orthogonal wavelet filter banks have approximate linear phase responses. Moreover, we have discussed the design problem of the proposed IIR orthogonal wavelet filter banks with the flat or equiripple frequency responses, and given some design examples to examine the filter property. Finally, we have applied the proposed IIR orthogonal wavelet filter banks to the wavelet-based image compression, and investigated the lossy and lossless coding performance. The coding performance results have been also compared with the wavelet transforms supported by the baseline codec of JPEG2000. It is seen from the experimental results that the proposed IIR orthogonal wavelet filter banks with approximate linear phase responses can achieve a better coding performance than the conventional wavelet transforms.

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