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Design of Orthonormal Wavelet Filter Banks Using the Remez Exchange Algorithm

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SUMMARY

This paper presents a new method for designing orthonormal wavelet filter banks using the Remez exchange algorithm. It is well known that orthonormal wavelet bases can be generated by paraunitary filter banks. Thus, synthesis of orthonormal wavelet bases can be reduced to the design of paraunitary filter banks. From the orthonormality and regularity of wavelets, we derive some conditions that must be satisfied for FIR paraunitary filter banks, and investigate the relationship between the filter coefficients and their zeros. According to the relationship, we apply the Remez exchange algorithm in the z-domain directly and formulate the design problem in the form of a linear problem. Therefore, we can easily get a set of filter coefficients by solving the linear equations. Optimal solutions with an arbitrary regularity can be obtained after using an iteration procedure. In the proposed method, the main advantage is that less computational complexity is required as compared with the conventional methods. © 1998 Scripta Technica. Electron Comm Jpn Pt 3, 81(6): 1–8, 1998

Key words: Orthonormal wavelet; paraunitary filter bank; Remez exchange algorithm; FIR filter.

1. Introduction

Wavelet transforms were initially introduced by Morlet in geophysical signal processing. The connection between compactly supported wavelets and perfect reconstruction filter banks was investigated by Daubechies. Thus, wavelets have received considerable attention in various fields of applied mathematics, signal processing, multiresolution theory, and so on [1–11]. Wavelet transforms can analyze signals in the time and frequency domains simultaneously, and can process signals in multiresolution space. It is well known that wavelet transforms can be realized by using filter banks. But wavelet bases can also be generated by designing perfect reconstruction filter banks [1–5]. In this paper, we will discuss the latter method. There are some procedures for designing paraunitary filter banks to construct orthonormal wavelet bases. In general, the design problem can be reduced to the design of half-band filters [2, 6–11]. However, only the design of maximally flat filters and equiripple filters is discussed in Refs. 2, 6, 8, and 9. In Refs. 7, 10, and 11, design methods for FIR filter banks with an arbitrary regularity are proposed. The methods of Refs. 10 and 11 use projection minimization techniques to optimize the filter, but require long computation times. Compared with these methods, the method proposed in Ref. 7 uses the Remez exchange algorithm and is computationally efficient. The Remez exchange algorithm is well known in the design of FIR linear phase filters, and the McClellan–Parks method is computationally efficient and widely used. However, in the design of paraunitary filter banks, the transfer function of the filter is constrained by the orthonormality and regularity conditions, and the Remez exchange algorithm can no longer be used. In Ref. 7, the transfer function that satisfies the orthonormality and regularity conditions is first variable-changed so that the Remez exchange algorithm can be applied, then the problem is optimized by using the Remez

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exchange algorithm in the variable-changed domain. The obtained solution is inversely changed to get a set of filter coefficients. But it requires the operations of variable change and inversion.

In this paper, we propose a new design method for orthonormal wavelet filter banks with arbitrary regularity by using the Remez exchange algorithm in the z-domain directly. First, from the orthonormality and regularity of wavelets, we derive some conditions that must be satisfied for FIR paraunitary filter banks, and we then investigate the relationship between the filter coefficients and their zeros. According to the relationship, we apply the Remez exchange algorithm in the stopband directly and formulate the design problem in the form of a linear problem. Therefore, we can easily get a set of filter coefficients by solving the linear equations. The optimal solution can be obtained after using an iteration procedure. In the proposed method, the regularity can be arbitrarily specified, and less computational complexity is required as compared with the conventional methods, because the Remez exchange algorithm is used in the z-domain directly and the operations of variable change and inversion of Ref. 7 are not required. Section 2 shows the relationship between orthonormal wavelets and paraunitary filter banks. Section 3 gives a design method for orthonormal wavelet filter banks. Finally, some examples are designed in order to demonstrate the effectiveness of the proposed method.

2. Orthonormal Wavelets and Paraunitary Filter Banks

Assume that $\psi(t)$ is a basic wavelet function, that the wavelet transform to a signal $f(t)$ ($f \in \mathbf{L}^2(\mathbf{R})$) is defined as

$$F_W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right) dt \quad (1)$$

where x^* denotes the complex conjugate of x, and that the dilation/contraction and translation parameters are $a \in \mathbf{R}^+$, $b \in \mathbf{R}$. When discretized, $a = 2^{-k}$ and $b = 2^{-k}m$ (k, m: integer), in general.

It is well known that wavelet bases can be generated by a paraunitary filter bank $\{H(z),G(z)\}$ as shown in Fig. 1. In Fig. 1, $H(z)$ is a lowpass filter, and $G(z)$ is highpass. When the filter bank is infinitely iterated on the lowpass branch at each step of decomposition, as shown in Fig. 2, a scaling function $\phi(t)$ and wavelet function $\psi(t)$ can be produced. Assume that $\hat{\phi}(\omega)$ and $\hat{\psi}(\omega)$ are the Fourier transforms of $\phi(t)$ and $\psi(t)$, respectively; the scaling and wavelet function are related to the paraunitary filter bank $\{H(z), G(z)\}$ in the frequency domain as follows:

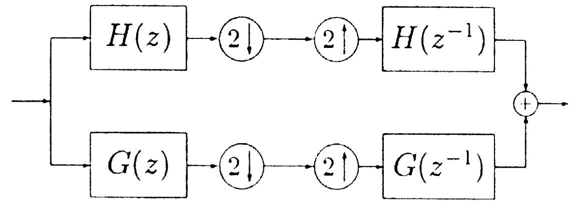

Fig. 1. Paraunitary filter bank (noncausal).

$$\hat{\phi}(\omega) = H(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) = \prod_{k=1}^{\infty} H(e^{j2^{-k}\omega}) \quad (2)$$

$$\hat{\psi}(\omega) = G(e^{j\frac{\omega}{2}})\hat{\phi}\left(\frac{\omega}{2}\right) \quad (3)$$

From the orthonormality of wavelets, the filter bank must satisfy the following constraints:

$$\begin{cases} H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 1 \\ G(z)G(z^{-1}) + G(-z)G(-z^{-1}) = 1 \\ H(z)G(z^{-1}) + H(-z)G(-z^{-1}) = 0 \end{cases} \quad (4)$$

Here, we define the product filter as

$$P(z) = H(z)H(z^{-1}) \quad (5)$$

To satisfy Eq. (4), $P(z)$ must be

$$P(z) = \frac{1}{2} + \sum_{n=0}^{N} c_{2n+1}[z^{2n+1} + z^{-(2n+1)}] \quad (6)$$

where the filter coefficients c_n are real. It is known from Eq. (6) that $P(z)$ is a half-band filter, and its degree is $4N + 2$. Since $P(z)$ has symmetric filter coefficients, its zeros occur on the unit circle or in mirror-image pairs. If all zeros on

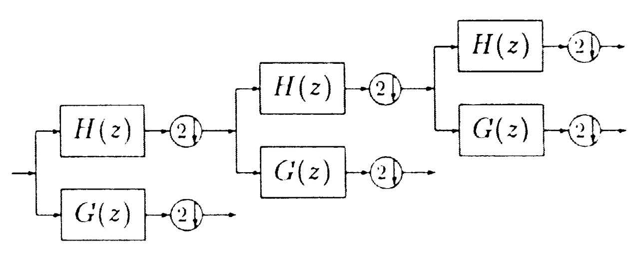

Fig. 2. Multiple-stage filter bank.

the unit circle are double zeros, we can decompose the mirror-image pairs and double zeros on the unit circle to get $H(z)$ as shown in Eq. (5). Then, the degree of $H(z)$ is $2N + 1$ and odd. We construct

$$G(z) = \pm z^{-2N-1} H(-z^{-1}) \tag{7}$$

thus Eq. (4) is satisfied, and the obtained wavelet basis is orthonormal. Therefore, the design problem of paraunitary filter banks will become the design of $P(z)$ in Eq. (6) with double zeros on the unit circle.

Scaling function $\phi(t)$ and wavelet function $\psi(t)$, when iterated to infinity, must converge to continuous functions, or possibly functions with several continuous derivatives. This condition is the so-called regularity condition. The simplest regularity condition for paraunitary filter banks is that $H(z)$ has at least one zero at $z = -1$. When $H(z)$ contains L zeros located at $z = -1$, we have

$$\left.\frac{d^i H(e^{j\omega})}{d\omega^i}\right|_{\omega=\pi} = 0 \qquad (i = 0, 1, \cdots, L-1) \tag{8}$$

and

$$\int_{-\infty}^{\infty} t^i \psi(t) dt = 0 \qquad (i = 0, 1, \cdots, L-1) \tag{9}$$

which means that the wavelet function has L consecutive vanishing moments. This property is potentially useful in some practical applications. In the next section, we consider the design of the product filter $P(z)$ that has the best possible frequency selectivity for a specified number of vanishing moments.

3. Design of Orthonormal Wavelet Filter Banks

3.1. Properties of product filters

Before designing the product filter $P(z)$, we first investigate the properties of $P(z)$. It is known from Eq. (6) that $P(z)$ is a half-band filter. All of its even-numbered coefficients are zero except for $c_0 = 0.5$, namely,

$$c_{2n} = 0 \qquad (n = \pm 1, \pm 2, \cdots, \pm N) \tag{10}$$

$P(z)$ has a total of $4N + 2$ zeros, where $2N$ zeros are used for satisfying the time-domain condition of Eq. (10), and the remaining $2(N + 1)$ zeros are independent. Therefore, the design problem of $P(z)$ becomes the location of $2(N + 1)$ independent zeros. We can obtain the magnitude response of $P(z)$ from Eq. (6) by

$$P(e^{j\omega}) = \frac{1}{2} + 2\sum_{n=0}^{N} c_{2n+1} \cos(2n+1)\omega \tag{11}$$

Then, we get

$$P(e^{j\omega}) + P(e^{j(\pi-\omega)}) \equiv 1 \tag{12}$$

which means that $P(z)$ has an antisymmetric magnitude response to point $(\pi/2, 1/2)$. Thus, the magnitude in passband $[0, \omega_p]$ is dependent on that in stopband $[\omega_s, \pi]$, where the passband edge frequency ω_p and stopband edge frequency ω_s must satisfy $\omega_p + \omega_s = \pi$. If the stopband response is optimized, we can obtain the total response from the magnitude symmetry. Therefore, the design problem of $P(z)$ is to approximate the stopband response by locating $2(N + 1)$ independent zeros.

3.2. Maximally flat filters

It is known from Eqs. (8) and (9) that to obtain the maximum number of vanishing moments, the magnitude response of $H(z)$ must be maximally flat. In Eq. (6), $P(z)$ has $2(N+1)$ independent zeros. Hence, to obtain maximally flat magnitude response, all $2(N + 1)$ independent zeros must be located at $z = -1$; then $P(z)$ is

$$P(z) = (1+z)^{N+1}(1+z^{-1})^{N+1} Q(z) \tag{13}$$

where $Q(z)$ is an FIR linear phase filter of degree $2N$. By expanding Eq. (13), we can obtain the coefficients of $Q(z)$ in such a way that the even-numbered filter coefficients of $P(z)$ satisfy Eq. (10). From Eq. (13), we have

$$\left.\frac{d^i P(e^{j\omega})}{d\omega^i}\right|_{\omega=\pi} = 0 \quad (i = 0, 1, \cdots, 2N+1) \tag{14}$$

Then,

$$\left.\frac{d^i H(e^{j\omega})}{d\omega^i}\right|_{\omega=\pi} = 0 \qquad (i = 0, 1, \cdots, N) \tag{15}$$

which means that the number of vanishing moments is N maximally. Substituting the magnitude response of Eq. (11) into Eq. (14), we get

$$\begin{cases} \sum_{n=0}^{N} c_{2n+1} = \frac{1}{4} & (i=0) \\ \sum_{n=0}^{N} (2n+1)^{2i} c_{2n+1} = 0 & (i=1,\cdots,N) \end{cases} \tag{16}$$

Therefore, we can obtain a set of filter coefficients c_n by solving linear equations (16), and we can construct the maximally flat filter $H(z)$ by decomposing the obtained $P(z)$ as shown in Eq. (5). In this paper, we construct $H(z)$ with a minimum phase response by using the zeros inside and on the unit circle. Then, we generate a scaling function $\phi(t)$ and wavelet function $\psi(t)$ by constructing $G(z)$ as shown in Eq. (7).

3.3. Filters with arbitrary regularity

In section 3.2, we have described the design of maximally flat filters. The filters are maximally flat at $\omega = 0$ and $\omega = \pi$ since all independent zeros are located at $z = -1$. However, the filters have a poor frequency selectivity. Of course, frequency selectivity is also thought of as a useful property for many applications in signal processing. However, regularity and frequency selectivity somewhat contradict each other. Maximally flat filters do not have optimal frequency selectivity, while filters with optimal frequency selectivity are not maximally flat. For this reason, we consider the design of the filters with the best possible frequency selectivity for a specified number of vanishing moments, that is, $H(z)$ has K zeros at $z = -1$. It is known from section 3.2 that K must be

$$0 \leqq K \leqq N+1 \tag{17}$$

From Eq. (5), $P(z)$ has $2K$ zeros at $z = -1$, that is,

$$P(z) = (1+z)^K (1+z^{-1})^K Q(z) \tag{18}$$

where $Q(z)$ is an FIR linear phase filter of degree $2(2N - K + 1)$. Therefore, we have

$$\left. \frac{d^i P(e^{j\omega})}{d\omega^i} \right|_{\omega=\pi} = 0 \quad (i = 0, 1, \cdots, 2K-1) \tag{19}$$

Similarly to Eq. (16),

$$\begin{cases} \sum_{n=0}^{N} c_{2n+1} = \frac{1}{4} & (i=0) \\ \sum_{n=0}^{N} (2n+1)^{2i} c_{2n+1} = 0 & (i=1,\cdots,K-1) \end{cases} \tag{20}$$

Since $P(z)$ has $2(N + 1)$ independent zeros, the number of remaining independent zeros is $2(N - K + 1)$ other than the zeros at $z = -1$. These remaining independent zeros must be located on the unit circle to minimize the magnitude error of the filter. The zeros of $P(z)$ on the unit circle, other than $z = \pm 1$, occur in complex conjugate pairs, and are required to be double zeros so that $P(z)$ can be decomposed as shown in Eq. (5); therefore,

$$2(N-K+1) = 4M \tag{21}$$

where M is an integer. Hence, when N is odd, K is even. When N is even, K is odd and $H(z)$ has at least one zero at $z = -1$. To force $4M$ independent zeros of $P(z)$ to be double zeros on the unit circle, and the magnitude response to be equiripple, we apply the Remez exchange algorithm.

Since the magnitude response of $P(z)$ is antisymmetric, we need to optimize the magnitude response in the stopband $[\omega_s, \pi]$ only. Here, we use the Remez exchange algorithm in the stopband only. Since $P(z)$ has $2M$ zeros on the upper unit semicircle of the z plane (other than $z = -1$), we can select $2M + 1$ extremal frequencies ω_i in the stopband as follows:

$$\omega_s = \omega_0 < \omega_1 < \cdots < \omega_{2M} \leqq \pi \tag{22}$$

Considering that all zeros on the unit circle must be double zeros, we use the Remez exchange algorithm to formulate

$$P(e^{j\omega_i}) = \begin{cases} \delta & (i = 0, 2, \cdots, 2M) \\ 0 & (i = 1, 3, \cdots, 2M-1) \end{cases} \tag{23}$$

where δ (> 0) is a magnitude error. Substituting Eq. (11) into Eq. (23), we can get

$$\sum_{n=0}^{N} c_{2n+1} \cos(2n+1)\omega_i - \frac{1+(-1)^i}{4}\delta = -\frac{1}{4} \tag{24}$$

We rewrite Eqs. (24) and (20) in matrix form as

$$\boldsymbol{A}\,\boldsymbol{C} = \boldsymbol{B} \tag{25}$$

where $\boldsymbol{B} = [-\frac{1}{4}, \ldots, -\frac{1}{4}, \frac{1}{4}, 0, \ldots, 0,]^T$, $\boldsymbol{C} = [c_1, c_3, \ldots, c_{2N+1}, \delta]^T$, and the elements A_{ij} of $\boldsymbol{A}$ are

$$A_{ij} = \begin{cases} \cos(2j+1)\omega_i & \begin{pmatrix} 0 \leqq i \leqq 2M \\ 0 \leqq j \leqq N \end{pmatrix} \\ (2j+1)^{2(i-2M-1)} & \begin{pmatrix} 2M+1 \leqq i \leqq N+1 \\ 0 \leqq j \leqq N \end{pmatrix} \\ -\frac{1}{2} & \begin{pmatrix} i = 0, 2, \cdots, 2M \\ j = N+1 \end{pmatrix} \\ 0 & \text{(else)} \end{cases} \tag{26}$$

It is clear that Eq. (25) is a set of linear equations. Because there are $N + 2$ unknown parameters (including $N + 1$ filter coefficients c_n and a magnitude error δ) for $K + 2M + 1 = N + 2$ equations, we can uniquely obtain a set of filter coefficients by solving the linear equations of Eq. (25). We compute the magnitude response of $P(z)$ in the stopband by using the obtained filter coefficients c_n, and search for the peak frequencies $\overline{\omega}_i$. As a result, the initially selected extremal frequencies ω_i cannot be guaranteed to be equal to the peak frequencies $\overline{\omega}_i$. Then, we set the obtained peak frequencies as the extremal frequencies in the next iteration, and solve the linear equations of Eq. (25) to obtain the filter coefficients c_n again. The above procedure is iterated until the extremal frequencies ω_i and the peak frequencies $\overline{\omega}_i$ are consistent. When the extremal frequencies do not change, we can obtain the optimal solution with an equiripple magnitude response. The design algorithm is presented in detail below.

3.4. Design algorithm

1. Read specifications of paraunitary filter banks N, K and cutoff frequency ω_s.
2. Select $2M + 1$ initial extremal frequencies ω_i equally spaced in the stopband as shown in Eq. (22).
3. Solve the linear equations of Eq. (25) to obtain a set of filter coefficients c_n.
4. Compute the magnitude response of $P(z)$ by using the obtained filter coefficients c_n, and search peak frequencies $\overline{\omega}_i$ in the stopband.
5. If $\sum_{i=0}^{2M} |(\overline{\omega}_i - \omega_i| < \varepsilon$ is satisfied, then go to Step 7. Else, go to Step 6, where ε is a prescribed small constant.
6. Set $\omega_i = \overline{\omega}_i$ $(i = 0, 1, \ldots, 2M)$, then go to Step 3.
7. Decompose the zeros of $P(z)$ to construct $H(z)$ as shown in Eq. (5).
8. Construct $G(z)$ from the obtained $H(z)$ as shown in Eq. (7) to generate a scaling function $\phi(t)$ and wavelet function $\psi(t)$.

3.5. Comparison with conventional methods

In this section, we compare the computational complexity of the design algorithm proposed above with that of the conventional methods. In the design of FIR paraunitary filter banks with arbitrary regularity, the methods of Refs. 10 and 11 use projection minimization techniques to optimize the magnitude response, after considering the orthonormality and regularity conditions of wavelet functions. However, the required computational complexity increases with the filter degree. Compared with these methods, the method proposed in Ref. 7 uses the Remez exchange algorithm, and thus is computationally efficient. In Ref. 7, the transfer function satisfying the orthonormality and regularity conditions is first variable-changed so that the Remez exchange algorithm can be applied, then the problem is formulated in the form of a linear problem by using the Remez exchange algorithm in the variable-changed domain. Finally, the obtained solution is inverted to get a set of filter coefficients. In this paper, we first investigate the properties of the product filters and the relationship between the magnitude response and the independent zeros. According to the relationship, we can obtain the linear equations of Eq. (20) from the regularity conditions, and the linear equations of Eq. (24) by using the Remez exchange algorithm in the stopband directly. Then, the design problem of the filter banks can be reduced to a linear problem. Similarly to Ref. 7, the proposed method can easily obtain a set of filter coefficients by solving the linear equations. Further, the operations of variable change and inversion such as are used in Ref. 7 are not required and the computational complexity can be reduced, since the Remez exchange algorithm is used in the z-domain directly. In the design algorithm proposed in section 3.4, the main computation is that of the linear equations of Step 3 and the magnitude response of Step 4 in each iteration. The same is true of the method of Ref. 7. However, in Ref. 7, the obtained solution must be inverted to get a set of filter coefficients when the iteration procedure is completed. This operation is used to solve the linear equations of the same size as that in Step 3. Considering the change of the design specifications before designing, the required computation is almost the same as that of one iteration. In the practical design, the proposed method requires three to five iterations in general. If the method of Ref. 7 converges with the same number of iterations, the operations of variable change and inversion are equal to one additional iteration. Therefore, the proposed method requires less computational complexity as compared with the conventional methods.

4. Design Examples

Example 1. {Maximally Flat Filters}

The specification of a maximally flat filter is $N = 10$. The degree of $P(z)$ is 42. By solving the linear equations of Eq. (16), we show the obtained filter coefficients c_n of $P(z)$ in Table 1. From these filter coefficients, we construct

Table 1. Filter coefficients of Example 1

c_1	0.3111590153	c_{13}	0.0004060484
c_3	−0.0864317979	c_{15}	−0.0000781986
c_5	0.0359014212	c_{17}	0.0000108942
c_7	−0.0146530546	c_{19}	−0.0000009747
c_9	0.0053182605	c_{21}	0.0000000420
c_{11}	−0.0016316557		

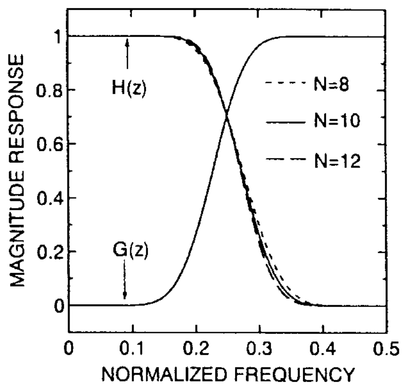

Fig. 3. Magnitude responses of Example 1.

maximally flat filter $H(z)$ with minimum phase response by computing the zeros of $P(z)$. The degree of $H(z)$ is 21, and the magnitude response is shown in Fig. 3 by the solid line. In Fig. 3, the magnitude responses of $N = 8$ and $N = 12$ are also shown. It is clear in Fig. 3 that the magnitude response of the filter becomes flatter with increasing N. When $N = 10$, we construct $G(z)$ from $H(z)$ as shown in Eq. (7), and show its magnitude response in Fig. 3. The scaling function $\phi(t)$ and wavelet function $\psi(t)$ generated by the above filter bank are shown in Figs. 4 and 5, respectively.

Example 2. {Filters with Arbitrary Regularity}

The filter specifications are $N = 10$, $K = 7$, and $\omega_s = 0.67\pi$. We design the filter by using the proposed procedure, and show the obtained filter coefficients c_n of $P(z)$ in Table 2. We construct $H(z)$ with minimum phase response from $P(z)$, and show the magnitude response in Fig. 6 as a solid line. The filters of $K = 5$ and $K = 9$ are also designed, and the magnitude responses are shown in Fig. 6. It is clear from Fig. 6 that the filters of $K = 7$ and $K = 5$ have two and three ripples in the stopband, while the filter of $K = 9$ has only one ripple. Therefore, the magnitude error will become smaller with decreasing K. When $K = 7$, we construct $G(z)$ as shown in Eq. (7), and show its magnitude

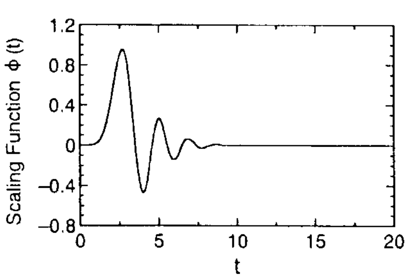

Fig. 4. Scaling function of Example 1.

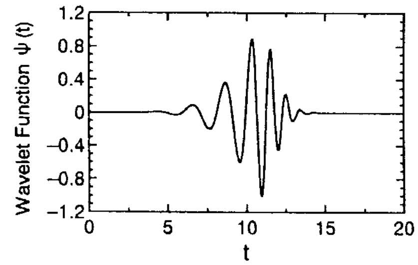

Fig. 5. Wavelet function of Example 1.

Table 2. Filter coefficients of Example 2

c_1	0.3156246210	c_{13}	0.0057576230
c_3	−0.0986920702	c_{15}	−0.0026337401
c_5	0.0524614950	c_{17}	0.0008764902
c_7	−0.0309334931	c_{19}	−0.0001801141
c_9	0.0182834505	c_{21}	0.0000168605
c_{11}	−0.0105811514		

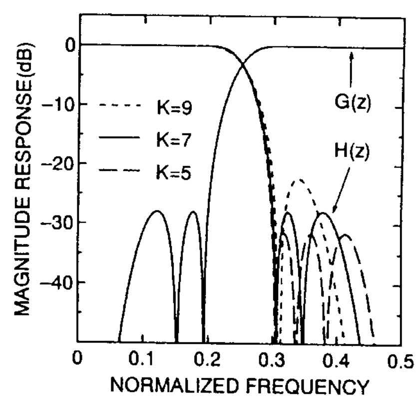

Fig. 6. Magnitude responses of Example 2.

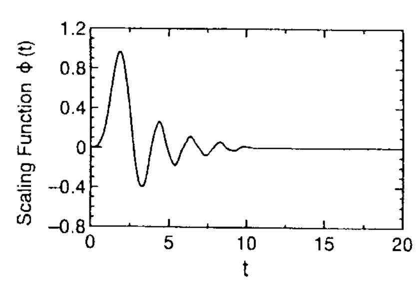

Fig. 7. Scaling function of Example 2.

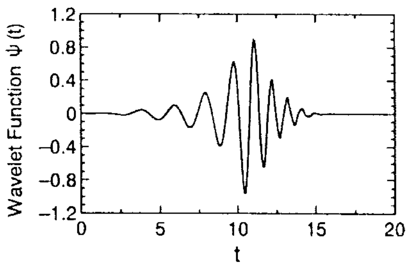

Fig. 8. Wavelet function of Example 2.

response in Fig. 6. From the magnitude responses of $H(z)$ and $G(z)$, it can be seen that they are paraunitary filter banks. The scaling function $\phi(t)$ and wavelet function $\psi(t)$ generated by the above filter bank are shown in Figs. 7 and 8, respectively. In this example, the required number of iterations is three to five.

5. Conclusions

In this paper, we have proposed a new method for designing orthonormal wavelet filter banks with arbitrary regularity by using the Remez exchange algorithm in the z-domain directly. From the orthonormality and regularity of wavelets, we have derived some conditions that must be satisfied for FIR paraunitary filter banks, and have investigated the relationship between the filter coefficients and the zeros of the filter. According to the relationship, we have used the Remez exchange algorithm in the stopband directly and have formulated the design problem in the form of a linear problem. Therefore, we can easily get a set of filter coefficients by solving the linear equations. The optimal solution can be obtained after using an iteration procedure. In the proposed method, the main advantages are that the regularity can be arbitrarily specified and that less computational complexity is required as compared with the conventional methods.

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