

Design of IIR Digital Filters with Flat Passband and Equiripple Stopband Responses

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SUMMARY

In the design of IIR digital filters, the method that utilizes the classic analog filter design theory to design analog filters and then obtain the corresponding digital filters by s - z transformation is well-known. However, IIR digital filters obtained via the bilinear s - z transformation have just equal-order numerator and denominator. Having unequal-order numerator and denominator will give more degrees of freedom in filter design. In this paper, we consider the design of IIR digital filters with unequal-order numerator and denominator, and propose a method for designing the flat passband and equiripple stopband filters in z -domain directly. First, we present a design method of IIR filters with flat stopband and equiripple passband responses. The flat stopband response can be easily obtained only by locating multiple zeros on the specified frequency points, while the equiripple passband response can be designed by using the Remez exchange algorithm and specifying the maximum magnitude error. Second, we can obtain IIR filters with flat passband and equiripple stopband responses via a magnitude transformation such that the passband and stopband become the corresponding stopband and passband, respectively. However, the numerator order of IIR filters obtained by the above method is equal to or higher than the denominator. Finally, we consider the design of IIR filters that have lower-order numerator than denominator, and present a method for designing the flat passband and equiripple stopband filters directly. © 2001 Scripta Technica, Electron Comm Jpn Pt 3, 84(11): 37–44, 2001

Key words: IIR digital filter; flat passband; equiripple stopband; Remez exchange algorithm.

1. Introduction

In the design problem of IIR digital filters for magnitude response, the method utilizing the classic analog filter design theory is well-known [1–4]. There are basically four types of filters—Butterworth (maximally flat), Chebyshev type I (equiripple passband and flat stopband) and II (flat passband and equiripple stopband), and elliptic (both passband and stopband are equiripple) filters—in the analog filters. Given the design specification, the desired type of analog filters is first designed, then the corresponding digital filters are obtained via s - z transformation. However, IIR digital filters obtained via the bilinear s - z transformation have just equal-order numerator and denominator. Consequently, IIR digital filters with unequal-order numerator and denominator cannot be designed from the analog filters. Hence, it is required to design IIR digital filters with unequal-order numerator and denominator in z -domain directly. Freely choosing the numerator and denominator order will increase one design parameter and give more degrees of freedom in filter design. Design of IIR filters having unequal-order numerator and denominator had been considered in Refs. 5 to 12. In Refs. 7 to 10, some methods for designing IIR filters with both equiripple passband and stopband had been proposed, and it was shown that compared with IIR filters with equal-order numerator and denominator, unequal-order filters have superior magnitude performance in narrow- and wide-band filters. Also, a design method for maximally

flat IIR filters was proposed in Ref. 11, and a closed-form solution was given. However, in the filter design where one passband or stopband is flat and another is equiripple, only FIR filters [5, 6] and IIR filters simultaneously considering its group delay [12] were given.

In this paper, we consider the design of IIR digital filters with unequal-order numerator and denominator, and propose a method for designing the flat passband and equiripple stopband filters in z -domain directly. First, we present a design method of IIR filters with flat stopband and equiripple passband responses. The flat stopband response can be easily obtained only by locating multiple zeros on the specified frequency points, while the equiripple passband response can be designed by using the Remez exchange algorithm and specifying the maximum magnitude error. Second, we can obtain IIR filters with flat passband and equiripple stopband responses via a magnitude transformation such that the passband and stopband become the corresponding stopband and passband, respectively. However, the numerator order of IIR filters obtained via the magnitude transformation is equal to or higher than the denominator. Therefore, an alternative method for designing the flat passband and equiripple stopband filters is needed in the design of IIR filters that have lower-order numerator than denominator. We give a transfer function that yields the flat response in passband, and present a method for designing the equiripple response in stopband by using the Remez exchange algorithm. Finally, some examples are designed to demonstrate the effectiveness of the proposed method.

2. Transfer Functions of IIR Digital Filters

The transfer function $H(z)$ of an IIR digital filter is defined as

$$H(z) = \frac{\sum_{n=0}^N a_n z^{-n}}{\sum_{m=0}^M b_m z^{-m}} \quad (1)$$

where N and M are order of numerator and denominator, respectively, filter coefficients a_n and b_m are real, and $b_0 = 1$ in general.

If the transfer function $H(z)$ and its inverse transfer function $H(z^{-1})$ are cascaded, then zero-phase filter $G(z)$ can be obtained by

$$G(z) = H(z)H(z^{-1}) = \frac{\sum_{n=-N}^N c_n z^{-n}}{\sum_{m=-M}^M d_m z^{-m}} \quad (2)$$

where filter coefficients c_n and d_m are real, and satisfy the following symmetrical conditions:

$$\begin{cases} c_n = c_{-n} & (1 \leq n \leq N) \\ d_m = d_{-m} & (1 \leq m \leq M) \end{cases} \quad (3)$$

Hence, the magnitude response of $G(z)$ is given by

$$G(e^{j\omega}) = |H(e^{j\omega})|^2 = \frac{c_0 + 2 \sum_{n=1}^N c_n \cos n\omega}{d_0 + 2 \sum_{m=1}^M d_m \cos m\omega} \quad (4)$$

It can be seen in Eq. (4) that since the magnitude response of $G(z)$ is equal to the squared magnitude of $H(z)$, the $G(e^{j\omega}) \geq 0$.

Also, we examine the relation of zeros and poles between $H(z)$ and $G(z)$. If a zero of $H(z)$ is located on the unit circle, then $G(z)$ has double zeros on the unit circle. If a zero of $H(z)$ is located inside or outside the unit circle, the zeros of $G(z)$ become the mirror-image pairs with respect to the unit circle. To obtain a stable filter, the poles of $H(z)$ must be located inside the unit circle, then $G(z)$ has the mirror-image poles with respect to the unit circle. Therefore, if $G(z)$ having the mirror-image zeros and poles with respect to the unit circle and the double zeros on the unit circle can be designed, then stable $H(z)$ can be obtained by factorizing these zeros and poles. The symmetrical conditions of Eq. (3) guarantee that $G(z)$ has the mirror-image poles and zeros, and the zeros on the unit circle. To force the zeros on the unit circle to be double zeros, $G(e^{j\omega}) \geq 0$ must be satisfied. In the following, we consider the design of $G(z)$ having double zeros on the unit circle.

3. Design of IIR Filters with Flat Stopband and Equiripple Passband Responses

In this section, we describe the design of IIR digital filters with flat stopband and equiripple passband responses. The transfer functions of zero-phase filters $F(z)$ are first defined as

$$F(z) = \frac{\sum_{n=-L_1}^{L_1} f_n z^{-n}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (5)$$

where filter coefficients f_n and g_m are real, and satisfy

$$\begin{cases} f_n = f_{-n} & (1 \leq n \leq L_1) \\ g_m = g_{-m} & (1 \leq m \leq L_2) \end{cases} \quad (6)$$

In the following, we present a method for designing $F(z)$.

3.1. Design specification

Now, we consider the design of low-pass filters. Its specification is that in stopband, the following flatness conditions at $\omega = \pi$ are required:

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=\pi} = 0 \quad (i = 0, 1, \dots, 2L_1 - 1) \quad (7)$$

where L_1 is a parameter denoting the degree of flatness, while in passband, the filter magnitude is required to meet the specified error range, that is,

$$1 - \delta \leq F(e^{j\omega}) \leq 1 \quad (0 \leq \omega \leq \omega_p) \quad (8)$$

where ω_p is a cutoff frequency of the passband, and δ is the specified maximum magnitude error and known.

3.2. Formulation using Remez exchange algorithm

To meet the flatness condition of Eq. (7), we must locate multiple zeros of order $2L_1$ at $z = -1$, that is,

$$F(z) = \frac{z^{L_1} (1 + z^{-1})^{2L_1}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (9)$$

Then, the magnitude response of $F(z)$ is

$$F(e^{j\omega}) = \frac{\left(2 \cos \frac{\omega}{2}\right)^{2L_1}}{g_0 + 2 \sum_{m=1}^{L_2} g_m \cos m\omega} \quad (10)$$

and satisfies the flatness condition of Eq. (7). In passband $[0, \omega_p]$, we first select $(L_2 + 1)$ sampling frequencies ω_i as follows:

$$\omega_p = \omega_0 > \omega_1 > \dots > \omega_{L_2} \geq 0 \quad (11)$$

and then use the Remez exchange algorithm to formulate $F(e^{j\omega_i})$ as

$$F(e^{j\omega_i}) = \begin{cases} 1 - \delta & (i : \text{even}) \\ 1 & (i : \text{odd}) \end{cases} \quad (12)$$

By substituting Eq. (10) into Eq. (12), we get

$$\begin{aligned} g_0 + 2 \sum_{m=1}^{L_2} g_m \cos m\omega_i &= \begin{cases} \frac{\left(2 \cos \frac{\omega_i}{2}\right)^{2L_1}}{1 - \delta} & (i : \text{even}) \\ \left(2 \cos \frac{\omega_i}{2}\right)^{2L_1} & (i : \text{odd}) \end{cases} \end{aligned} \quad (13)$$

Therefore, a set of filter coefficients g_m can be easily obtained by solving the linear equations of Eq. (13). We compute the magnitude response of $F(z)$ and search for the peak frequencies $\bar{\omega}_i$ in passband. Then, we set the obtained peak frequencies as the sampling frequencies in the next iteration and solve the linear equations again. The above procedure is iterated until the sampling frequencies ω_i and the peak frequencies $\bar{\omega}_i$ are consistent. When the peak frequencies do not change, we can obtain the optimal solution with an equiripple magnitude response in passband. The design algorithm is shown in detail as follows.

3.3. Design algorithm

1. Read filter specifications L_1, L_2, δ and the cutoff frequency ω_p .
2. Select $(L_2 + 1)$ initial sampling frequencies ω_i equally spaced in passband as shown in Eq. (11).
3. Solve the linear equations of Eq. (13) to obtain a set of filter coefficients g_m .
4. Compute the magnitude response of $F(z)$ by using the obtained filter coefficients g_m , and search for the peak frequencies $\bar{\omega}_i$ in passband.
5. If $\sum_{i=0}^{L_2} |\bar{\omega}_i - \omega_i| < \varepsilon$, then exit. Else, go to step 6, where ε is a prescribed small constant.
6. Set $\omega_i = \bar{\omega}_i$ ($i = 0, 1, \dots, L_2$), then go to step 3.

3.4. Design of bandpass and bandstop filters

In Section 3.2, we described the design of low-pass filters. High-pass filters can be designed the same as low-pass filters. Also, the design can be easily obtained from the transfer function of the obtained low-pass filters via a frequency transformation replacing z with $-z$. Here, we describe the design of bandpass and bandstop filters.

In the case of bandpass filters, there are two stopbands and the flatness conditions are

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=0} = 0 \quad (i = 0, 1, \dots, 2K - 1) \quad (14)$$

and

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=\pi} = 0 \quad (i = 0, 1, \dots, 2(L_1 - K) - 1) \quad (15)$$

Therefore, to meet the flatness conditions of Eqs. (14) and (15), we have

$$F(z) = \frac{z^{L_1} (1 - z^{-1})^{2K} (1 + z^{-1})^{2(L_1 - K)}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (16)$$

In passband, the filter magnitude is required to meet

$$1 - \delta \leq F(e^{j\omega}) \leq 1 \quad (\omega_{p1} \leq \omega \leq \omega_{p2}) \quad (17)$$

where ω_{p1} , ω_{p2} are the cutoff frequency of the passband. Similarly, the equiripple passband can be designed by using the Remez exchange algorithm. Note that since the number of peak frequencies must be odd in passband, L_2 is required to be even.

In the case of bandstop filters, the flatness conditions in stopband are

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=\omega_{sf}} = 0 \quad (i = 0, 1, \dots, L_1 - 1) \quad (18)$$

where ω_{sf} is the specified frequency point. Therefore, $F(z)$ becomes

$$F(z) = \frac{z^{L_1} (e^{j\omega_{sf}} - z^{-1})^{L_1} (e^{-j\omega_{sf}} - z^{-1})^{L_1}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (19)$$

where L_1 is required to be even. In two passbands, the filter magnitude must satisfy

$$1 - \delta \leq F(e^{j\omega}) \leq 1 \quad \begin{pmatrix} 0 \leq \omega \leq \omega_{p1} \\ \omega_{p2} \leq \omega \leq \pi \end{pmatrix} \quad (20)$$

and can be designed by using the Remez exchange algorithm. The design algorithm is the same as in Section 3.3.

4. Design of IIR Filters with Flat Passband and Equiripple Stopband Responses

In this section, we describe the design of IIR digital filters $G(z)$ with flat passband and equiripple stopband responses. First, we consider the design of low-pass filters. In passband, the flatness conditions at $\omega = 0$ are

$$\begin{cases} G(1) = 1 \\ \left. \frac{\partial^i G(e^{j\omega})}{\partial \omega^i} \right|_{\omega=0} = 0 \end{cases} \quad (i = 1, 2, \dots, 2L_1 - 1) \quad (21)$$

In stopband, the filter magnitude must satisfy

$$0 \leq G(e^{j\omega}) \leq \delta \quad (\omega_s \leq \omega \leq \pi) \quad (22)$$

where ω_s is a cutoff frequency of the stopband.

4.1. Design of filters with higher-order numerator

First, we transform the stopband and passband of $G(z)$ into the passband and stopband of $F(z)$, respectively, as

$$F(e^{j\omega}) = 1 - G(e^{j\omega}) \quad (23)$$

Then the flatness conditions of Eq. (21) become

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=0} = 0 \quad (i = 0, 1, \dots, 2L_1 - 1) \quad (24)$$

In stopband of $G(z)$, we have

$$1 - \delta \leq F(e^{j\omega}) \leq 1 \quad (\omega_s \leq \omega \leq \pi) \quad (25)$$

Therefore, by using the method proposed in Section 3, the high-pass filter $F(z)$ with flat stopband and equiripple passband can be designed to satisfy the conditions of Eqs. (24) and (25). From Eq. (23), we get

$$G(z) = 1 - F(z) = \frac{\sum_{m=-L_2}^{L_2} g_m z^{-m} - \sum_{n=-L_1}^{L_1} f_n z^{-n}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (26)$$

where the denominator order of $G(z)$ is $M = L_2$, and the numerator order is decided by the larger one between L_1 and L_2 , that is, $N = \text{Max}\{L_1, L_2\}$. Hence, the numerator order is equal to or larger than the denominator regardless of L_1 , that is, $N \geq M$. Consequently, the filters with lower-order numerator cannot be designed by the above method. In the design of IIR filters with higher-order denominator, an alternative method is needed for designing the flat passband and equiripple stopband filters. In the following, we consider the design of IIR filters with higher-order denominator. Also, the design of high-pass, bandpass, and bandstop filters $G(z)$ can be similarly obtained by designing the low-pass, bandstop, and bandpass filters $F(z)$.

4.2. Design of filters with higher-order denominator

Now, we describe the design of IIR filters with higher-order denominator. Similarly, we consider the case of low-pass filters.

To meet the flatness conditions of Eq. (21), we rewrite $G(z)$ as follows [1, 2]:

$$F(e^{j\omega}) = \frac{1}{G(e^{j\omega})} - 1 \quad (27)$$

Hence, the flatness conditions of Eq. (21) are equivalent to

$$\left. \frac{\partial^i F(e^{j\omega})}{\partial \omega^i} \right|_{\omega=0} = 0 \quad (i = 0, 1, \dots, 2L_1 - 1) \quad (28)$$

To meet the conditions of Eq. (28), it is required to locate multiple zeros of order $2L_1$ at $z = 1$, and $F(z)$ must be

$$F(z) = \frac{(1-z)^{L_1}(1-z^{-1})^{L_1}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}} \quad (29)$$

Therefore, from Eq. (27), $G(z)$ becomes

$$G(z) = \frac{1}{1 + \frac{(1-z)^{L_1}(1-z^{-1})^{L_1}}{\sum_{m=-L_2}^{L_2} g_m z^{-m}}} \quad (30)$$

and it is clear that the flatness conditions of Eq. (21) are satisfied. In the stopband $[\omega_s, \pi]$, we select $(L_2 + 1)$ sampling frequencies ω_i as

$$\omega_s = \omega_0 < \omega_1 < \dots < \omega_{L_2} \leq \pi \quad (31)$$

then use the Remez exchange algorithm to formulate $G(e^{j\omega_i})$ as

$$G(e^{j\omega_i}) = \begin{cases} \delta & (i : \text{even}) \\ 0 & (i : \text{odd}) \end{cases} \quad (32)$$

By substituting Eq. (30) into Eq. (32), we get

$$\begin{aligned} g_0 + 2 \sum_{m=1}^{L_2} g_m \cos m\omega_i \\ = \begin{cases} \left(2 \sin \frac{\omega_i}{2}\right)^{2L_1} \frac{\delta}{1-\delta} & (i : \text{even}) \\ 0 & (i : \text{odd}) \end{cases} \end{aligned} \quad (33)$$

Therefore, a set of filter coefficients g_m can be obtained by solving the linear equations of Eq. (33). The design algorithm is the same as in Section 3.3. Also, the bandstop and bandpass filters can be similarly designed by using $F(z)$ of Eqs. (16) and (19) instead of Eq. (29). It is omitted here. From Eq. (27), we have

$$G(z) = \frac{1}{1+F(z)} = \frac{\sum_{m=-L_2}^{L_2} g_m z^{-m}}{\sum_{m=-L_2}^{L_2} g_m z^{-m} + \sum_{n=-L_1}^{L_1} f_n z^{-n}} \quad (34)$$

where the numerator order of $G(z)$ is $N = L_2$, and the denominator order is decided by the larger one between L_1 and L_2 , that is, $M = \text{Max}\{L_1, L_2\}$. Therefore, it can be seen that the denominator order is not lower than the numerator, that is, $M \geq N$.

5. Design Examples

[Example 1] {High-pass filters}

The specifications of high-pass filters with flat passband and equiripple stopband are $N = L_1 = 8$, $M = L_2 = 6$, and $\omega_s = 0.3\pi$. First, we give the maximum magnitude errors $\delta = 10^{-4}$, $\delta = 10^{-5}$, and $\delta = 10^{-6}$, respectively, and have designed the low-pass filters with flat stopband and equiripple passband by using the proposed method. The

obtained magnitude responses are shown in Fig. 1. We then obtained the high-pass filters with flat passband and equiripple stopband from the low-pass filters via the magnitude transformation, and show their magnitude responses in Fig. 2. It can be seen in Fig. 2 that these filters have the minimum stopband attenuations of 40, 50, and 60 dB, respectively. Given $\delta = 10^{-4}$, we have also varied the degree of flatness as $N = L_1 = 6$ and $N = L_1 = 10$, and designed the high-pass filters. The obtained magnitude responses are shown in Fig. 3, where it can be seen that the magnitude responses become more flat as L_1 increases. Note that the filter with $N = M = 6$ is the same as that obtained from the analog filters, and is shown for comparison.

[Example 2] {Bandstop filters}

The filter specification is $K = 4$, $L_1 = 10$, $L_2 = 8$, $\omega_{s1} = 0.3\pi$, $\omega_{s2} = 0.5\pi$, and $\delta = 10^{-4}$. First, we designed the bandpass filter with flat stopband and equiripple passband, and obtained the bandstop filter with flat passband and equiripple stopband via the magnitude transformation. The obtained filter has numerator of order $N = 10$ and denominator of order $M = 8$. Its magnitude response is shown in Fig. 4 by the solid line. Second, we directly designed the bandstop filter with flat passband and equiripple stopband by using the method proposed in Section 4.2. The obtained filter has numerator of order $N = 8$ and denominator of order $M = 10$. Its magnitude response is shown in Fig. 4 by the dotted line, and is the same. Also, we varied the maximum magnitude error as $\delta = 10^{-5}$ in stopband and designed the bandstop filter by using the method proposed in Section 4.2. The obtained magnitude response is shown in Fig. 4 by the dashed line.

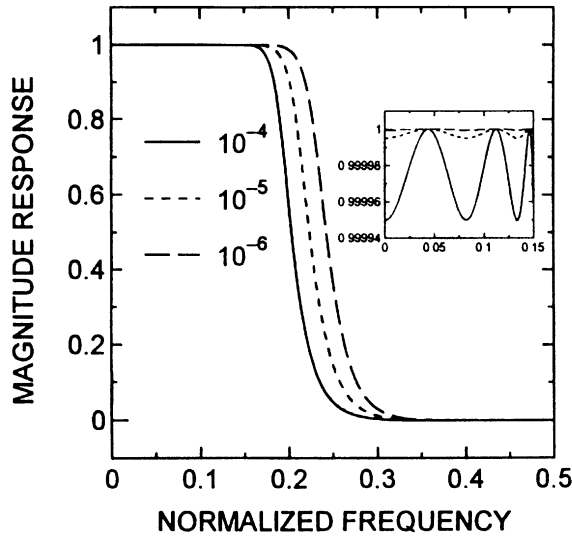


Fig. 1. Magnitude responses of low-pass filters.

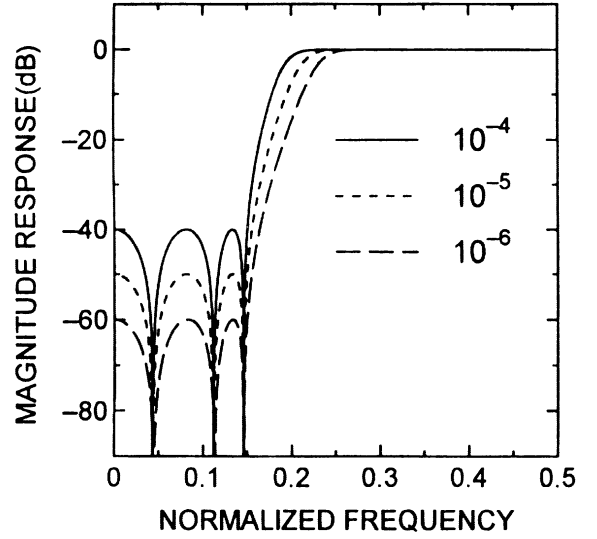


Fig. 2. Magnitude responses of high-pass filters.

[Example 3] {Bandpass filters}

The filter specification is that the degree of flatness at $\omega_{pf} = 0.5\pi$ is $L_1 = 10$, the cutoff frequencies are $\omega_{s1} = 0.3\pi$, $\omega_{s2} = 0.65\pi$, respectively, $L_2 = 6$, and $\delta = 10^{-4}$. We designed the bandpass filter with flat passband and equiripple stopband. The obtained filter has numerator of order $N = 6$ and denominator of order $M = 10$. Its magnitude response is shown in Fig. 5 by the solid line. We varied the passband and stopband as $\omega_{pf} = 0.6\pi$, $\omega_{s1} = 0.4\pi$,

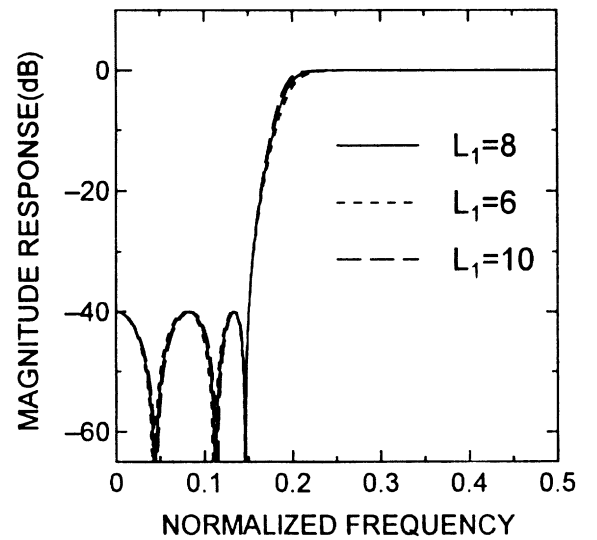


Fig. 3. Magnitude responses of high-pass filters.

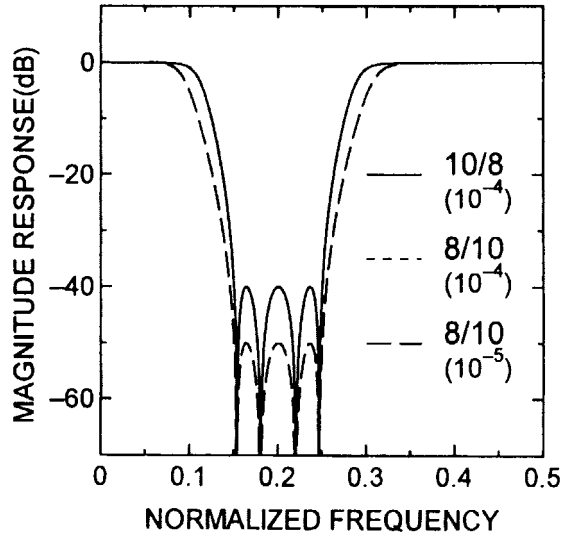


Fig. 4. Magnitude responses of bandstop filters.

$\omega_{s2} = 0.75\pi$, and designed the filter. Its magnitude response is shown in Fig. 5 by the dotted line. Also, we varied the maximum magnitude errors in the first and second stopbands as $\delta_1 = 10^{-4}$, $\delta_2 = 10^{-5}$ and $\delta_1 = 10^{-5}$, $\delta_2 = 10^{-4}$, respectively, and designed the bandpass filters. The obtained magnitude responses are shown in Fig. 6. It is clear that the magnitude error can be arbitrarily specified.

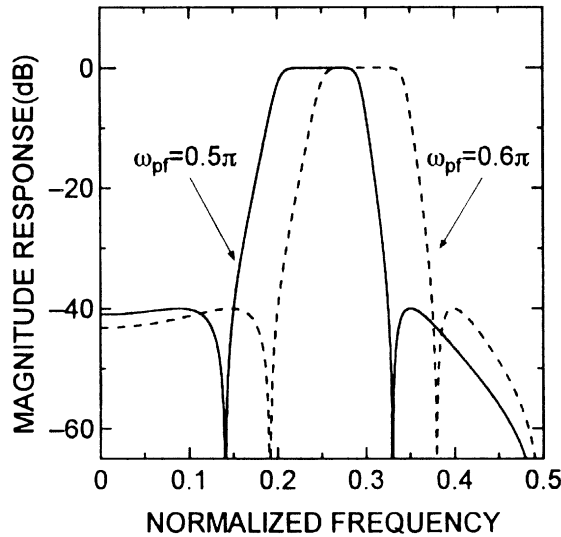


Fig. 5. Magnitude responses of bandpass filters.

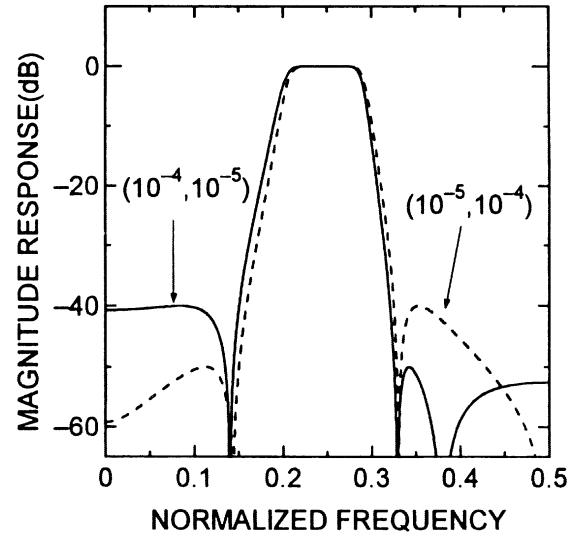


Fig. 6. Magnitude responses of bandpass filters.

6. Conclusions

In this paper, we have considered the design of IIR digital filters with unequal-order numerator and denominator, and proposed a method for designing the flat passband and equiripple stopband filters in z -domain directly. First, we have presented a design method of IIR filters with flat stopband and equiripple passband responses. The flat stopband response can be easily obtained only by locating multiple zeros on the specified frequency points, while the equiripple passband response can be designed by using the Remez exchange algorithm and specifying the maximum magnitude error. Second, we obtained IIR filters with flat passband and equiripple stopband responses via a magnitude transformation such that the passband and stopband become the corresponding stopband and passband, respectively. However, the numerator order of IIR filters obtained via the magnitude transformation is not lower than the denominator. Therefore, we have also presented an alternative method for designing IIR filters with lower-order numerator. Since the efficient Remez exchange algorithm is used in the proposed method, the filter coefficients can be easily obtained only by solving the linear equations. The feature of this method is that the degree of flatness in passband and the maximum magnitude error in stopband can be arbitrarily specified.

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