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# Design of FIR Nyquist Filters Using the Remez Exchange Algorithm

Xi Zhang

Faculty of Engineering, Nagaoka University of Technology, Nagaoka, Japan 940-21

Hiroshi Iwakura

Faculty of Electro-Communications, University of Electro-Communications, Chofu, Japan 182

## SUMMARY

In this paper, we propose a method of designing FIR Nyquist filters with zero intersymbol interference by using the Remez exchange algorithm directly. First, we present some magnitude properties of FIR Nyquist filters with zero intersymbol interference. It is known from the time-domain condition of zero intersymbol interference that the magnitude response of Nyquist filters in the passband is mainly dependent on the stopband magnitude. Therefore, the design problem of Nyquist filters becomes optimization of the magnitude response in the stopband. By using the Remez exchange algorithm in the stopband directly, we formulate the design problem in the form of a linear problem. The filter coefficients can be computed by simply solving the linear equations, and the optimal coefficients with equiripple stopband response are easily obtained after a few iterations. The proposed procedure is computationally more efficient than the existing procedures. Finally, we extend the proposed procedure to the design of matched Nyquist filter pairs, multistage Nyquist filters, and so on. Some design examples are presented to demonstrate the effectiveness of the proposed procedure.  Electron Comm Jpn Pt 3, **80**(6): 22–29, 1997

**Key words:** FIR linear phase filter; zero intersymbol interference; Remez exchange algorithm; Nyquist filter.

## 1. Introduction

Nyquist filters play an important role in designing data transmission systems and filter banks. Nyquist filters are required in order to band-limit the data spectrum and minimize intersymbol interference. Therefore, with the exception of one point, the impulse response must exactly cross zero at the Nyquist rate. Several procedures have been proposed for designing FIR Nyquist filters with zero intersymbol interference [2–10]. The procedures using linear programming techniques and nonlinear optimization methods [3–6] require a large amount of computer time, and it is difficult to obtain an equiripple stopband response. Compared with these procedures, the design method proposed in Ref. 8 is computationally efficient. In this approach, the transfer function of Nyquist filters is split into two parts. One takes care of the time-domain condition of zero intersymbol interference and is determined by solving a set of linear equations. The other provides an equiripple stopband response for the overall filter, and is designed using the McClellan–Parks method [1]. Two parts are alternately adjusted until the time-domain and the frequency-domain conditions are satisfied simultaneously. Therefore, design-

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ing an FIR Nyquist filter requires several applications of the McClellan–Parks design program and repeated solution of a set of linear equations.

In this paper, we propose a method of designing FIR Nyquist filters with zero intersymbol interference by using the Remez exchange algorithm directly. First, we investigate some magnitude properties of FIR Nyquist filters with zero intersymbol interference. From the time-domain condition of zero intersymbol interference, the magnitude response of Nyquist filters in the passband is mainly dependent on the stopband magnitude. Therefore, the design problem of Nyquist filters becomes the optimization of the magnitude response in stopband. In this paper, by using the magnitude response of Nyquist filters with zero intersymbol interference, we apply the Remez exchange algorithm in the stopband directly and formulate the design problem in the form of a linear problem. Then the filter coefficients can be computed by solving a set of linear equations, and the optimal coefficients with an equiripple stopband response can be easily obtained after a few iterations. The proposed procedure requires almost the same computation time as the McClellan–Parks method, and is computationally more efficient than the existing procedures. Finally, we extend the proposed procedure to the design of matched Nyquist filter pairs, multistage Nyquist filters, and so on. Some design examples are presented to demonstrate the effectiveness of the proposed procedure.

## 2. Properties of FIR Nyquist Filters

Let the transfer function $H(z)$ of a linear phase FIR filter of order $2N$ be

$$H(z) = \sum_{n=0}^{2N} h_n z^{-n} \tag{1}$$

where $h_n = h_{2N-n}$ are real. When $H(z)$ is used as Nyquist filter, from the time-domain condition of zero intersymbol interference, its impulse response is required to exactly cross zero at the Nyquist rate except for one point, that is,

$$\begin{cases} h_N = \dfrac{1}{M} & \\ h_{N+iM} = 0 & (i = \pm 1, \pm 2, \cdots) \end{cases} \tag{2}$$

where $M$ is an integer. To band-limit the data spectrum, the desired magnitude response of Nyquist filters is

$$H_d(\omega) = \begin{cases} 1 & (0 \leqq \omega \leqq \omega_p) \\ 0 & (\omega_s \leqq \omega \leqq \pi) \end{cases} \tag{3}$$

where $\omega_p = (1 - \rho)\pi/M$ is the cut-off frequency of the passband, $\omega_s = (1 + \rho)\pi/M$ is the cut-off frequency of the stopband, and $\rho$ is the rolloff rate. Using the time-domain condition of Eq. (2), the magnitude response of FIR Nyquist filters with zero intersymbol interference is given by

$$|H(e^{j\omega})| = \frac{1}{M} + \sum_{\substack{n=1 \\ n \neq iM}}^{N} a_n \cos(n\omega) \tag{4}$$

where $a_n = 2h_{N+n}$ [$n = 1,2, \ldots, N$ ($\neq iM$)]. Therefore, the problem of designing FIR Nyquist filters with zero intersymbol interference will become the problem of approximating the magnitude response of Eq. (4) to the desired magnitude response of Eq. (3).

Before designing FIR Nyquist filters, we investigate some properties of FIR Nyquist filters with zero intersymbol interference. The magnitude response of FIR Nyquist filters with zero intersymbol interference satisfies

$$\sum_{k=0}^{M-1} |H(e^{j(\frac{2k\pi}{M}+\omega)})| \equiv 1 \tag{5}$$

Since, by Eq. (4), $|H(e^{j(2\pi-\omega)})| = |H(e^{j\omega})|$, we can obtain

$$\sum_{k=0}^{M-1} |H(e^{j\omega_k})| \equiv 1 \tag{6}$$

where

$$\omega_k = \left[\frac{k+1}{2}\right]\frac{2\pi}{M} + (-1)^k\omega_0$$

and $[\cdot]$ denotes the integer part of $\cdot$. Equation (6) means that the sum of the magnitude responses of FIR Nyquist filters with zero intersymbol interference at the frequency points $\omega_0$,

$$\omega_1 = \frac{2\pi}{M} - \omega_0,$$

$$\omega_2 = \frac{2\pi}{M} + \omega_0, \ldots, \omega_{M-1} = \left[\frac{M}{2}\right]\frac{2\pi}{M} - (-1)^M\omega_0$$

is always unity regardless of the values of the coefficients $a_n$. Equation (6) can be rewritten as

$$|H(e^{j\omega_0})| = 1 - \sum_{k=1}^{M-1} |H(e^{j\omega_k})| \tag{7}$$

Therefore, as shown in Fig. 1, it is clear that if its stopband response is approximated to 0, then the magnitude response in passband will be 1. Let $\delta_s$ be the maximum magnitude

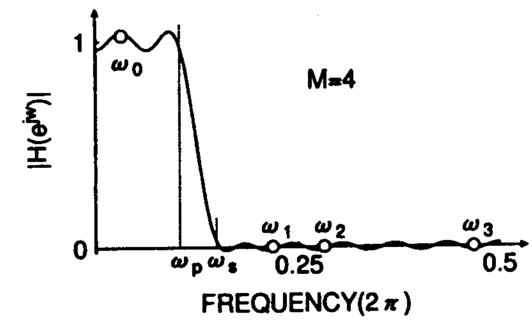


Fig. 1. Performance of FIR Nyquist filters.

error in the stopband; the maximum magnitude error $\delta_p$ in passband is then

$$\delta_p \leqq (M-1)\delta_s \tag{8}$$

In practical designs, the passband error is usually much smaller than this upper limit. Since $\delta_p$ is guaranteed to be relatively small for a small value of $\delta_s$, filter design can concentrate on shaping the stopband response. It can also be explained according to the zero locations. There are $L(= N - [N/M])$ unknown coefficients $a_n$ in Eq. (4), and thus the Nyquist filter has $2L$ independent zeros. These independent zeros are used to provide the desired stopband response and hence must be located on the unit circle to minimize stopband error. The $2[N/M]$ remaining zeros are used to satisfy the time-domain condition of zero intersymbol interference, so that passband response is formed in a natural manner. Therefore, the design of FIR Nyquist filters with zero intersymbol interference requires that we approximate the magnitude response of Eq. (4) in the stopband only.

## 3. Design of FIR Nyquist Filters

### 3.1. Formulation using the remez exchange algorithm

FIR Nyquist filters with zero intersymbol interference have $2L$ independent zeros. To minimize the magnitude error in the stopband, these independent zeros must be located on the unit circle. When all $2L$ zeros are located on the unit circle, there necessarily exist $(L + 1)$ extremal frequencies in the stopband $[\omega_s, \pi]$. Hence, we can select $(L + 1)$ extremal frequencies in the stopband as follows;

$$\omega_s = \omega_0 < \omega_1 < \cdots < \omega_L = \pi \tag{9}$$

Using the Remez exchange algorithm, we formulate the condition for the magnitude response of Eq. (4) in such a way that the amplitudes are equal and the sign alternating at these extremal frequencies $\omega_m$, that is,

$$W(\omega_m)|H(e^{j\omega_m})| = (-1)^m \delta \tag{10}$$

where $W(\omega)$ is a weighting function and $\delta(> 0)$ is the magnitude error. Substituting Eq. (4) into Eq. (10), we have

$$\sum_{\substack{n=1 \\ n \neq iM}}^{N} a_n \cos(n\omega_m) - \frac{(-1)^m}{W(\omega_m)}\delta = -\frac{1}{M} \tag{11}$$

which is a set of linear equations. Since there are $(L + 1)$ unknowns ($L$ filter coefficients $a_n$ and one magnitude error $\delta$) and $(L + 1)$ extremal frequencies, the filter coefficients can be uniquely determined by solving the linear equations of Eq. (11). Using the obtained filter coefficients $a_n$, we compute the magnitude response and search for the peak frequencies $\bar{\omega}_m$ in the stopband. However, the initially selected extremal frequencies $\omega_m$ cannot be guaranteed to be equal to the peak frequencies $\bar{\omega}_m$. Therefore, we use the obtained peak frequencies as the extremal frequencies in the next iteration and solve the linear equations of Eq. (11) so as to obtain the filter coefficients $a_n$ again. The above procedure is iterated until the extremal frequencies $\omega_m$ and the peak frequencies $\bar{\omega}_m$ are consistent. When the extremal frequencies do not change, we can obtain the optimal solution with an equiripple stopband response. The design algorithm is shown in detail as follows.

### 3.2. Design algorithm

1. Read Nyquist filter specifications $N$, $M$, $\rho$ and weighting function $W(\omega)$.
2. Select $(L + 1)$ initial extremal frequencies $\omega_m$ equally spaced in the stopband as shown in Eq. (9).
3. Solve the linear equations of Eq. (11) to obtain a set of filter coefficients $a_n$.
4. Compute the magnitude response of the filter by using the obtained filter coefficients $a_n$, and search for the peak frequencies $\bar{\omega}_m$ in the stopband.
5. If $|\bar{\omega}_m - \omega_m| < \varepsilon$ ($m = 0,1,\ldots, L$), where $\varepsilon$ is a specified small constant, are satisfied, then stop. Otherwise, go to the next step.
6. Set $\omega_m = \bar{\omega}_m$ ($m = 0,1, \ldots, L$), then go to step 3.

### 3.3. Comparison of computations

In this section, we compare the computation time of the design algorithm proposed in section 3.2 with that of the conventional methods. In the design of general FIR linear phase filters, the McClellan–Parks method is well-known and often used. In this method, the Remez exchange algorithm is used to formulate the design problem, and the magnitude response of the filter is computed by using the Lagrange interpolation formula without solving the linear equations directly. The peak frequencies are sought via the obtained magnitude response, and are used as the extremal frequencies in the next iteration. The above procedure is iterated until the peak frequencies do not change. Then an equiripple response is obtained, and a set of filter coefficients is computed by using the inverse discrete Fourier transform (IDFT). However, in the proposed method, since the time-domain condition of zero intersymbol interference has been included in the magnitude response of the Nyquist filters, the Lagrange interpolation formula cannot be used. Thus, we must solve the linear equations of Eq. (11) directly. From the time-domain condition of zero intersymbol interference, the number of unknown coefficients in the Nyquist filters is $M - 1/M$ of the same-order FIR filters, and the peak frequencies are sought in the stopband only. The filter coefficients are directly computed by solving the linear equations, and the IDFT need not be used. After considering these factors, although the computation time of the proposed method is slightly higher than in the McClellan–Parks method, they are nearly the same.

In the conventional methods for design of Nyquist filters, it is clear that procedures using linear programming techniques and nonlinear optimization methods [3–5] require large amounts of computation time. In the method using eigenfilters [6], the optimal solution in the least-square sense is obtained by finding the minimum eigenvalue. Its computation time is almost the same as the McClellan–Parks method. However, to obtain an equiripple response, an iteration procedure with weighting of the magnitude error is needed. Therefore, its computation time increases with the number of iterations. In the design method proposed in Ref. 8, the transfer function of a Nyquist filter is split into two parts. One deals with the time-domain condition of zero intersymbol interference and is determined by solving a set of linear equations. The other provides an equiripple stopband response for the overall filter and is designed using the McClellan–Parks method. Two parts are alternately adjusted until the time-domain and the frequency-domain conditions are satisfied simultaneously. Therefore, designing an FIR Nyquist filter requires several applications of the McClellan–Parks design program and repeated solution of a set of linear equations. From the above comparison, it is clear that the proposed method is computationally more efficient than the conventional methods.

## 4. Design of Matched Nyquist Filter Pairs

In data transmission systems, low-pass filters are used in the transmitted and received terminals to band-limit the spectra of the transmitted and received signals, respectively, while the overall impulse response is required to have zero intersymbol interference [3, 4, 9]. In this section, we consider design of the matched Nyquist filter pairs that satisfy the above conditions. Let a pair including the filters used in the transmitted and received terminals be the overall filter. The overall filter is an FIR linear phase filter, and its impulse response must satisfy the time-domain condition of zero intersymbol interference in Eq. (2). Therefore, the magnitude response of the overall filter with zero intersymbol interference can be expressed by Eq. (4). To obtain transmitted and received filters with the same magnitude response, the overall filter is required to have double zeros on the unit circle [3, 9]. If an overall filter with double zeros on the unit circle is obtained, we can split the double zeros and mirror-image pairs into the transmitted and received filters, respectively; then transmitted and received filters with the same magnitude response can be obtained. An overall filter with double zeros on the unit circle cannot be designed by using the design method proposed in section 3, because the filter obtained by the design algorithm in section 3.2 does not have double zeros on the unit circle. Hence, we must increase the magnitude of the obtained filter so that the zeros on the unit circle become double zeros [9]. Let $H(z)$ be the FIR Nyquist filter with zero intersymbol interference designed by using the design algorithm in section 3.2; its stopband error is

$$-\delta_s \leqq |H(e^{j\omega})| \leqq \delta_s \tag{12}$$

From the filter coefficients $h_n$ of $H(z)$, we construct a new transfer function $\bar{H}(z)$ as follows:

$$\begin{cases} \bar{h}_N = h_N + \delta_s = \dfrac{1}{M} + \delta_s \\ \bar{h}_n = h_n & (n \neq N) \end{cases} \tag{13}$$

Then the magnitude response of $\bar{H}(z)$ is

$$|\bar{H}(e^{j\omega})| = |H(e^{j\omega})| + \delta_s \geqq 0 \tag{14}$$

and an overall filter $\bar{H}(z)$ with double zeros on the unit circle is obtained. It is clear from Eq. (13) that if $H(z)$ satisfies the time-domain condition of zero intersymbol interference, then $\bar{H}(z)$ satisfies it also. We can rewrite Eq. (10) as

$$W(\omega_m)|H(e^{j\omega_m})| = \begin{cases} \delta & (m \ : \ \text{even}) \\ 0 & (m \ : \ \text{odd}) \end{cases} \tag{15}$$

then the overall filter with double zeros on the unit circle can be designed directly. The design algorithm is the same as that described in section 3.2.

## 5. Design of Multistage Nyquist Filters

When a sharp magnitude response (high stopband attenuation and narrow transition band) is required in the design of FIR Nyquist filters, the order of the filter will increase rapidly and many multipliers will be needed in order to implement it. In Ref. 8, multistage Nyquist filters are proposed as a means of splitting the filter into multiple subfilters, thus allowing an implementation with a decreasing number of multipliers when $M$ can be decomposed into several integers, for example,

$$M = M_1 M_2 \cdots M_K = \prod_{k=1}^{K} M_k \tag{16}$$

The transfer function of a multistage Nyquist filter can be expressed as

$$\begin{aligned} H(z) = H_1(z^{M_2 M_3 \cdots M_K}) H_2(z^{M_3 \cdots M_K}) \\ \cdots H_{K-1}(z^{M_K}) H_K(z) \end{aligned} \tag{17}$$

where the $k$-th subfilter $H_k(z)$ is a Nyquist filter of order $2N_k$. If $H_k(z)$ is designed to satisfy the time-domain condition of zero intersymbol interference, then the overall filter $H(z)$ satisfies the condition of zero intersymbol interference also. In the frequency domain, let the passband and stopband cut-off frequencies of $H(z)$ be $\omega_p = \{1-\rho\}/M\pi, \omega_s = \{1+\rho\}/M\pi$, and let the stopband error be $\delta_s$. Then the frequency specifications of subfilters $H_k(z)$ are as shown in Fig. 2, the passband $\Omega_{p1}$ and stopband $\Omega_{s1}$ of $H_1(z)$ are

$$\begin{cases} \Omega_{p1} = [0, \dfrac{1-\rho}{M_1}\pi] \\ \Omega_{s1} = [\dfrac{1+\rho}{M_1}\pi, \pi] \end{cases} \tag{18}$$

and the passband $\Omega_{pk}$ and stopband $\Omega_{sk}$ of $H_k(z)$ are

$$\begin{cases} \Omega_{pk} = [0, \dfrac{(1-\rho)\pi}{M_1 M_2 \cdots M_k}] \\ \Omega_{sk} = \bigcup_{i=1}^{\lfloor M_k/2 \rfloor} [\dfrac{2i\pi}{M_k} - \dfrac{(1+\rho)\pi}{M_1 M_2 \cdots M_k}, \\ \qquad \min\{\dfrac{2i\pi}{M_k} + \dfrac{(1+\rho)\pi}{M_1 M_2 \cdots M_k}, \pi\}] \quad (k \geq 2) \end{cases} \tag{19}$$

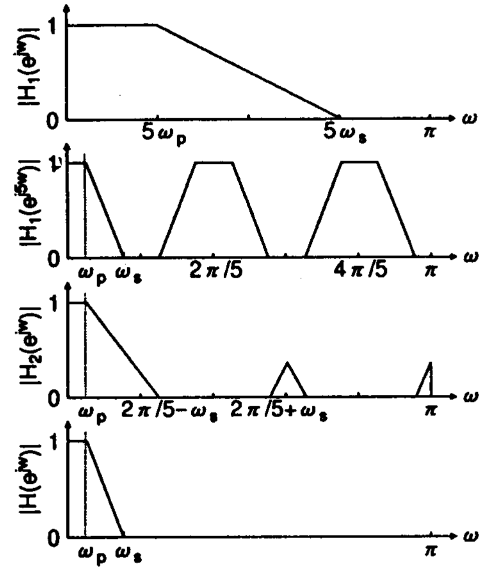


Fig. 2. Specifications of multistage Nyquist filters.

Therefore, we can obtain a multistage Nyquist filter by using the design method proposed in section 3 to design $H_k(z)$. The order of each subfilter is chosen in such a way that the magnitude error in the stopband is smaller than $\delta_s$, so that the order of the overall filter $H(z)$ is $2N$:

$$N = \sum_{i=1}^{K} \{N_i \prod_{k=i+1}^{K} M_k\} \tag{20}$$

When $M$ can be decomposed into several integers, the number of multipliers required in order to obtain the same attenuation in the stopband differs greatly depending on the ordering sequence. Hence, the ordering is very important. In practical designs, the wider the transition band of each subfilter $H_k(z)$, the higher the stopband attenuation, and thus the optimal ordering sequence is

$$M_1 \leqq M_2 \leqq \cdots \leqq M_{K-1} \leqq M_K \tag{21}$$

and the minimum number of multipliers is needed.

## 6. Design Examples

[ Example 1 ] { FIR Nyquist Filters }

The specifications of the Nyquist filters are $N = 19$, $M = 4$, and $\rho = 0.15$, and the weighting function in the stopband is $W(\omega) = 1$. We use the design algorithm proposed in section 3.2 to design the filter. The magnitude

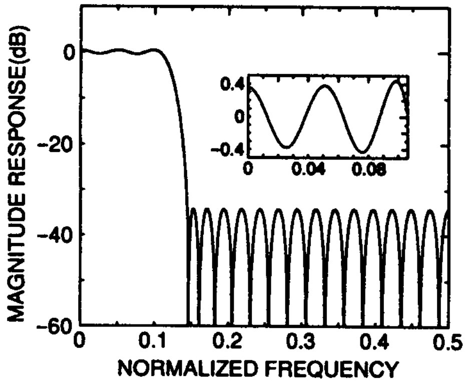


Fig. 3. Magnitude response of FIR Nyquist filter.

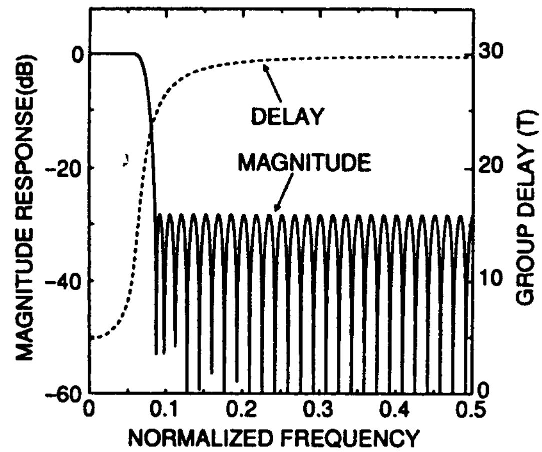


Fig. 4. Frequency responses of matched Nyquist filter pair.

response, shown in Fig. 3, is equiripple. Since the time-domain condition of zero intersymbol interference has been included in Eq. (4), the impulse response of the resulting filter must be an exact zero crossing, and is omitted here. We compare the magnitude error with that of the conventional methods in Table 1. It is clear from Table 1 that the proposed method has the same result as that in Ref. 8 and a smaller error than those in Refs. 5 and 6.

[ Example 2 ] { Matched Nyquist Filter Pairs }

The order of the transmitted and received filters is $N = 60$, the specifications are $M = 7$, $\rho = 0.2$, and the weighting function in the stopband is $W(\omega) = 1$. The order of the overall filter is then $2N = 120$. We design the overall filter and retain one of its double zeros on the unit circle and the zeros that lie inside the unit circle in the mirror-image pairs as zeros of the transmitted filter. The resulting transmitted filter has minimum phase response, and its magnitude response and group delay are shown in Fig. 4. The receiving filter has zeros outside the unit circle in the mirror-image pairs and thus has maximum phase response. Its magnitude response is the same as the transmitted filter, and its group delay plus one of the transmitted filter is constant $N$ and omitted here.

Table 1. Comparison of magnitude error in Example 1

| | Passband attenuation (dB) | Stopband attenuation (dB) |
|---|---|---|
| Ref. 5 | 0.45 | 33.0 |
| Ref. 6 | 0.45 | 33.2 |
| Ref. 8 | 0.44 | 34.3 |
| Proposed method | 0.44 | 34.3 |

[ Example 3 ] { Multistage Nyquist Filters }

The specifications are $M = 10$, $\rho = 0.1$, and the stopband attenuation is more than 40dB. First we design a two-stage filter with $M_1 = 2$ and $M_2 = 5$. To obtain more than 40dB attenuation in the stopband, the orders of $H_1(z)$ and $H_2(z)$ are chosen as $2N_1 = 42$, $2N_2 = 18$. The magnitude response of the resulting filter is shown in Fig. 5. Assume that each subfilter is implemented by direct configuration. The number of multipliers required is then 21 after considering the condition of zero intersymbol interference and symmetry of filter coefficients. If the above specifications

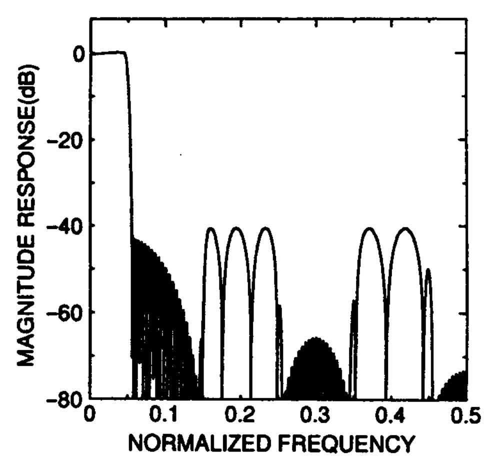


Fig. 5. Magnitude response of multistage Nyquist filter.

are designed by using one-stage Nyquist filters, the order will be more than $2N = 160$, and the number of multipliers is 73. We also design another two-stage filter with $M_1 = 5$ and $M_2 = 2$. The orders of $H_1(z)$ and $H_2(z)$ are $2N_1 = 104$, $2N_2 = 6$, and 46 multipliers are required.

## 7. Conclusions

In this paper we have proposed a method of designing FIR Nyquist filters with zero intersymbol interference by using the Remez exchange algorithm directly. We have investigated some magnitude properties of FIR Nyquist filters with zero intersymbol interference. We know from the time-domain condition of zero intersymbol interference that the magnitude response of Nyquist filters in the passband is mainly dependent on the stopband magnitude. Therefore, the design problem of Nyquist filters becomes minimization of the magnitude response in the stopband. We have formulated the design problem as a linear problem by using the Remez exchange algorithm directly in the stopband. Then the filter coefficients can be computed by simply solving linear equations, and the optimal coefficients with an equiripple stopband response can be easily obtained by a small number of iterations. The proposed procedure is computationally more efficient than the existing procedures. Finally, we have extended the proposed procedure to the design of matched Nyquist filter pairs, multistage Nyquist filters, and so on, in order to demonstrate its effectiveness.

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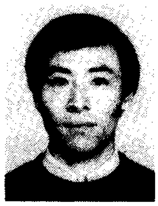
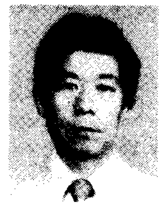
## **AUTHORS** (from left to right)





**Xi Zhang** was born in Changshu, China, on December 23, 1963. He received his B.E. degree in electronic engineering from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 1984, and his M.E. and Ph.D. degrees in communication engineering from the University of Electro-Communications, Tokyo, in 1990 and 1993, respectively. He was in the Department of Electronic Engineering, NUAA, from 1984 to 1987, and the Department of Communications and Systems, UEC, from April 1993 to March 1996, as a research assistant. Since April 1996, he has been an associate professor in the Department of Electrical Engineering, Nagaoka University of Technology, Niigata, Japan. He was a recipient of the Science and Technology Progress Award in China in 1987. His research interests are in the areas of digital signal processing, approximation theory, wavelets, and neural networks. He is a member of the IEEE.

**Hiroshi Iwakura** was born in Nagano, Japan, on September 3, 1939. He graduated from the Faculty of Electro-Communications, University of Electro-Communications, Tokyo, in 1963. He received his M.E. degree in electrical engineering from Tokyo Metropolitan University in 1968 and his Ph.D. degree in electronic engineering from Tokyo Institute of Technology in 1990. In 1968, he joined the Faculty of Electro-Communications, UEC, and since then, as a faculty member, he has been engaged in teaching and research in the fields of microwave theory, circuit and system theory, digital signal processing, and others. He is now a professor in the Department of Communications and Systems, UEC, and a member of the IEEE.