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# Design of Biorthogonal FIR Linear Phase Filter Banks with Structurally Perfect Reconstruction

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## SUMMARY

In the design of two channel perfect reconstruction filter banks, most of the conventional methods optimize the frequency response of each filter to meet the perfect reconstruction condition. However, quantization of the filter coefficients results in some errors in the frequency response, so it is not guaranteed that the perfect reconstruction condition is still satisfied. In this paper, we present a new method for designing biorthogonal FIR linear phase filter banks with structurally perfect reconstruction. From the perfect reconstruction condition, we first describe a class of structurally perfect reconstruction implementations. Since the proposed filter banks structurally satisfy the perfect reconstruction condition, the design problem becomes the magnitude approximation of the analysis or synthesis filters. Design of these filters can be reduced to the design of half-band filters. We then give a new method to design FIR linear phase half-band filters with arbitrary flatness. Therefore, the proposed filter banks can be designed easily by using the proposed method. Additionally, the magnitude responses of the low- and high-pass filters can be arbitrarily controlled by using two different half-band filters. © 1998 Scripta Technica, Electron Comm Jpn Pt 3, 82(1): 1–8, 1999

**Key words**: Structurally perfect reconstruction; biorthogonal filter bank; half-band filter; FIR linear phase filter.

## 1. Introduction

In recent years, two channel perfect reconstruction (PR) filter banks have received considerable attention in many signal processing applications, including subband coding of speech and image signals [1–15]. In the design of two channel PR filter banks using FIR linear phase filters, some design methods are known [2–15]. Most of these methods optimize the frequency response of each filter to meet the perfect reconstruction condition. Then, the filter coefficients obtained satisfy the PR condition with infinite accuracy. However, when implemented with finite accuracy, coefficient quantization results in errors in the frequency responses, hence, it is not guaranteed that the perfect reconstruction condition is still satisfied. Compared with these methods, lattice structures [4, 5] have been proposed that structurally satisfy the PR condition even though the filter coefficients are quantized. However, to optimize the filter coefficients in lattice structures a nonlinear optimization procedure is required and this entails a large amount of computation. One additional method has been also proposed that first designs the PR filter bank and then transforms it into a lattice structure [9, 13]. In Ref. 15, a class of structurally PR filter banks was described for which the design problem can be reduced to the design of one half-band filter. Hence, the filter banks can be easily designed by using the conventional design method for half-band filters. However, the magnitude responses of the low- and high-pass filters cannot be separately designed, since they are dependent on the same transfer function. In Ref. 11, the PR filter banks are similarly designed by using half-band filters, but the maximum stopband attenuation in

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the high pass filter is not obtained since the maximally flat function is used.

In this paper, we propose a new method for designing biorthogonal FIR linear phase filter banks that structurally satisfy the PR condition. From the PR condition, we first show a class of structurally perfect reconstruction implementations. Since the proposed filter banks are basically the same as those of Ref. 15, the PR condition is structurally satisfied even though the filter coefficients are quantized. In contrast to the half-band filter used in Ref. 15, we use two different half-band filters in this paper. This is a different issue from Ref. 15. By using two different transfer functions, we can arbitrarily control the magnitude responses of the low- and high-pass filters, which cannot be designed separately in Ref. 15. The design problem of the proposed filter banks can be reduced to the magnitude approximation of two half-band filters. We then give a new method to design FIR half-band filters. In the conventional methods for designing half-band filters, only maximally flat and equiripple filters are considered. In this paper, from the regularity condition of wavelets, we consider the design of FIR linear phase half-band filters with arbitrary flatness. According to the symmetric property of the magnitude response of half-band filters, after considering the given flatness condition, we use the Remez exchange algorithm and formulate the design problem in linear form. Therefore, we can easily obtain a set of filter coefficients by solving a set of linear equations, and the optimal solution is obtained by applying an iteration procedure. Finally, some examples are employed to demonstrate the effectiveness of the proposed method.

## 2. Structurally PR Filter Banks

In the two channel filter bank shown in Fig. 1, assume that $H_i(z)$, $G_i(z)$ are analysis and synthesis filters, respectively. To obtain a PR filter bank, these filters must satisfy

$$\begin{cases} H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K-1} \\ H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \end{cases} \tag{1}$$

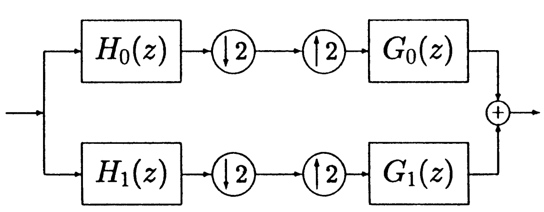


Fig. 1. Two channel filter bank.

where $K$ is integer. By using the polyphase matrix description of $H_i(z)$ and $G_i(z)$,

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} = \boldsymbol{H}(z^2) \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix}^T = \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^T \begin{bmatrix} G_{00}(z^2) & G_{01}(z^2) \\ G_{10}(z^2) & G_{11}(z^2) \end{bmatrix} = \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}^T \boldsymbol{G}(z^2) \tag{3}$$

the PR condition of Eq. (1) becomes

$$\boldsymbol{G}(z)\boldsymbol{H}(z) = \frac{z^{-K}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{z^{-K}}{2}\boldsymbol{I} \tag{4}$$

It is well known that

$$\begin{bmatrix} z^{-N} & 0 \\ -A(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ A(z) & z^{-N} \end{bmatrix} = z^{-N}\boldsymbol{I} \tag{5}$$

$$\begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix} \begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix} = z^{-M}\boldsymbol{I} \tag{6}$$

Therefore, if

$$\boldsymbol{H}(z) = \begin{bmatrix} z^{-M} & -B(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{A(z)}{2} & \frac{z^{-N}}{2} \end{bmatrix} \tag{7}$$

$$\boldsymbol{G}(z) = \begin{bmatrix} \frac{z^{-N}}{2} & 0 \\ -\frac{A(z)}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & B(z) \\ 0 & z^{-M} \end{bmatrix} \tag{8}$$

then the PR condition of Eq. (4) is satisfied regardless of the values of $A(z)$ and $B(z)$, i.e., the PR filter banks can be still obtained even when the filter coefficients of $A(z)$ and $B(z)$ are quantized. The structurally PR implementation is shown in Fig. 2. Therefore, the design problem of the filter banks becomes a design for analysis or synthesis filters. The transfer functions for analysis filters are

$$\begin{cases} H_0(z) = z^{-2M} - B(z^2)H_1(z) \\ H_1(z) = \frac{1}{2}\{z^{-2N-1} + A(z^2)\} \end{cases} \tag{9}$$

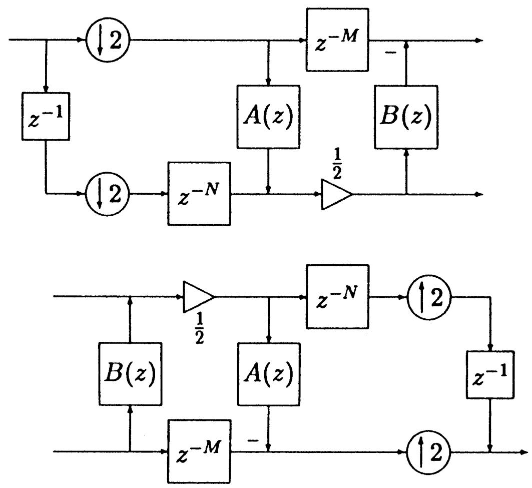


Fig. 2. Structurally perfect reconstruction filter bank.

and the transfer functions for synthesis filters are

$$\begin{cases} G_0(z) = \frac{1}{2}\{z^{-2N-1} - A(z^2)\} = -H_1(-z) \\ G_1(z) = z^{-2M} + B(z^2)G_0(z) = H_0(-z) \end{cases} \tag{10}$$

In the following, we consider design of the analysis filters $H_i(z)$.

## 3. Design of PR Filter Banks

In this section, we describe the design of $H_i(z)$ using FIR filters. The transfer functions $A(z)$ and $B(z)$ are defined as

$$\begin{cases} A(z) = \sum_{n=0}^{L_1} a_n z^{-n} \\ B(z) = \sum_{n=0}^{L_2} b_n z^{-n} \end{cases} \tag{11}$$

where the filter coefficients $a_n$ and $b_n$ are real. To obtain an exact linear phase, the filter coefficients of $A(z)$ and $B(z)$ must be symmetric, i.e., $a_n = a_{L_1-n}$, $b_n = b_{L_2-n}$.

### 3.1. Desired magnitude responses

From Eq. (9), the transfer function $H_1(z)$ is

$$H_1(z) = \frac{1}{2}\left\{z^{-2N-1} + \sum_{n=0}^{L_1} a_n z^{-2n}\right\} \tag{12}$$

If $L_1 = 2N + 1$, then it is clear that $H_1(z)$ is a linear phase half-band filter. $H_1(z)$ can be designed as low- or high-pass filter. Here, we consider design of lowpass filter. When highpass filter is needed, it can be obtained by replacing $A(z^2)$ with $-A(z^2)$ in the lowpass filter obtained. Assume that the passband of $H_1(z)$ is $[0, \omega_p]$ and the stopband is $[\omega_s, \pi]$, where $\omega_p + \omega_s = \pi$. It is well known that the half-band filter $H_1(z)$ has an antisymmetric magnitude response. Therefore, the desired magnitude response of $A(z)$ is

$$|A_d(e^{j\omega})| = 1 \qquad (0 \leqq \omega \leqq 2\omega_p) \tag{13}$$

From Eq. (9), the transfer function $H_0(z)$ is

$$H_0(z) = z^{-2M} - H_1(z)\sum_{n=0}^{L_2} b_n z^{-2n} \tag{14}$$

In the stopband $[\omega_s, \pi]$ of $H_1(z)$, since the magnitude of $H_1(z)$ is 0, the magnitude of $H_0(z)$ is 1, and it becomes the passband. In the passband $[0, \omega_p]$ of $H_1(z)$, since $H_1(z) = z^{-2N-1}$, ideally, then,

$$H_0(z) = z^{-2N-1}\left\{z^{-2(M-N)+1} - \sum_{n=0}^{L_2} b_n z^{-2n}\right\} \tag{15}$$

Hence, if we choose $L_2 = 2(M - N) - 1$, $H_0(z)$ of Eq. (15) will become a linear phase half-band filter. To force $H_0(z)$ to be the stopband, the desired magnitude response of $B(z)$ is

$$|B_d(e^{j\omega})| = 1 \qquad (0 \leqq \omega \leqq 2\omega_p) \tag{16}$$

Therefore, approximating $A(z)$ and $B(z)$ by the desired magnitude response of Eqs. (13) and (16), the PR filter banks can be easily designed.

### 3.2. Half-band filters with arbitrary flatness

In this section, we describe design of half-band filters. In the conventional methods for designing half-band filters, only maximally flat and equiripple filters have been considered. In recent years, wavelet transforms have been applied in various fields of signal processing. In many applications, the wavelet functions are required to be continuous. This is the so-called regularity condition. The regularity corresponds to a flatness condition for the filter banks [2, 8]. Therefore, we consider the design of half-band filters with an arbitrary flatness. The magnitude response of $A(z)$ is given by

$$|A(e^{j\omega})| = 2\sum_{n=0}^{N} a_n \cos\left(N - n + \frac{1}{2}\right)\omega \tag{17}$$

Assume that $A(z)$ has flatness of order $2J_1$ at $\omega = 0$, i.e.,

$$\begin{cases} |A(1)| = 1 & (i = 0) \\ \left.\dfrac{d^i |A(e^{j\omega})|}{d\omega^i}\right|_{\omega=0} = 0 & (i = 1, 2, \cdots, 2J_1 - 1) \end{cases} \tag{18}$$

where $J_1$ must satisfy

$$0 \leqq J_1 \leqq N + 1 \tag{19}$$

When $J_1 = 0$, it is an equiripple filter that minimizes the maximum magnitude error. When $J_1 = N + 1$, it becomes a maximally flat filter. Therefore, from Eq. (18), we have

$$\begin{cases} \displaystyle\sum_{n=0}^{N} a_n = \frac{1}{2} & (i = 0) \\ \displaystyle\sum_{n=0}^{N} \left(N - n + \frac{1}{2}\right)^{2i} a_n = 0 & (i = 1, 2, \cdots, J_1 - 1) \end{cases} \tag{20}$$

To force the magnitude response of $A(z)$ to be equiripple, we use the Remez exchange algorithm. First, we select $I(= N - J_1 + 2)$ extremal frequencies $\omega_i$ in the band $[0, 2\omega_p]$

$$0 \leqq \omega_0 < \omega_1 < \cdots < \omega_{I-1} = 2\omega_p \tag{21}$$

Then, we formulate $|A(e^{j\omega})|$ at these extremal frequencies, i.e.,

$$|A(e^{j\omega_i})| = 1 - (-1)^i \delta \tag{22}$$

where $\delta$ is a magnitude error. By substituting the magnitude response of Eq. (17) into Eq. (22), we obtain

$$\sum_{n=0}^{N} a_n \cos\left(N - n + \frac{1}{2}\right)\omega_i + \frac{(-1)^i}{2}\delta = \frac{1}{2} \tag{23}$$

We can rewrite Eqs. (23) and (20) in matrix form

$$\boldsymbol{P}\ \boldsymbol{A} = \boldsymbol{Q} \tag{24}$$

where $\boldsymbol{A} = [a_0, a_1, \ldots, a_N, \delta]^T, \boldsymbol{Q} = [\frac{1}{2}, \ldots, \frac{1}{2}, \frac{1}{2}, 0, \ldots, 0,]^T$, and the elements $P_{ij}$ of $\boldsymbol{P}$ are

$$P_{ij} = \begin{cases} \cos(N - j + \frac{1}{2})\omega_i & \begin{pmatrix} 0 \leqq i \leqq I - 1 \\ 0 \leqq j \leqq N \end{pmatrix} \\ (N - j + \frac{1}{2})^{2(i-I)} & \begin{pmatrix} I \leqq i \leqq N + 1 \\ 0 \leqq j \leqq N \end{pmatrix} \\ \dfrac{(-1)^i}{2} & \begin{pmatrix} 0 \leqq i \leqq I - 1 \\ j = N + 1 \end{pmatrix} \\ 0 & (\text{else}) \end{cases} \tag{25}$$

It is clear that Eq. (24) is a set of linear equations. Because there are $N + 2$ unknown parameters (including $N + 1$ filter coefficients $a_n$ and one magnitude error $\delta$) from the $I + J_1 = N + 2$ equations, we can uniquely obtain a set of filter coefficients by solving the linear equations of Eq. (24). We compute the magnitude response of $A(z)$ by using the filter coefficients obtained $a_n$, and search for the peak frequencies $\overline{\omega}_i$. As a result, the initially selected extremal frequencies $\omega_i$ cannot be guaranteed to be equal to the peak frequencies $\overline{\omega}_i$. We then set the peak frequencies obtained to be the extremal frequencies in the next iteration, and solve the linear equations of Eq. (24) to obtain the filter coefficients $a_n$ again. The above procedure is iterated until the extremal frequencies $\omega_i$ and the peak frequencies $\overline{\omega}_i$ are consistent. When the extremal frequencies do not change, we obtain the optimal solution with an equiripple magnitude response. The design algorithm is described in detail in the next section.

### 3.3. Design algorithm

1. Read filter specifications $N, J_1$ and the cutoff frequency $\omega_p$.
2. Select $I$ initial extremal frequencies $\omega_i$ equally spaced in the band $[0, 2\omega_p]$ as shown in Eq. (21).
3. Solve the linear equations of Eq. (24) to obtain a set of filter coefficients $a_n$.
4. Compute the magnitude response of $A(z)$ by using the obtained filter coefficients $a_n$, and search for the peak frequencies $\overline{\omega}_i$.
5. If

$$\sum_{i=0}^{I-1} |\overline{\omega}_i - \omega_i| < \varepsilon,$$

then exit. Else, go to 6, where $\varepsilon$ is a prescribed small constant.

6. Set $\omega_i = \overline{\omega}_i (i = 0, 1, \ldots, I - 1)$, then go to 3.

### 3.4. Design of $H_0(z)$

We can design $A(z)$ by using the method proposed in section 3.2 to obtain $H_1(z)$ with arbitrary flatness. The obtained $H_1(z)$ has the maximum magnitude error $\delta_{s1} = \delta_a/2$ in the stopband, where $\delta_a$ is the maximum magnitude error of $A(z)$. According to the symmetric property of the magnitude response of half-band filters, the magnitude error of $H_1(z)$ in the passband is the same as the stopband error. We can design $B(z)$ similarly. However, even when the magnitude responses of $A(z)$ and $B(z)$ are designed to be equiripple, it can be seen from Eq. (9) that the magnitude response of $H_0(z)$ cannot be guaranteed to be equiripple. In the most cases, we cannot obtain an equiripple response for $H_0(z)$. In Ref. 15, $H_0(z)$ becomes equiripple by setting $B(z) = A(z)$, but both $H_0(z)$ and $H_1(z)$ are dependent on the same transfer function $A(z)$. Hence, the magnitude responses of both cannot be designed separately. In particular, the maximum magnitude error $\delta_{s0}$ of $H_0(z)$ in the stopband is

$$\delta_{s0} = 1-(1-\delta_a)\left(1-\frac{\delta_a}{2}\right) = \frac{3\delta_a}{2} - \frac{\delta_a^2}{2} \simeq 3\delta_{s1} \tag{26}$$

In other words, the stopband error of $H_0(z)$ is three times larger than that of $H_1(z)$, even though the filter order is higher than for $H_1(z)$. In this paper, we directly design $B(z)$ to force $H_0(z)$ to have an equiripple magnitude response. By using different $B(z)$, we can arbitrarily control the magnitude error of $H_0(z)$. To force $H_0(z)$ to have an equiripple response in the stopband, we have to design an equiripple magnitude response of $B(z^2)H_1(z)$. Hence, we can consider the magnitude response of $H_1(z)$ as a weighting function $W(\omega)$, i.e., we can reformulate

$$\begin{aligned} W(\omega)|B(e^{j\omega_i})| &= |H_1(e^{j\frac{\omega_i}{2}})||B(e^{j\omega_i})| \\ &= 1 + (-1)^i\delta \end{aligned} \tag{27}$$

hence $H_0(z)$ has an equiripple response. The design algorithm is the same as that described in section 3.3. From Eq. (9), the magnitude response of $H_0(z)$ is

$$|H_0(e^{j\omega})| = 1 - \frac{1}{2}|B(e^{j2\omega})|\{1 + |A(e^{j2\omega})|\} \tag{28}$$

Therefore, in the passband of $H_0(z)$, its maximum magnitude error $\delta_{p0}$ is

$$\delta_{p0} = (1+\delta_b)\left(\frac{\delta_a}{2}\right) = \frac{\delta_a}{2} + \frac{\delta_a\delta_b}{2} \simeq \frac{\delta_a}{2} = \delta_{s1} \tag{29}$$

where $\delta_b$ is the maximum magnitude error of $B(z)$. Therefore, in the passband of $H_0(z)$, since its magnitude error is mainly determined by $A(z)$, it is seen that an almost equiripple response can be obtained. Assume that $B(z)$ has flatness of order $2J_2$ at $\omega = 0$. It is clear from Eq. (28) that the flatness of $H_0(z)$ is determined by the smallest flatness between $A(z)$ and $B(z)$. That is, it has flatness of order 2 $\mathrm{Min}\{J_1, J_2\}$. In other words, the flatness of $H_0(z)$ is not higher than $H_1(z)$, and at most is the same as $H_1(z)$, even when $J_2$ is larger than $J_1$. Therefore, in the practical design it is required that the larger flatness filter between low- and high-pass filters must be designed by $H_1(z)$, and the lower flatness filter by $H_0(z)$.

Table 1. Filter coefficients of $A(z)$ in Example 1

| | | | |
|---|---|---|---|
| $a_0 = a_{17}$ | $2.312876e-03$ | $a_5 = a_{12}$ | $-6.218851e-02$ |
| $a_1 = a_{16}$ | $-5.497636e-03$ | $a_6 = a_{11}$ | $1.052088e-01$ |
| $a_2 = a_{15}$ | $1.153720e-02$ | $a_7 = a_{10}$ | $-1.982618e-01$ |
| $a_3 = a_{14}$ | $-2.147420e-02$ | $a_8 = a_9$ | $6.318541e-01$ |
| $a_4 = a_{13}$ | $3.718555e-02$ | | |

## 4. Design Examples

**Example 1** (Equiripple Filters)

The specifications for the filter banks are $N = 8$, $M = 19$ and $\omega_p = 0.4\pi$. The orders of $A(z)$ and $B(z)$ are 17 and 21, respectively. We design the equiripple filters by setting $J_1 = J_2 = 0$. The filter coefficients obtained $a_n$, $b_n$ of $A(z)$ and $B(z)$ are shown in Tables 1 and 2, and their magnitude responses are shown in Fig. 3. It is clear from Fig. 3 that the magnitude response of $A(z)$ is equiripple, while the magnitude response of $B(z)$ is not equiripple. However, it can be seen in Fig. 4 that the magnitude responses of both $H_0(z)$ and $H_1(z)$ are equiripple. For comparison purposes, the magnitude response of $H_0(z)$ obtained by setting $B(z) = A(z)$ in the conventional method of Ref. 15 is also shown in Fig. 4. In that case, the order of $B(z)$ was 17, and $M = 17$. It can be seen in Fig. 4 that there is a difference of about 10 dB in the stopband attenuation between $H_0(z)$ and $H_1(z)$ when $B(z) = A(z)$. Therefore, by directly designing

Table 2. Filter coefficients of $B(z)$ in Example 1

| | | | |
|---|---|---|---|
| $b_0 = b_{21}$ | $1.064541e-03$ | $b_6 = b_{15}$ | $4.384128e-02$ |
| $b_1 = b_{20}$ | $-2.908241e-03$ | $b_7 = b_{14}$ | $-6.837364e-02$ |
| $b_2 = b_{19}$ | $5.413201e-03$ | $b_8 = b_{13}$ | $1.102419e-01$ |
| $b_3 = b_{18}$ | $-1.013660e-02$ | $b_9 = b_{12}$ | $-2.015517e-01$ |
| $b_4 = b_{17}$ | $1.726910e-02$ | $b_{10} = b_{11}$ | $6.329982e-01$ |
| $b_5 = b_{16}$ | $-2.793873e-02$ | | |

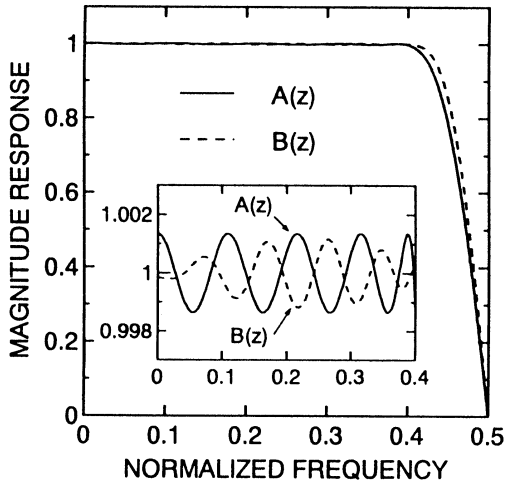


Fig. 3. Magnitude responses of $A(z)$ and $B(z)$ in Example 1.

Table 3. Filter coefficients of $A(z)$ in Example 2

| | | | |
|---|---|---|---|
| $a_0 = a_{17}$ | $7.045959e-04$ | $a_5 = a_{12}$ | $-6.030876e-02$ |
| $a_1 = a_{16}$ | $-3.920778e-03$ | $a_6 = a_{11}$ | $1.033390e-01$ |
| $a_2 = a_{15}$ | $1.015700e-02$ | $a_7 = a_{10}$ | $-1.970177e-01$ |
| $a_3 = a_{14}$ | $-1.929778e-02$ | $a_8 = a_9$ | $6.316505e-01$ |
| $a_4 = a_{13}$ | $3.469393e-02$ | | |

Table 4. Filter coefficients of $B(z)$ in Example 2

| | | | |
|---|---|---|---|
| $b_0 = b_{21}$ | $4.058596e-04$ | $b_6 = b_{15}$ | $4.301973e-02$ |
| $b_1 = b_{20}$ | $-2.155584e-03$ | $b_7 = b_{14}$ | $-6.751611e-02$ |
| $b_2 = b_{19}$ | $5.269806e-03$ | $b_8 = b_{13}$ | $1.096822e-01$ |
| $b_3 = b_{18}$ | $-9.406072e-03$ | $b_9 = b_{12}$ | $-2.011674e-01$ |
| $b_4 = b_{17}$ | $1.618753e-02$ | $b_{10} = b_{11}$ | $6.328189e-01$ |
| $b_5 = b_{16}$ | $-2.713879e-02$ | | |

$B(z)$ to be different from $A(z)$, the magnitude error of $H_0(z)$ can be arbitrarily controlled.

**Example 2** (Filters with Arbitrary Flatness)

The specifications of filter banks are $N = 8$, $M = 19$, and $\omega_p = 0.4\pi$. The order of $A(z)$ and $B(z)$ are 17 and 21, respectively. First, we designed the filter bank by setting $J_1 = J_2 = 4$ and using the proposed method. The obtained filter coefficients $a_n$, $b_n$ of $A(z)$ and $B(z)$ are shown in Tables 3 and 4, and their magnitude responses are shown in Fig. 5 in the solid line. In this case, $H_0(z)$ and $H_1(z)$ have the same flatness, their magnitude responses are shown in Fig. 6 in the solid line, and both are equiripple. We also made the design with $J_1 = 9$ and $J_2 = 4$. When $J_1 = 9$, $H_1(z)$ is a maximally flat filter, and has a higher flatness than $H_0(z)$. The obtained magnitude responses of $A(z)$ and $B(z)$ are shown in Fig. 5, and those of $Ho(z)$ and $H_1(z)$ in Fig. 6 using dotted solid line, respectively. It is clear from Fig. 6 that the magnitude responses of $H_0(z)$ are equiripple.

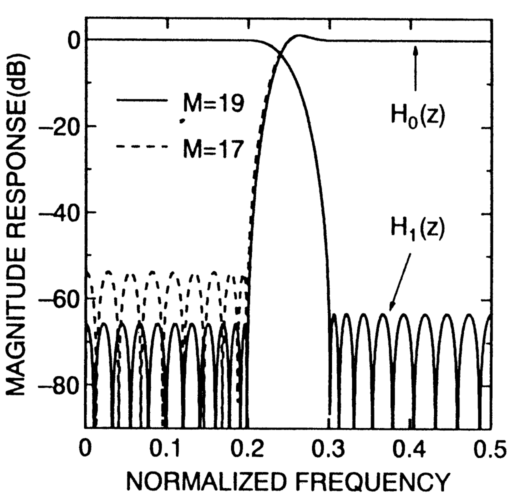


Fig. 4. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 1.

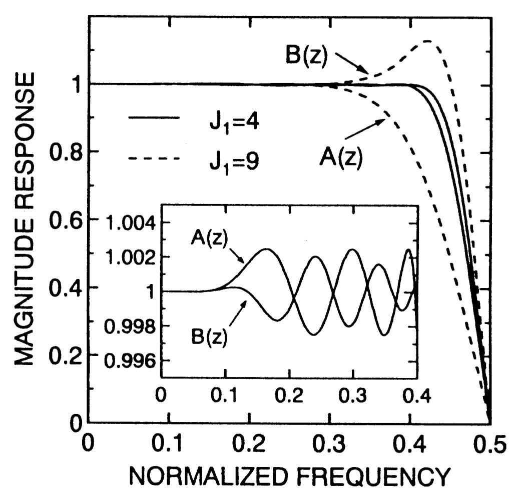


Fig. 5. Magnitude responses of $A(z)$ and $B(z)$ in Example 2.

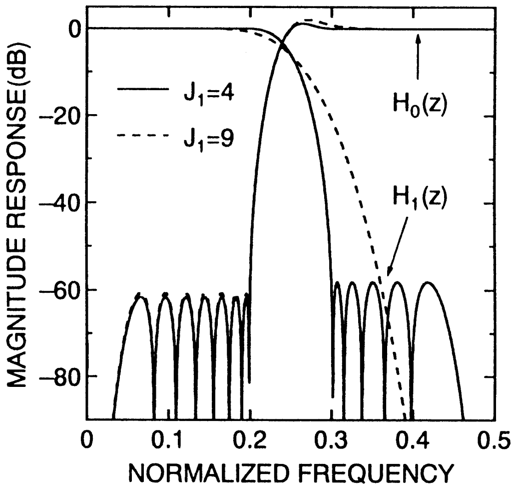


Fig. 6. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 2.

## 5. Conclusions

In this paper, we have proposed a new method for designing two channel biorthogonal FIR linear phase filter banks that structurally satisfy the PR condition. We first showed a class of structurally perfect reconstruction implementations. Since the PR condition is structurally satisfied even though the filter coefficients are quantized in the proposed filter banks, the design problem becomes the magnitude approximation of analysis or synthesis filters. Their design can be reduced to the design of half-band filters. Then a new design method of linear phase FIR half-band filters with an arbitrary flatness was presented. After considering the given flatness condition, we used the Remez exchange algorithm and formulated the design of FIR half-band filters as a linear problem. Therefore, we can easily obtain a set of filter coefficients by solving the linear equations, and the optimal solution is obtained by applying an iteration procedure. Additionally, by using two half-band filters, we can arbitrarily design the magnitude responses of the low- and high-pass filters. Design of stable PR filter banks using allpass filters remains to be investigated in future.

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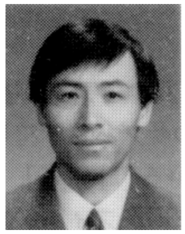

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