Electronics and Communications in Japan, Part 3, Vol. 83, No. 10, 2000
Translated from Denshi Joho Tsushin Gakkai Ronbunshi, Vol. J82-A, No. 10, October 1999, pp. 1529–1537

Design of Low-Delay FIR Half-Band Filters with Arbitrary Flatness and Its Application to Filter Banks

Xi Zhang, Kentaro Inotsume, and Toshinori Yoshikawa

Department of Electrical Engineering, Nagaoka University of Technology, Nagaoka, Japan 940-2188

SUMMARY

Half-band filters are a class of important filters among digital filters, and have been widely used in many applications such as filter banks and wavelets. The conventional methods are mainly concerned with FIR half-band filters with exactly linear phase. However, the exactly linear phase filters have a drawback of large group delay when high-order filters are required. In this paper, the design of FIR half-band filters with a lower group delay is considered. In some applications of filter banks and wavelets, a flat magnitude response is required for the half-band filters. Therefore, a new method for designing low-delay FIR half-band filters with arbitrary flatness is proposed. In the proposed method, while taking the specified flatness condition into account, the design problem is formulated by using the complex Remez exchange algorithm in the stopband. Then, a set of filter coefficients can be easily obtained by solving a simple system of linear equations. The optimal solution with an equiripple response in the stopband is obtained by applying an iteration process. Finally, the proposed method is applied to the design of two-channel perfect reconstruction filter banks with low group delay to demonstrate its effectiveness. © 2000 Scripta Technica, Electron Comm Jpn Pt 3, 83(10): 1–9, 2000

Key words: FIR half-band filter; low-delay filter; flat magnitude response; filter bank.

1. Introduction

Half-band filters are a class of important filters among digital filters, and have been widely used in many applications such as filter banks and wavelets [1–3]. The conventional methods for designing half-band filters are mainly concerned with FIR half-band filters with exactly linear phase [1–5]. However, the group delay is equal to half the filter order for FIR filters with exactly linear phase, since the filter coefficients are symmetric. Thus, the group delay becomes larger as the filter order increases. When a sharp magnitude response is required and high-order filters are needed, the large group delay of FIR linear phase filters will present a serious problem. In particular, it will have a negative influence on the overall system in cases of real-time signal processing [7, 9]. Therefore, it is essential to design FIR half-band filters with a lower group delay.

In this paper, we consider the design of FIR half-band filters with low group delay. First, we investigate the frequency response property and constraint from the time-domain condition of FIR half-band filters. According to this property, it is shown that the design problem of low-delay FIR half-band filters can be reduced to the minimization of the magnitude response in the stopband. Also, in some applications of filter banks and wavelets [6–9], a flat magnitude response is required for the half-band filters. Therefore, we propose a new method for designing low-delay FIR half-band filters with arbitrary flatness. In the proposed method, while taking the specified flatness condition into account, the design problem of FIR half-band filters is formulated as a linear problem by using the complex Remez exchange algorithm in the stopband. Then, a set of filter coefficients can be easily obtained by solving a simple system of linear equations. The optimal solution with an equiripple response in the stopband is obtained by applying an iteration process. The advantages of the proposed method are that a lower group delay can be realized and flatness can be arbitrarily specified for FIR half-band filters. Finally, we apply the proposed method to the design of two-channel perfect reconstruction filter banks with low group delay to demonstrate its effectiveness.

2. Low-Delay FIR Half-Band Filters

The transfer function $H(z)$ of FIR digital filter of order $2N$ is defined as

$$H(z) = \sum_{n=0}^{2N} h_n z^{-n} \tag{1}$$

where h_n are real filter coefficients. In the design of half-band filters, the impulse response is required to satisfy the following constraint in the time domain:

$$\begin{cases} h_K = \dfrac{1}{2} \\ h_{K+2k} = 0 \quad (k = \pm 1, \pm 2, \ldots) \end{cases} \tag{2}$$

where K is odd. In the frequency domain, the desired frequency response is given by

$$H_d(e^{j\omega}) = \begin{cases} e^{-jK\omega} & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases} \tag{3}$$

where ω_p and ω_s are the passband and stopband cutoff frequencies, respectively, and $\omega_p + \omega_s = \pi$. It is seen in Eq. (3) that the desired group delay of half-band filters is K in the passband, that is, the group delay can be controlled by selecting a smaller or larger K. In the conventional design methods of FIR half-band filters, the filter coefficients must be symmetric to obtain an exact linear phase, so that the group delay is equal to half the filter order, that is, $K = N$. Thus, the group delay K becomes larger as the filter order $2N$ increases. When a sharp magnitude response is required and high-order filters are needed, the large group delay of FIR linear phase filters will pose a serious problem. In particular, it will have a negative influence on the overall system in some applications of real-time signal processing [7, 9]. Therefore, we consider the design of FIR half-band filters with a lower group delay in this paper. That is, we select a smaller K by relaxing the symmetry condition of filter coefficients required in the linear phase filters, as shown in Fig. 1. While it is only possible to design half-band filters of order $2N = 2, 6, 10, \ldots$, since N is odd due to $K = N$ in FIR linear-phase half-band filters, filters of not only $2N = 2, 6, 10, \ldots$ but also $2N = 4, 8, 12, \ldots$ can be designed, because the symmetry condition of filter coefficients is relaxed in this paper.

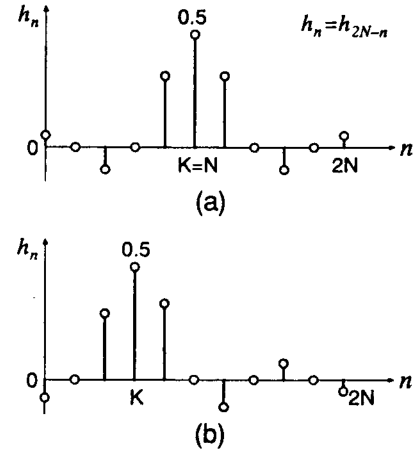


Fig. 1. Impulse responses of FIR half-band filters.

By substituting the time-domain condition of Eq. (2) into Eq. (1) the transfer function of FIR half-band filters becomes

$$H(z) = \frac{1}{2} z^{-K} + \sum_{n=0}^{N} a_n z^{-2n} \tag{4}$$

where $a_n = h_{2n}$. Assume that $\hat{H}(z)$ is a noncausal shifted version of $H(z)$ in Eq. (4):

$$\hat{H}(z) = z^K H(z) = \frac{1}{2} + \sum_{n=0}^{N} a_n z^{K-2n} \tag{5}$$

Hence, the frequency response of $\hat{H}(z)$ is given by

$$\hat{H}(e^{j\omega}) = e^{jK\omega} H(e^{j\omega}) = \frac{1}{2} + \sum_{n=0}^{N} a_n e^{j(K-2n)\omega} \tag{6}$$

From Eq. (3), the desired frequency response of $\hat{H}(z)$ is

$$\hat{H}_d(e^{j\omega}) = \begin{cases} 1 & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases} \tag{7}$$

It is clear in Eq. (6) that the frequency response of $\hat{H}(z)$ satisfies the following relation:

$$\hat{H}(e^{j\omega}) + \hat{H}^*(e^{j(\pi-\omega)}) \equiv 1 \tag{8}$$

where x^* denotes the complex conjugate of x. Equation (8) means that a certain constraint is imposed on the frequency response of $\hat{H}(z)$ at two frequency points ω and $\pi - \omega$ from the time-domain condition of Eq. (2). By rewriting Eq. (8), we have

$$\hat{H}(e^{j\omega_0}) = 1 - \hat{H}^*(e^{j(\pi-\omega_0)}) \tag{9}$$

where ω_0 is one frequency point in the passband. Then $\pi - \omega_0$ is located in the stopband. It is seen in Eq. (9) that the frequency response of $\hat{H}(z)$ in the passband is dependent on the stopband response. Therefore, if its stopband response is 0, then $\hat{H}(e^{j\omega}) = 1$, that is, $H(e^{j\omega}) = e^{-jK\omega}$ in the passband. Let δ_s be the maximum magnitude error in the stopband; the maximum magnitude error δ_p and phase error $\Delta\theta_p$ in the passband are

$$\begin{cases} \delta_p \le \delta_s \\ \Delta\theta_p \le \sin^{-1}\delta_s \end{cases} \tag{10}$$

Then both the passband magnitude and phase errors are decided by the stopband magnitude error. Therefore, the design problem of low-delay FIR half-band filters is reduced to the minimization of the stopband magnitude error. In the following, we consider the approximation of $\hat{H}(z)$ in the stopband.

3. Design of Low-Delay FIR Half-Band Filters with Arbitrary Flatness

In many applications of filter banks and wavelets [6–9], a flat magnitude response is required for half-band filters from the regularity of wavelets. Now, we consider the design of low-delay FIR half-band filters with arbitrary flatness. There are $N + 1$ unknown filter coefficients in the transfer function $H(z)$ of Eq. (4). Hence. $H(z)$ has $N + 1$ independent zeros. In the following, we will use these $N + 1$ independent zeros to approximate the stopband response.

3.1. Flatness condition

For FIR half-band filters, a flat magnitude response is required at $\omega = 0$ and $\omega = \pi$. The flatness conditions are given by

$$\begin{cases} \hat{H}(1) = 1 \\ \left.\dfrac{\partial^m \hat{H}(e^{j\omega})}{\partial\omega^m}\right|_{\omega=0} = 0 \quad (m = 1, 2, \ldots, M-1) \end{cases} \tag{11}$$

$$\left.\frac{\partial^m \hat{H}(e^{j\omega})}{\partial\omega^m}\right|_{\omega=\pi} = 0 \quad (m = 0, 1, \ldots, M-1) \tag{12}$$

where $0 \le M \le N + 1$. It is clear in Eq. (9) that if the flatness condition in Eq. (12) is satisfied, the flatness condition in Eq. (11) is automatically satisfied. Therefore, only the flatness condition in Eq. (12) is taken into account in the following. To satisfy the flatness condition in Eq. (12), M independent zeros must be located at $z = -1$.

By differentiating the frequency response of Eq. (6),

$$\frac{\partial^m \hat{H}(e^{j\omega})}{\partial\omega^m} = \sum_{n=0}^{N} j^m a_n (K-2n)^m e^{j(K-2n)\omega} \tag{13}$$

Therefore, from the flatness condition in Eq. (12), we have

$$\sum_{n=0}^{N}(K-2n)^m a_n = \begin{cases} 0.5 & (m=0) \\ 0 & (m = 1, 2, \cdots, M-1) \end{cases} \tag{14}$$

When $M = N + 1$, that is, the maximally flat half-band filters are designed, the filter coefficients can be easily obtained by solving the linear equations in (14). Also the maximally flat half-band filters can be analytically solved. See Ref. 2 for details.

3.2. A choice of initial zeros

When $M \le N$, the number of remaining independent zeros other than $z = -1$ is $N - M + 1$. As shown in Fig. 2, all of the remaining independent zeros must be located on the unit circle in the z plane to minimize the magnitude error in the stopband. In the z plane, the zeros on the unit circle other than $z = \pm 1$ must be complex conjugate pairs in the case of real filters. Then $N - M + 1$ is required to be even, that is, $N - M + 1 = 2I$. Here, we first select $2I$ independent zeros as an initial guess in the stopband as follows;

$$z_i = e^{\pm j\bar{\omega}_i} \quad (\omega_s < \bar{\omega}_1 < \bar{\omega}_2 < \cdots < \bar{\omega}_I < \pi) \tag{15}$$

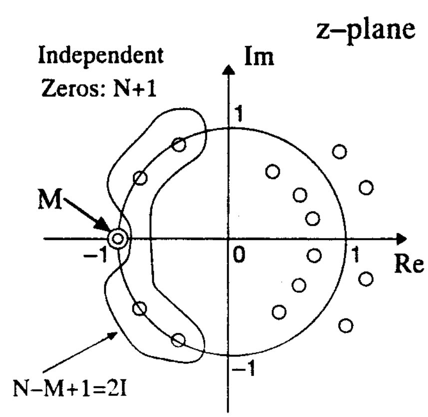


Fig. 2. Zero location of FIR half-band filters.

Therefore, we have from Eq. (6)

$$\hat{H}(z_i) = \frac{1}{2} + \sum_{n=0}^{N} a_n e^{\pm j(K-2n)\bar{\omega}_i} = 0 \tag{16}$$

By dividing Eq. (16) into real and image parts, we get

$$\begin{cases} \sum_{n=0}^{N} a_n \cos(K-2n)\bar{\omega}_i = -0.5 \\ \sum_{n=0}^{N} a_n \sin(K-2n)\bar{\omega}_i = 0 \end{cases} \tag{17}$$

Summarizing Eqs. (14) and (17), there are a total of $N + 1$ equations. Therefore, a set of filter coefficients can be obtained by solving the linear equations in (14) and (17).

3.3. Formulation using complex Remez exchange algorithm

In Section 3.2, we equally select a set of initial zeros in the stopband to obtain a set of filter coefficients. As a result, the obtained magnitude response may not be equiripple in the stopband. Here, we use the filter coefficients obtained in Section 3.2 as an initial value, and then formulate the design problem in such a way that the stopband response becomes equiripple by using the complex Remez exchange algorithm. We calculate the magnitude response of $\hat{H}(z)$ from the initial filter coefficients, then search for the $I + 1$ extremal frequencies ω_i in the stopband as follows and compute the corresponding phase $\theta(\omega_i)$:

$$\omega_s = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_I < \pi \tag{18}$$

Next, we apply the complex Remez exchange algorithm at these extremal frequencies in the stopband, and formulate $\hat{H}(e^{j\omega})$ as

$$\hat{H}(e^{j\omega_i}) = \delta\, e^{j\{\theta(\omega_i)+\Delta\theta\}} \tag{19}$$

where δ is the magnitude error and $\Delta\theta$ is the phase error. Substituting Eq. (6) into Eq. (19), we get

$$\begin{aligned} \hat{H}(e^{j\omega_i}) &= \frac{1}{2} + \sum_{n=0}^{N} a_n e^{j(K-2n)\omega_i} \\ &= \delta\, e^{j\{\theta(\omega_i)+\Delta\theta\}} \end{aligned} \tag{20}$$

By dividing Eq. (20) into real and image parts,

$$\begin{cases} \sum_{n=0}^{N} a_n \cos(K-2n)\omega_i - \delta_1 \cos\theta(\omega_i) \\ \qquad\qquad + \delta_2 \sin\theta(\omega_i) = -0.5 \\ \sum_{n=0}^{N} a_n \sin(K-2n)\omega_i - \delta_1 \sin\theta(\omega_i) \\ \qquad\qquad - \delta_2 \cos\theta(\omega_i) = 0 \end{cases} \tag{21}$$

where $\delta_1 = \delta \cos \Delta\theta$ and $\delta_2 = \delta \sin \Delta\theta$. There are a total of $N + 3$ equations in (14) and (21). Therefore, a set of filter coefficients can be obtained by solving the linear equations in (14) and (21). We calculate the magnitude response of $\hat{H}(z)$ by using the obtained filter coefficients, then search for the peak frequencies Ω_i in the stopband and compute the corresponding phase $\theta(\Omega_i)$. As a result, the obtained peak frequencies Ω_i may not be consistent with the extremal frequencies ω_i. We then use the obtained peak frequencies as the extremal frequencies in the next iteration and solve the linear equations in (14) and (21) to obtain a set of filter coefficients again. The algorithm is iterated until the equiripple stopband response is attained. In this paper, the proposed algorithm converges with a few iterations, since the filter coefficients obtained in Section 3.2 are used as an initial value.

3.4. Design algorithm

1. Read half-band filter specifications N, M, K, and the passband and stopband cutoff frequencies ω_p and ω_s.
2. Select the initial zeros $\bar{\omega}_i$ equally spaced in the stopband as shown in Eq. (15).
3. Solve the linear equations in (14) and (17) to obtain a set of initial filter coefficients a_n.
4. Compute the magnitude response of $\hat{H}(z)$ by using the obtained initial coefficients a_n, then search for the peak frequencies Ω_i in the stopband and compute the corresponding phase $\theta(\Omega_i)$.
5. Set $\omega_i = \Omega_i (i = 0, 1, \ldots I)$.
6. Solve the linear equations in (14) and (21) to obtain a set of coefficients a_n.
7. Compute the magnitude response of $\hat{H}(z)$ by using the obtained filter coefficients a_n, then search for the peak frequencies Ω_i in the stopband and compute the corresponding phase $\theta(\Omega_i)$.
8. If $|\Omega_i - \omega_i| < \varepsilon (i = 0, 1, \ldots, I)$ is satisfied, then exit. Else, go to step 5, where ε is a prescribed small constant.

4. Application to Filter Banks

Recently, filter banks and wavelets have been exhaustively studied and applied in signal processing and so on. Here, we describe the design of two-channel filter banks as an application of FIR half-band filters. A class of biorthogonal filter banks with structurally perfect reconstruction have been proposed in Ref. 8. In this class of two-channel filter banks, the perfect reconstruction condition is structurally satisfied, that is, reversible, regardless of the quantization of filter coefficients and roundoff noise by the multiplier. In Ref. 8, since FIR linear-phase filters are used, the overall delay of the filter banks becomes large when high-order filters are required. In real-time signal processing applications, it is essential to have a lower delay. Therefore, the design of low-delay filter banks has been attempted in Refs. 7 and 9. In this paper, we apply the design method of low-delay FIR half-band filters described in Section 3 to the structurally perfect reconstruction filter banks proposed in Ref. 8, and design two-channel perfect reconstruction filter banks with low delay. Assume that $H_1(z)$, $H_2(z)$ are analysis filters, and $F_1(z)$, $F_2(z)$ synthesis filters, respectively, in two-channel filter banks. The perfect reconstruction condition of filter banks is

$$\begin{cases} H_1(z)F_1(z) + H_2(z)F_2(z) = z^{-2L-1} \\ H_1(-z)F_1(z) + H_2(-z)F_2(z) = 0 \end{cases} \tag{22}$$

where L is an integer. In Ref. 8, the analysis and synthesis filters are constructed as

$$\begin{cases} H_1(z) = \frac{1}{2}\{z^{-2K_1-1} + A(z^2)\} = -F_2(-z) \\ H_2(z) = z^{-2K_2} - B(z^2)H_1(z) = F_1(-z) \end{cases} \tag{23}$$

where K_1, K_2 are integers, and $L = K_1 + K_2$. Hence, the perfect reconstruction condition of Eq. (22) is satisfied. Let $A(z)$ and $B(z)$ be FIR filters of order N_1 and N_2, respectively:

$$\begin{cases} A(z) = \sum_{n=0}^{N_1} a_n z^{-n} \\ B(z) = \sum_{n=0}^{N_2} b_n z^{-n} \end{cases} \tag{24}$$

where a_n, b_n are real filter coefficients. By comparing the transfer function in Eq. (4) with $H_1(z)$ in Eq. (23), it is clear that $H_1(z)$ is a half-band filter. Therefore, it is possible to design $H_1(z)$ with low group delay by using the design method proposed in Section 3. In the stopband $[\omega_s, \pi]$ of $H_1(z)$, since $H_1(e^{j\omega}) = 0$, then $H_2(e^{j\omega}) = e^{-j2K_2\omega}$, and it is a passband. In the passband $[0, \omega_p]$ of $H_1(z)$, $H_1(e^{j\omega}) = e^{-j(2K_1+1)\omega}$ ideally, then

$$\begin{aligned} H_2(z) &= z^{-2K_2} - B(z^2)z^{-2K_1-1} \\ &= -2z^{-2K_1-1}\tilde{H}_2(-z) \end{aligned} \tag{25}$$

where

$$\tilde{H}_2(z) = \frac{1}{2}\{z^{-2(K_2-K_1)+1} + B(z^2)\} \tag{26}$$

Since the band $[0, \omega_p]$ is the stopband of $H_2(z)$, $\tilde{H}_2(z)$ must be 0 in the band $[\omega_s, \pi]$. Note that $\omega_p + \omega_s = \pi$. $\tilde{H}_2(z)$ in Eq. (26) is clearly a half-band filter, and then can be designed by using the method proposed in Section 3. However, $H_2(z)$ may not be equiripple in the stopband since it is influenced by $H_1(z)$, although $\tilde{H}_2(z)$ is designed to be equiripple in the stopband [12]. In the practical design, the influence of error of $H_1(z)$ must be considered. Here, we consider $\hat{H}_2(z)$ as

$$\begin{aligned} \hat{H}_2(z) &= \frac{1}{2}z^{2K_2}H_2(-z) \\ &= \frac{1}{2}\{1 + z^{2(K_2-K_1)-1}B(z^2)\hat{H}_1(-z)\} \end{aligned} \tag{27}$$

where

$$\begin{aligned} \hat{H}_1(z) &= z^{2K_1+1}H_1(z) \\ &= \frac{1}{2}\{1 + z^{2K_1+1}A(z^2)\} \end{aligned} \tag{28}$$

When using the complex Remez exchange algorithm to design $H_2(z)$, the equiripple response in the stopband can be obtained by formulating $\hat{H}_2(z)$ in Eq. (27) [12]. The design algorithm is the same as in Section 3.4.

5. Design Examples

[Example 1]

The filter specifications are $N = 19$, $M = 10$, $\omega_p = 0.4\pi$, and $\omega_s = 0.6\pi$, and the filter order is $2N = 38$. We have designed FIR half-band filters with various K by using the design method described in Section 3. The range of designable K is $1 \le K \le 37$. The impulse response of $K = 15$ is shown in Fig. 3(a). For comparison, the impulse response of the exact linear phase filter with $K = 19$ is shown in Fig. 3(b) also. It can be seen that the impulse response of low (high) delay filters is not symmetric, while the impulse response of the exact linear phase filters is symmetric. There are $(N-1)/2$ zeros inside and outside the unit circle, respectively, for the exact linear phase filters. However, low (high) delay filters have $N - (K+1)/2$ zeros inside the unit circle, and $(K-1)/2$ zeros outside the unit circle. For filters with $K = N \pm 2D (D = 1, 2, \ldots)$, their zeros satisfy the image-mirror relation with respect to the unit circle, and thus the magnitude responses are the same and the group delays are symmetric with respect to $K = N$. The obtained

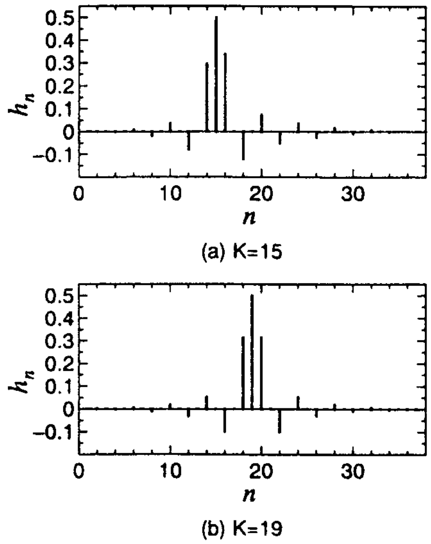


Fig. 3. Impulse responses of FIR half-band filters.

magnitude responses are shown in Fig. 4. It is clear that the equiripple response in the stopband is obtained while the given degrees of flatness are met. According to the symmetry of group delay, only the group delays with $K \le N$ are shown in Fig. 5, and ones with $K > N$ are omitted. It is seen that low (high) group delay can be realized. To examine the influence of group delay K on the frequency response, a chart of the stopband minimum attenuation versus K is shown in Fig. 6. The filters with $K = N \pm 2D$ have the same stopband attenuation, and thus only those to $K = N = 19$ are shown in Fig. 6; when $K = 19$, it is seen that the exact linear phase filters have the maximal stopband attenuation, and the stopband attenuation becomes smaller as K decreases.

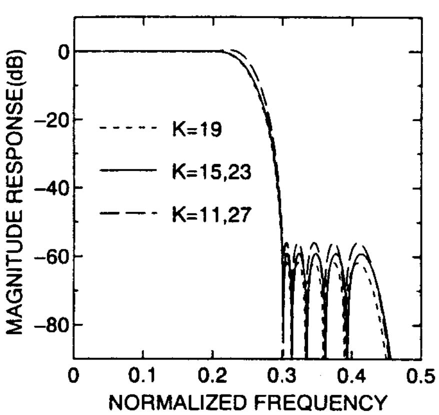


Fig. 4. Magnitude responses of FIR half-band filters.

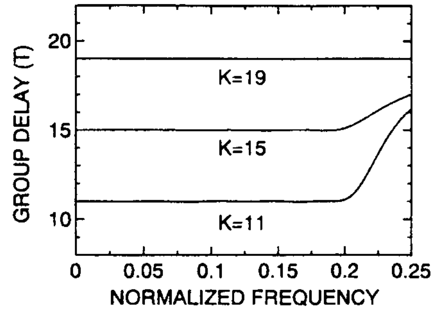


Fig. 5. Group delays of FIR half-band filters.

Here, we designed an exact linear phase filter with filter order $2N = 30$ and group delay $K = 15$, then compared it with the low group delay filter. Their magnitude responses are shown in Fig. 7. Compared with the low-delay filter, the stopband attenuation of the exact linear phase filter with lower filter order becomes smaller. To obtain the same stopband attenuation as the low-delay filter, an exact linear phase filter with at least $2N = 38$ is needed. However, since the filter coefficients are symmetric in the exact linear phase filters, the number of multipliers required in implementation is about half the filter order. In this design example, we needed about five or six iterations to obtain the equiripple response.

[Example 2]

The filter specifications are $N = 18$, $K = 15$, $\omega_p = 0.4\pi$, and $\omega_s = 0.6\pi$, and the filter order is $2N = 36$. The exact linear phase half-band filter with the same order does not exist. We have designed low-delay FIR half-band filters by varying the degree of flatness M. The obtained magnitude responses and group delays are shown in Figs. 8 and 9, respectively. $M = 19$ corresponds to the maximally flat filter, while $M = 1$ is the minimax solution with equirip-

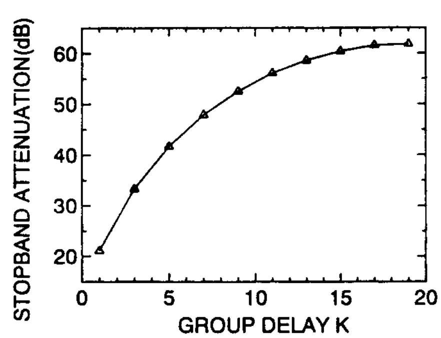


Fig. 6. Stopband minimum attenuation versus group delay K.

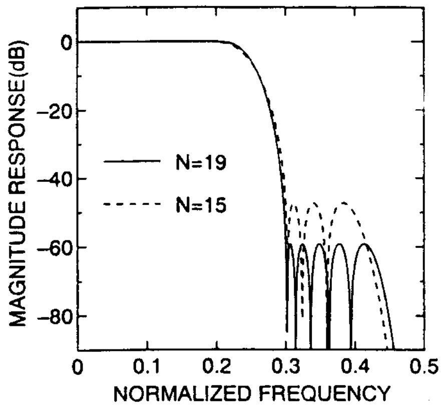


Fig. 7. Magnitude responses of FIR half-band filters.

ple response. It is seen in Fig. 8 that flatness can be arbitrarily specified. A plot of the stopband minimum attenuation versus M is shown in Fig. 10. As M increases, the stopband attenuation becomes smaller. In this design example, we needed about five or six iterations.

[Example 3]

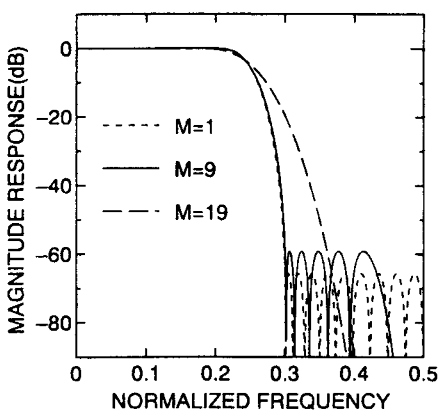


Fig. 8. Magnitude responses of FIR half-band filters.

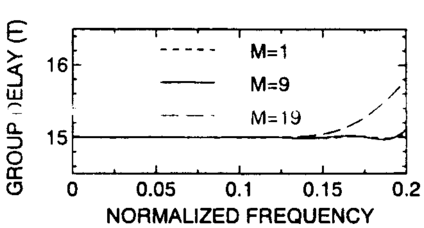


Fig. 9. Group delays of FIR half-band filters.

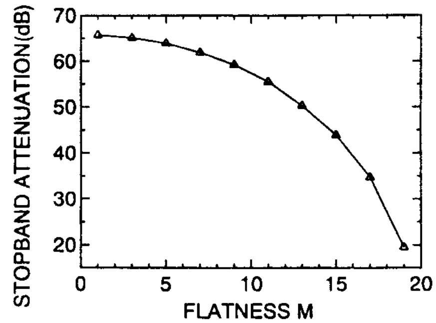


Fig. 10. Stopband minimum attenuation versus flatness M.

The specifications of filter banks are $K_1 = 6$, $K_2 = 13$, $\omega_p = 0.4\pi$, and $\omega_s = 0.6\pi$. The order of $A(z)$, $B(z)$ are $N_1 = 15$, $N_2 = 17$. We have designed the filter bank with a degree of flatness $M_1 = M_2 = 12$. The obtained magnitude

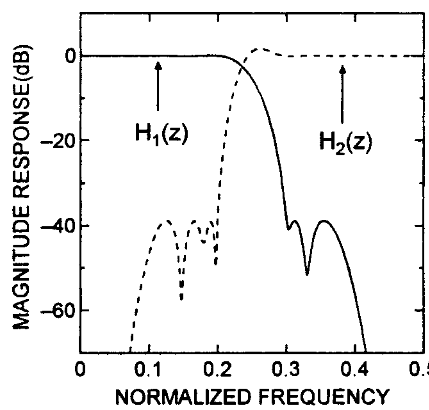


Fig. 11. Magnitude responses of analysis filters.

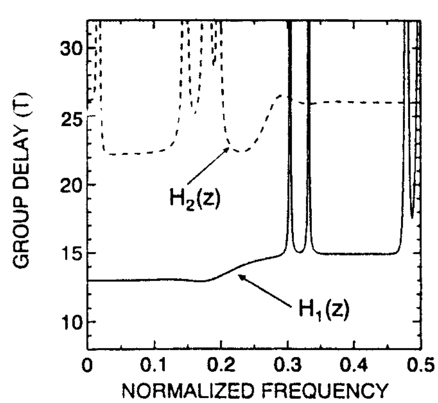


Fig. 12. Group delays of analysis filters.

responses and group delays are shown in Figs. 11 and 12, respectively. The overall delay of the filter bank is 39 samples, that is, $L = 19$. However, when the exact linear phase filters with the same order are used, the overall delay is 47 samples ($L = 23$).

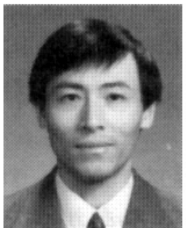
6. Conclusions

In this paper, we have proposed a new method for designing low-delay FIR half-band filters with arbitrary flatness. First, we have investigated the frequency response property and constraint from the time-domain condition of FIR half-band filters, and have shown that the design problem of FIR half-band filters can be reduced to the minimization of the magnitude response in the stopband. Next, the design problem of FIR half-band filters is formulated as a linear problem by using the complex Remez exchange algorithm in the stopband, while taking the specified flatness condition into account. Therefore, a set of filter coefficients can be easily obtained by solving a simple system of linear equations. The optimal solution with an equiripple response in the stopband is obtained by applying an iteration process. The advantages of the proposed method are that a lower group delay can be realized and flatness can be arbitrarily specified for FIR half-band filters. Finally, we have applied the proposed method to the design of two-channel perfect reconstruction filter banks with low delay to demonstrate its effectiveness.

REFERENCES

1. Takebe T. Design of digital filters. Tohkaidaigaku Shupansha; 1986.
2. Mitra SK, Kaiser JF. Handbook for digital signal processing. John Wiley & Sons; 1993.
3. Vaidyanathan PP. Multirate systems and filter banks. Prentice–Hall; 1993.
4. Mintzer F. On half-band, third-band, and Nth-band FIR filters and their design. IEEE Trans Acoust Speech Signal Process 1982;ASSP-30:734–738.
5. Vaidyanathan PP, Nguyen TQ. A “trick” for the design of FIR half-band filters. IEEE Trans Circuits Syst 1987;CAS-34:297–300.
6. Vetterli M, Herley C. Wavelets and filter banks: Theory and design. IEEE Trans Signal Process 1992;40:2207–2232.
7. Nayebi K, Barnwell TP, Smith MJT. Low delay FIR filter banks: Design and evaluation. IEEE Trans Signal Process 1994;42:24–31.
8. Phoong SM, Kim CW, Vaidyanathan PP, Ansari R. A new class of two channel biorthogonal filter banks and wavelet bases. IEEE Trans Signal Process 1995;43:649–665.
9. Raheem EA, Guibaly FE, Antoniou A. Design of low-delay two channel FIR filter banks using constrained optimization. Signal Process 1996;48:183–192.
10. Zhang X, Iwakura H. Design of FIR Nyquist filters using Remez exchange algorithm. Trans IEICE 1996;J79-A:1378–1384.
11. Wisutmethangoon Y, Nguyen TQ. Nonlinear phase *M*th band filter and applications in filter band design. Asilomar Conference on SSC, p 696–700, 1997.
12. Zhang X, Yoshikawa T. Design of biorthogonal FIR linear phase filter banks with structurally perfect reconstruction. Trans IEICE 1998;J81-A:17–23.
13. Zhang X, Yoshikawa T. Design of FIR Nyquist filters with low group delay. IEEE Trans Signal Process 1999;47:1454–1458.

AUTHORS (from left to right)



Xi Zhang (member) received his B.E. degree in electronics engineering from Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China, in 1984, and his M.E. and Ph.D. degrees in communication engineering from the University of Electro-Communications (UEC) in 1990 and 1993. He was with the Department of Electronics Engineering at NUAA from 1984 to 1987, and the Department of Communications and Systems at UEC from 1993 to 1996, all as a research assistant. Since 1996, he has been with the Department of Electrical Engineering, Nagaoka University of Technology, as an associate professor. He was a recipient of the 1987 Science and Technology Progress Award in China. His research interests are in the areas of digital signal processing, filter design theory, filter banks and wavelets, and their applications to image coding. He is a member of IEEE.

Kentaro Inotsume (student member) received his B.E. degree in electrical engineering from Nagaoka University of Technology in 1997. He is currently pursuing his M.E. degree at Nagaoka University of Technology. His research interest is digital signal processing.

Toshinori Yoshikawa (member) received his B.E., M.E., and D.Eng. degrees from Tokyo Institute of Technology in 1971, 1973, and 1976. From 1976 to 1983, he was with Saitama University, engaging in research on signal processing and related software development. Since 1983, he has been with Nagaoka University of Technology, where he is currently a professor. His main research area is digital signal processing. He is a member of the IEEE Computer Society.