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Design for Stable IIR Perfect Reconstruction Filter Banks Using Allpass Filters

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SUMMARY

This paper presents a new method for designing two-channel IIR linear phase filter banks that satisfy both the perfect reconstruction and stability conditions, using allpass filters. Using the perfect reconstruction condition for filter banks, we first offer a class of structurally perfect reconstruction implementations. Since the proposed filter banks must satisfy the perfect reconstruction condition even though the filter coefficients are quantized, the design problem becomes the design of half-band filters using a delay section and an allpass filter. Therefore, stable IIR perfect reconstruction filter banks can be easily obtained by approximating the phase responses of just the allpass filters. From the viewpoint of wavelets, a new method is presented for designing half-band filters with arbitrary flatness based on the parallel connection between the delay section and the allpass filter. Furthermore, by using two different allpass filters in the filter banks, the magnitude responses of low- and high-pass filters can be designed arbitrarily. © 1998 Scripta Technica, Electron Comm Jpn Pt 2, 81(5): 24–32, 1998

Key words: Structurally perfect reconstruction; stable IIR filter bank; allpass filter; approximately linear phase.

1. Introduction

In recent years, perfect reconstruction (PR) filter banks have received considerable attention in many applications of signal processing, including subband coding of speech and image signals [1–15]. There are FIR and IIR PR filter banks using a two channel filter bank design. FIR filter banks are always stable and have an exact linear phase response, thus many design methods have been proposed [2, 3, 5, 7]. In comparison with FIR filter banks, IIR filter banks usually need a lower filter order to meet the same magnitude specifications, and there are also design methods for these filter banks [4, 6, 11–15]. In particular, IIR filters based on parallel connection of allpass filters exhibit very low passband sensitivity and can be constructed with fewer multiplications than conventional IIR filters [4, 6, 12–14]. In the parallel structure using causal allpass filters [4, 6, 13, 14], however, phase error in the output of the filter banks cannot be completely removed, so that it cannot be guaranteed that the PR condition is satisfied. Therefore, noncausal allpass filters have to be used to obtain PR filter banks. In Ref. 12, a class of structurally PR implementations is described in which the design problem is reduced to the design of a half-band filter using a delay section and an allpass filter. Therefore, by approximating the phase response of the allpass filter, IIR filter banks that satisfy both the PR and stability conditions can be easily obtained. The analysis and synthesis filters obtained have approximately linear phase responses, because the parallel structure of delay section and allpass filter has been employed. However, there is a large bump in the transition band, and the magnitude responses of the low- and high-pass filters cannot be designed separately, since only one allpass filter is used.

In this paper, we propose a new method for designing two channel IIR linear phase filter banks that satisfy both the PR and stability conditions using allpass filters. From

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the PR condition of the filter banks, we first describe a class of structural PR implementations. Since the proposed filter banks are basically the same as in Ref. 12, the PR condition must be satisfied even though the filter coefficients are quantized. A difference from Ref. 12, is that we use two different allpass filters in this paper, whereas only one allpass filter is used in Ref. 12. By using two different allpass filters, we can arbitrarily design the magnitude responses of low- and high-pass filters, and suppress the large bump that arises in the transition band in Ref. 12. The design problem for the proposed filter banks can be reduced to the design of half-band filters using a delay section and an allpass filter. We offer a new design method for the half-band filters. In conventional design methods, only the design of maximally flat and equiripple filters are considered. In this paper, from the regularity condition of wavelets, we consider a design for IIR half-band filters with arbitrary flatness. We present a design method based on the eigenvalue problem by using the Remez exchange algorithm [9]. Finally, we offer some design examples to demonstrate the effectiveness of the proposed method.

2. Structurally PR Filter Banks

In two channel filter banks shown in Fig. 1, assume that $H_i(z)$, and $G_i(z)$ are the analysis and synthesis filters, respectively. To obtain a PR filter bank, these filters must satisfy

$$\begin{cases} H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K-1} \\ H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \end{cases} \tag{1}$$

where K is an integer. By using the polyphase matrix description for $H_i(z)$ and $G_i(z)$,

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} = \boldsymbol{H}(z^2) \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} \tag{2}$$

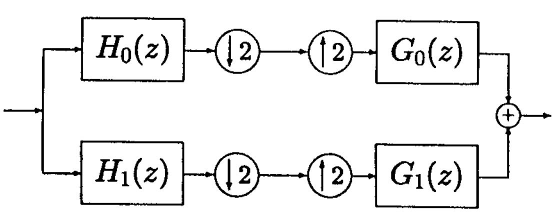

Fig. 1. Two channel filter bank.

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix}^T = \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^T \begin{bmatrix} G_{00}(z^2) & G_{01}(z^2) \\ G_{10}(z^2) & G_{11}(z^2) \end{bmatrix} = \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^T \boldsymbol{G}(z^2) \tag{3}$$

the PR condition of Eq. (1) becomes

$$\boldsymbol{G}(z)\boldsymbol{H}(z) = \frac{z^{-K}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{z^{-K}}{2}\boldsymbol{I} \tag{4}$$

It is well known that

$$\begin{bmatrix} z^{-N} & 0 \\ -A(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ A(z) & z^{-N} \end{bmatrix} = z^{-N}\boldsymbol{I} \tag{5}$$

$$\begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix} \begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix} = z^{-M}\boldsymbol{I} \tag{6}$$

Therefore, if we have

$$\boldsymbol{H}(z) = \begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{A(z)}{2} & \frac{z^{-N}}{2} \end{bmatrix} \tag{7}$$

$$\boldsymbol{G}(z) = \begin{bmatrix} \frac{z^{-N}}{2} & 0 \\ -\frac{A(z)}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix} \tag{8}$$

then the PR condition of Eq. (4) must be satisfied regardless of the form of $A(z)$ and $B(z)$, i.e., PR filter banks can be still obtained even when the filter coefficients of $A(z)$ and $B(z)$ are quantized. A structurally PR implementation is shown in Fig. 2. Therefore, the design problem for filter banks becomes the design of analysis or synthesis filters. The transfer functions for analysis filters are given by

$$\begin{cases} H_0(z) = z^{-2M} - B(z^2)H_1(z) \\ H_1(z) = \frac{1}{2}\{z^{-2N-1} + A(z^2)\} \end{cases} \tag{9}$$

and the transfer functions for synthesis filters are

$$\begin{cases} G_0(z) = \frac{1}{2}\{z^{-2N-1} - A(z^2)\} = -H_1(-z) \\ G_1(z) = z^{-2M} + B(z^2)G_0(z) = H_0(-z) \end{cases} \tag{10}$$

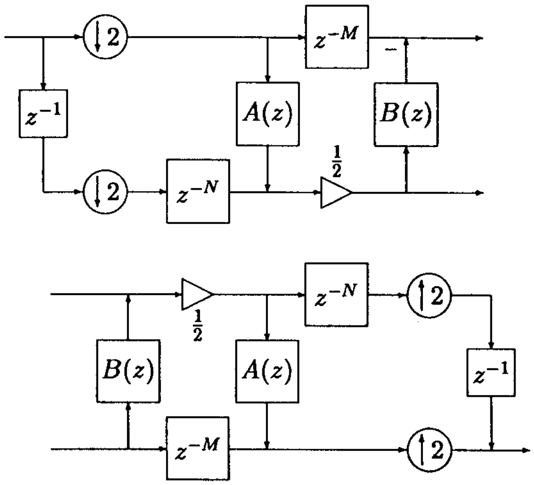

Fig. 2. Structurally perfect reconstruction filter bank.

In the following, we consider the design of analysis filters $H_i(z)$.

3. Design of IIR PR Filter Banks

In this section, we describe the design of $H_i(z)$ using allpass filters. Assume that $A(z)$ and $B(z)$ are allpass filters of order L_1 and L_2, respectively. Their transfer functions are defined by

$$A(z) = z^{-L_1} \frac{\sum_{n=0}^{L_1} a_n z^n}{\sum_{n=0}^{L_1} a_n z^{-n}} \tag{11}$$

$$B(z) = z^{-L_2} \frac{\sum_{n=0}^{L_2} b_n z^n}{\sum_{n=0}^{L_2} b_n z^{-n}} \tag{12}$$

where the filter coefficients a_n, b_n are real, and $a_0 = b_0 = 1$. Let the phase responses of $A(z)$ and $B_{(z)}$ be $\theta_1(\omega)$, $\theta_2(\omega)$, respectively. For stable allpass filters, $\theta_i(0) = 0$, and $\theta_i(\pi) = -L_i\pi$, the phase response is required to decrease monotonically with increasing frequency [9, 10].

3.1. Desired phase responses

From Eq. (9), the transfer function $H_1(z)$ is

$$H_1(z) = \frac{1}{2}\{z^{-2N-1} + A(z^2)\} \tag{13}$$

To force $H_1(z)$ to be the lowpass filter, the phase response of $A(z^2)$ must be

$$\theta_1(2\omega) = \begin{cases} -(2N+1)\omega & (0 \leqq \omega \leqq \omega_p) \\ -(2N+1)\omega \pm \pi & (\omega_s \leqq \omega \leqq \pi) \end{cases} \tag{14}$$

where $\omega_p + \omega_s = \pi$. From the stable condition for allpass filters, the order of $A(z)$ is required to be $L_1 = N$ or $L_1 = N + 1$. Therefore, the desired phase response of $A(z)$ is

$$\theta_1(\omega) = -\left(N + \frac{1}{2}\right)\omega \qquad (0 \leqq \omega \leqq 2\omega_p) \tag{15}$$

By making $A(z)$ approximate the desired phase response of Eq. (15), a lowpass filter $H_1(z)$ having an approximately linear phase response can be obtained. Additionally, a high-pass filter can be obtained by replacing $A(z^2)$ with $-A(z^2)$ in the lowpass filter obtained. From Eq. (9), the transfer function $H_0(z)$ is

$$H_0(z) = z^{-2M} - H_1(z)B(z^2) \tag{16}$$

In the stopband $[\omega_s, \pi]$ of $H_1(z)$, since the magnitude of $H_1(z)$ is 0, the magnitude of $H_0(z)$ will be 1 and it becomes passband. Thus, the phase response of $H_0(z)$ is linear. In the passband $[0, \omega_p]$ of $H_1(z)$, due to $H_1(z) = z^{-2N-1}$, ideally, then,

$$\begin{aligned} H_0(z) &= z^{-2N-1}\{z^{-2(M-N)+1} - B(z^2)\} \\ &= z^{-2N-1}\hat{H}_0(z) \end{aligned} \tag{17}$$

Hence, to make $H_0(z)$ be stopband, the desired phase response of $B(z)$ is

$$\theta_2(\omega) = -\left(M - N - \frac{1}{2}\right)\omega \quad (0 \leqq \omega \leqq 2\omega_p) \tag{18}$$

To get a stable $B(z)$, its order must be $L_2 = M - N - 1$ or $L_2 = M - N$. Therefore, by making $A(z)$ and $B(z)$ approximate the desired phase responses of Eqs. (15) and (18), we can design stable IIR PR filter banks in which both the analysis and synthesis filters exhibit approximately linear phase responses.

3.2. Half-band filters with arbitrary flatness

In this section, we describe design of half-band filters using a delay section and an allpass filter. In conventional design methods for half-band filters, only the design of maximally flat and equiripple filters has been considered. In recent years, wavelet transforms have been applied in various fields of signal processing. In many applications, wavelet functions are required to be continuous. This condition is the so-called regularity condition. The regularity condition is equivalent to the flatness condition for filter banks [2, 8]. Therefore, maximally flat filters are desirable from the regularity condition. In addition, filter frequency selectivity is also very important from the viewpoint of signal band-splitting, and minimization of the magnitude error is required. However, the flatness condition and frequency selectivity somewhat contradict each other, i.e., maximally flat filters do not have the best frequency selectivity, and the filters with the best frequency selectivity cannot possess maximal flatness. For this reason, we consider a design for IIR half-band filters that minimizes the maximum magnitude error while meeting a specified flatness. (In this paper, we will refer to this as a filter with arbitrary flatness.) We consider the complementary pair $\hat{H}_1(z)$ in place of $H_1(z)$ of Eq. (13);

$$\hat{H}_1(z) = \frac{1}{2}\{z^{-2N-1} - A(z^2)\} \tag{19}$$

The magnitude response of $\hat{H}_1(z)$ is

$$= \frac{\sum_{n=0}^{L_1} a_n \sin\left(2n - L_1 + N + \frac{1}{2}\right)\omega}{\sqrt{\left(\sum_{n=0}^{L_1} a_n \cos 2n\omega\right)^2 + \left(\sum_{n=0}^{L_1} a_n \sin 2n\omega\right)^2}} = \sin\theta_{1e}(\omega) \tag{20}$$

where

$$\theta_{1e}(\omega) = \tan^{-1}\frac{\sum_{n=0}^{L_1} a_n \sin\left(2n - L_1 + N + \frac{1}{2}\right)\omega}{\sum_{n=0}^{L_1} a_n \cos\left(2n - L_1 + N + \frac{1}{2}\right)\omega} \tag{21}$$

The magnitude response of $H_1(z)$ is given by

$$|H_1(e^{j\omega})| = \cos\theta_{1e}(\omega) \tag{22}$$

thus,

$$|H_1(e^{j\omega})|^2 + |\hat{H}_1(e^{j\omega})|^2 = 1 \tag{23}$$

which means that if the highpass filter $\hat{H}_1(z)$ is designed, then the lowpass filter $H_1(z)$ can be obtained. It is known from Eqs. (20), (21), and (22) that since $|H_1(e^{j\omega})| = |\hat{H}_1(e^{j(\pi-\omega)})|$, we have

$$|\hat{H}_1(e^{j\omega})|^2 + |\hat{H}_1(e^{j(\pi-\omega)})|^2 = 1 \tag{24}$$

Therefore, we can obtain the overall frequency response when $\hat{H}_1(z)$ is approximated only in the stopband. Assume that $\hat{H}_1(z)$ has a flatness of order $2J_1 = 1$ at $\omega = 0$,

$$\left.\frac{d^i|\hat{H}_1(e^{j\omega})|}{d\omega^i}\right|_{\omega=0} = 0 \qquad (i = 0, 1, \cdots, 2J_1) \tag{25}$$

which is equivalent to locating $2J_1 + 1$ zeros at $z = 1$. From Eq. (21), $\hat{H}_1(z)$ has at least one zero at $z = 1$ due to $\theta_{1e}(0) = 0$. Since the number of the unknown coefficients of $\hat{H}_1(z)$ is L_1, the number of the independent zeros is $2L_1$ after considering the filter coefficient symmetry. Therefore, J_1 must satisfy

$$0 \leqq J_1 \leqq L_1 \tag{26}$$

When $J_1 = 0$, it is an equiripple filter that minimizes the maximum magnitude error. When $J_1 = L_1$, it becomes a maximally flat filter. Since $\hat{H}_1(z)$ has $2J_1 + 1$ zeros at $z = 1$, from the numerator polynomial of Eq. (20), we obtain

$$\sum_{n=0}^{L_1}(2n - I_1)^{2i-1} a_n = 0 \quad (i = 1, 2, \cdots, J_1) \tag{27}$$

where $I_1 = L_1 - N - 1/2$. We next design the equiripple magnitude response of $\hat{H}_1(z)$ by using $2(L_1 - J_1)$ the remaining independent zeros. To minimize the magnitude error, all remaining independent zeros are required to be located on the unit circle. Then, we can select $L_1 - J_1 + 1$ extremal frequencies ω_i in the band $[0, \omega_p]$ as follows;

$$0 < \omega_0 < \omega_1 < \cdots < \omega_{L_1 - J_1} = \omega_p \tag{28}$$

We use the Remez exchange algorithm to formulate the magnitude response of Eq. (20),

$$|\hat{H}_1(e^{j\omega_i})| = \sin\theta_{1e}(\omega_i) = (-1)^i\delta_m \tag{29}$$

where δ_m is a magnitude error. From Eqs. (21) and (29), we obtain

$$\frac{\sum_{n=0}^{L_1} a_n \sin(2n - I_1)\omega_i}{\sum_{n=0}^{L_1} a_n \cos(2n - I_1)\omega_i} = (-1)^i\delta \tag{30}$$

where $\delta = \tan(\sin^{-1}\delta_m) = \delta_m/\sqrt{1-\delta_m^2}$. We rewrite Eqs. (27) and (30) in matrix form as

$$\boldsymbol{PA} = \delta \boldsymbol{QA} \tag{31}$$

where $\boldsymbol{A} = [a_0, a_1, \ldots, a_{L1}]^T$, and the elements P_{ij} and Q_{ij} of $\boldsymbol{P}$ and $\boldsymbol{Q}$, when $0 \le i \le L_1 - J_1$, are

$$\begin{cases} P_{ij} = \sin(2j - I_1)\omega_i \\ Q_{ij} = (-1)^i \cos(2j - I_1)\omega_i \end{cases} \tag{32}$$

and, when $L_1 - J_1 + 1 \le i \le L_1$, are

$$\begin{cases} P_{ij} = (2j - I_1)^{2(i-L_1+J_1)-1} \\ Q_{ij} = 0 \end{cases} \tag{33}$$

where $0 \le j \le L_1$. It is clear that Eq. (31) corresponds to a generalized eigenvalue problem. Therefore, as shown in Ref. 9, the optimal solution can be obtained by solving the eigenvalue problem of Eq. (31). The design algorithm is described in detail in the next section.

3.3. Design algorithm

1. Read filter specifications N, L_1, J_1, and the cutoff frequency ω_p.
2. Select equally $L_1 - J_1 + 1$ initial extremal frequencies ω_i as shown in Eq. (28).
3. Solve the eigenvalue problem of Eq. (31) to obtain a set of filter coefficients a_n.
4. Compute the magnitude response of $\hat{H}_1(z)$ by using the a_n values obtained, and search for the peak frequencies $\overline{\omega}_i$ in the stopband.
5. If $\sum_{i=0}^{L_1-J_1} |\overline{\omega}_i - \omega_i| < \varepsilon$, then exit, else go to 6, where ε is a prescribed small constant.
6. Set $\omega_i = \overline{\omega}_i (i = 0, 1, \ldots, L_1 - J_1)$, then go to 3.

3.4. Design of $H_0(z)$

We can design $A(z)$ by using the method proposed in section 3.2 to obtain $H_1(z)$ with an arbitrary flatness. The $H_1(z)$ obtained has maximum magnitude errors $\delta_{s1} \simeq \delta_a/2$ in the stopband and $\delta_{p1} \simeq \delta_a^2/8$ in the passband, where δ_a is the maximum phase error of $A(z)$. $B(z)$ can be similarly designed. However, even when the phase responses of both $A(z)$ and $B(z)$ are designed to be equiripple, it is seen from Eq. (16) that the magnitude response of $H_0(z)$ cannot be guaranteed to be equiripple. In the most cases, we cannot obtain an equiripple response for $H_0(z)$. In Ref. 12, the magnitude response of $H_0(z)$ becomes equiripple by setting $B(z) = A(z)$. By rewriting $H_0(z)$ of Eq. (16),

$$H_0(z) = z^{-2M}\left\{1 - \frac{B(z^2)}{z^{2N-2M+1}}\frac{1}{2}\left\{1 + \frac{A(z^2)}{z^{-2N-1}}\right\}\right\} \tag{34}$$

In [12], due to $M = 2N + 1$,

$$\frac{B(z^2)}{z^{2N-2M+1}} = \frac{A(z^2)}{z^{-2N-1}} \tag{35}$$

To meet the stability condition, the order of $A(z)$ must be $L_1 = N$ or $L_1 = N + 1$. Thus, the phase difference between $A(z^2)$ and z^{-2N-1} becomes $\pm\pi/2$ at $\omega = \pi/2$, i.e.,

$$\frac{A(e^{j\pi})}{e^{-j(N+1/2)\pi}} = e^{\pm j\frac{\pi}{2}} = \pm j \tag{36}$$

Hence we obtain

$$|H_0(e^{j\frac{\pi}{2}})| = \left\|1 \mp \frac{j}{2}(1 \pm j)\right\| = \left\|\frac{3}{2} \mp \frac{j}{2}\right\| = \frac{\sqrt{10}}{2} \tag{37}$$

which means that there is a large bump in the transition band. In addition, the maximum magnitude error δ_{s0} of $H_0(z)$ in the stopband is

$$\delta_{s0} = \left\|1 - e^{j\delta_a}\frac{1}{2}(1 + e^{j\delta_a})\right\| \simeq \frac{3\delta_a}{2} \simeq 3\delta_{s1} \tag{38}$$

In other words, the stopband magnitude error of $H_0(z)$ is three times larger than that of $H_1(z)$, even though its order is higher than for $H_1(z)$. This is because both $H_0(z)$ and $H_1(z)$ are dependent on one allpass filter $A(z)$ and thus their magnitude responses cannot be designed separately. In this paper, we directly design $B(z)$ to make the magnitude response of $H_0(z)$ be equiripple. By using $B(z)$ different from $A(z)$, we can arbitrarily control the magnitude error of $H_0(z)$, and suppress the large bump that is observed in the transition band. To get a stable $B(z)$, its order is required to be $L_2 = M - N - 1$ or $L_2 = M - N$. Hence, there are four combinations with $A(z)$. First, consider the case when the order of $A(z)$ is $L_1 = N$. In this case, the phase difference between $A(z^2)$ and the delay section z^{-2N-1} is $\pi/2$ at $\omega = \pi/2$. By removing the linear phase $-(2N + 1)\omega$, the phase difference of $H_1(z)$ becomes $\pi/4$, and its magnitude is $1/\sqrt{2}$ at $\omega = \pi/2$. If we choose the order of $B(z)$ as $L_2 = M - N - 1$, the phase difference between $B(z^2)$ and $z^{2N-2M+1}$ is $\pi/2$ at $\omega = \pi/2$ also. Hence, the phase difference of $B(z^2)H_1(z)$ becomes $3\pi/4$, and the large bump occurs as shown in Ref. 12. In contrast, when $L_2 = M - N$, since the phase difference of $B(z^2)$ is $-\pi/2$, the phase difference of $B(z^2)H_1(z)$ will become $-\pi/4$, and we obtain

$$|H_0(e^{j\frac{\pi}{2}})| = \left\|1 - (-j)\frac{1}{2}(1+j)\right\| = \frac{\sqrt{2}}{2} < 1 \tag{39}$$

Therefore, when $L_1 = N$, the large bump can be suppressed in the transition band by setting $L_2 = M - N$. Similarly, when $L_1 = N + 1$, the large bump will occur if we choose $L_2 = M - N$, and thus we must choose $L_2 = M - N - 1$. It is known that the stopband of $H_0(z)$ corresponds to the passband of $H_1(z)$. The magnitude error in the passband of $H_1(z)$ is much smaller than that in the stopband. Therefore, in the stopband, the magnitude response of $H_0(z)$ is

$$\begin{aligned} |H_0(e^{j\omega})| &= \|1 - e^{j(2\theta_{2e}(\omega)+\theta_{1e}(\omega))}\cos\theta_{1e}(\omega)\| \\ &\simeq \|1 - e^{j(2\theta_{2e}(\omega)+\theta_{1e}(\omega))}\| \\ &= 2\sin\left(\theta_{2e}(\omega) + \frac{\theta_{1e}(\omega)}{2}\right) \end{aligned} \tag{40}$$

where

$$\theta_{2e}(\omega) = \tan^{-1}\frac{\sum_{n=0}^{L_2} b_n \sin(2n - I_2)\omega}{\sum_{n=0}^{L_2} b_n \cos(2n - I_2)\omega} \tag{41}$$

and $I_2 = L_2 + N - M + 1/2$. To force $H_0(z)$ to have an equiripple response in the stopband, the phase response $\theta_{2e}(\omega) + \theta_{1e}(\omega)/2$ is required to be equiripple. To obtain an equiripple response, we reformulate

$$\theta_{2e}(\omega_i) + \frac{\theta_{1e}(\omega_i)}{2} = (-1)^i\delta_{ph} \tag{42}$$

where δ_{ph} is phase error. By rewriting Eq. (42), we obtain

$$\frac{\sum_{n=0}^{L_2} b_n \sin\left\{(2n - I_2)\omega_i + \frac{\theta_{1e}(\omega_i)}{2}\right\}}{\sum_{n=0}^{L_2} b_n \cos\left\{(2n - I_2)\omega_i + \frac{\theta_{1e}(\omega_i)}{2}\right\}} = (-1)^i\delta \tag{43}$$

where $\delta = \tan\delta_{ph}$. In addition, assume that $H_0(z)$ has a flatness of order $2J_2 + 1$ at $\omega = 0$. We can rewrite $H_0(z)$ as

$$\begin{aligned} H_0(z) &= z^{-2M} - B(z^2)\frac{1}{2}\{z^{-2N-1} + A(z^2)\} \\ &= z^{-2M} - B(z^2)z^{-2N-1} \\ &\quad + B(z^2)\frac{1}{2}\left\{z^{-2N-1} - A(z^2)\right\} \\ &= z^{-2N-1}\hat{H}_0(z) + B(z^2)\hat{H}_1(z) \end{aligned} \tag{44}$$

which means that the flatness of $H_0(z)$ is the same as the lower flatness between $\hat{H}_0(z)$ and $\hat{H}_1(z)$, and $J_2 \le J_1$ must be satisfied. It is required in the practical design that the higher flatness filter is designed by $H_1(z)$ and the lower flatness filter by $H_0(z)$. It can be seen from Eq. (44) that the flatness condition of $H_0(z)$ is equal to that of $\hat{H}_0(z)$. Therefore, this flatness condition is similar to Eq. (27), and

$$\sum_{n=0}^{L_2}(2n - I_2)^{2i-1}b_n = 0 \quad (i = 1, 2, \cdots, J_2) \tag{45}$$

The design algorithm is the same as that shown in section 3.3. In the passband of $H_0(z)$, its maximum magnitude error δ_{p0} is

$$\delta_{p0} \simeq \frac{3\delta_a^2}{8} + \frac{\delta_a\delta_b}{2} \tag{46}$$

and is very small, where δ_b is the maximum phase error of $B(z)$.

4. Design Examples

Example 1 (Equiripple Filters)

The specifications of filter banks are $N = 8$, $M = 16$, and $\omega_p = 0.4\pi$. The order of $A(z)$ and $B(z)$ are $L_1 = L_2 = 8$. We have set $J_1 = J_2 = 0$ and designed the equiripple filters by using the proposed method. The obtained phase responses of $A(z)$ and $B(z)$ are shown in Fig. 3, and the phase errors in Fig. 4. The magnitude and phase responses of $H_0(z)$ and $H_1(z)$ are shown in Figs. 5 and 6, respectively. It

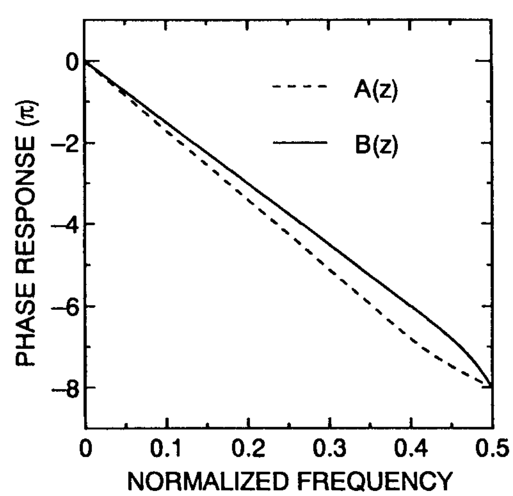

Fig. 3. Phase responses of $A(z)$ and $B(z)$ in Example 1.

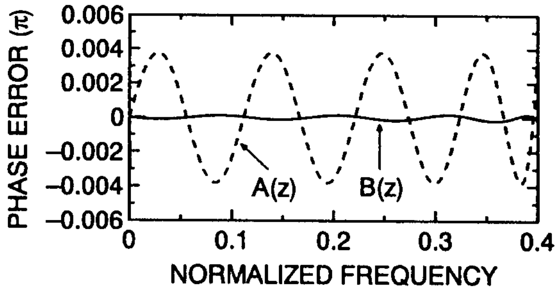

Fig. 4. Phase errors of $A(z)$ and $B(z)$ in Example 1.

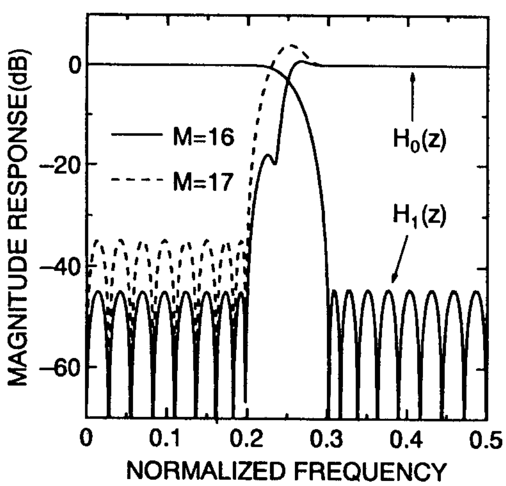

Fig. 5. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 1.

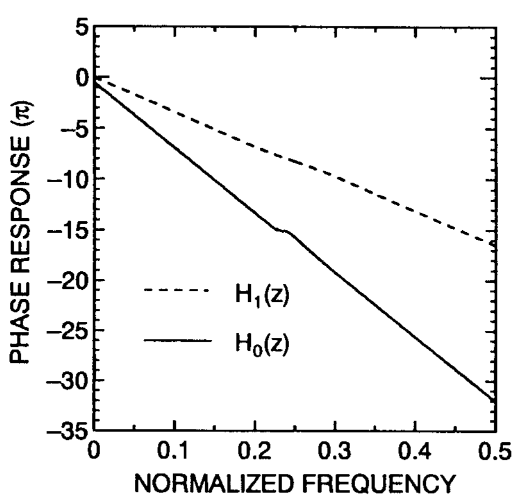

Fig. 6. Phase responses of $H_0(z)$ and $H_1(z)$ in Example 1.

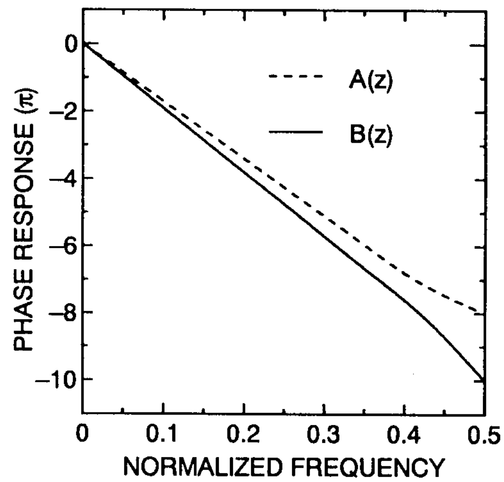

Fig. 7. Phase responses of $A(z)$ and $B(z)$ in Example 2.

can be seen from Fig. 5 that the magnitude response of both $H_0(z)$ and $H_1(z)$ are equiripple. It is clear from Fig. 6 that approximately linear phase responses have been obtained. For comparison purposes, the magnitude response of $H_0(z)$ obtained by setting $B(z) = A(z)$ in the conventional method of Ref. 12 is also shown in Fig. 5. In that case, the order of $B(z)$ is $L_2 = 8$, and $M = 17$. It can be seen in Fig. 5 that when $B(z) = A(z)$, there is a large bump in the transition band and a difference of about 10 dB in the stopband attenuation between $H_0(z)$ and $H_1(z)$. Therefore, by using $B(z)$ different from $A(z)$, the large bump in the transition band of $H_0(z)$ can be suppressed.

Example 2 (Filters with Arbitrary Flatness)

The specifications of filter banks are $N = 8$, $M = 18$, and $\omega_p = 0.4\pi$. The orders of $A(z)$ and $B(z)$ are $L_1 = 8$, $L_2 = 10$. We have set $J_1 = J_2 = 4$ and designed the filter bank by using the proposed method. The obtained

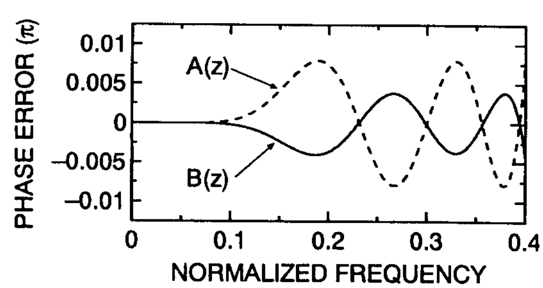

Fig. 8. Phase errors of $A(z)$ and $B(z)$ in Example 2.

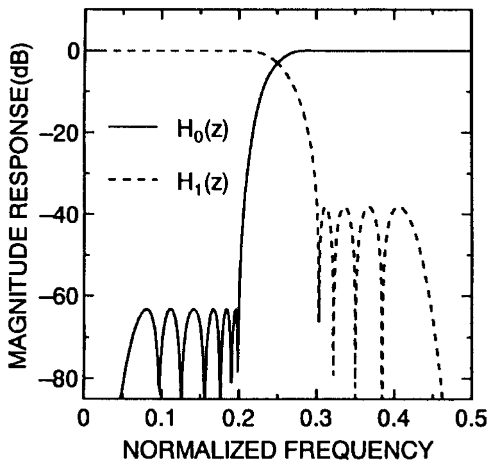

Fig. 9. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 2.

phase responses of $A(z)$ and $B(z)$ are shown in Fig. 7, and the phase errors in Fig. 8. $H_0(z)$ and $H_1(z)$ have the same flatness and their magnitude responses are shown in Fig. 9 and are equiripple. By increasing the order of $B(z)$, we can decrease the magnitude error of $H_0(z)$. The phase responses of $H_0(z)$ and $H_1(z)$ are shown in Fig. 10, and it is clear that they are approximately linear phase.

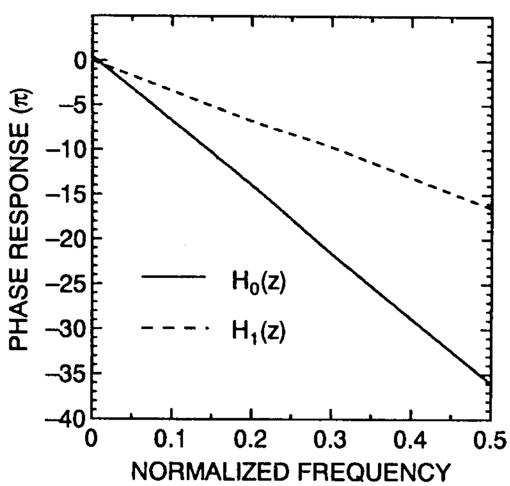

Fig. 10. Phase responses of $H_0(z)$ and $H_1(z)$ in Example 2.

5. Conclusions

In this paper, we have proposed a new method for designing two channel IIR linear phase filter banks that satisfy both the PR and stability conditions using allpass filters. From the PR condition of the filter banks, we first described a class of structurally PR implementations. Since the proposed filter banks must satisfy the PR condition even though the filter coefficients are quantized, the design problem became that of the design of half-band filters using a delay section and an allpass filter. For design of IIR half-band filters based on a parallel connection between the delay section and the allpass filter, we have given a design method for half-band filters with arbitrary flatness. In addition, by using two different allpass filters in the filter banks, we can arbitrarily design the magnitude responses of low- and high-pass filters, and suppress the large bump that arises in the transition band.

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