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Design of IIR Half-Band Filters with Arbitrary Flatness and Its Application to Filter Banks

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SUMMARY

Half-band filters are important for applications to multirate signal processing and wavelets. Previously, FIR half-band filters have mainly been treated. However, it should be recognized that the IIR filter can produce frequency characteristics similar to those of the FIR filter with a low order. Also, in the applications of filter banks and wavelets, design of a half-band filter is needed in which the degree of flatness can be specified arbitrarily. In the present paper, a new design method is proposed for an IIR half-band filter with an arbitrary degree of flatness. In the present design method, the analytical solution of the filter coefficients is given in the case of a maximally flat filter. Also, in the case of a specified degree of flatness, the amplitude error in the stopband can be specified in the design. Further, the stability of the filter is studied and then the minimum group delay for causal stability is clarified. Finally, the IIR half-band filter is applied to the design of a two-channel filter bank and the effectiveness of the present design method is proven. © 2003 Wiley Periodicals, Inc. Electron Comm Jpn Pt 3, 87(1): 10–18, 2004; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ecjc.10114

Key words: half-band filter; IIR filter; filter bank; flatness characteristic.

1. Introduction

Half-band filters are important in many applications such as multirate signal processing and wavelets [1–3]. Many design methods have been proposed to date in regard to the design of half-band filters [4–13]. In many of these design methods, the FIR half-band filters have mainly been dealt with because of such advantages as easy realization of always stable and perfect linear phase characteristic [4, 5, 10–13]. On the other hand, in comparison with the FIR filter, the IIR filter can realize the frequency characteristics similar to those of the FIR filters with a low order. There are fewer design methods for the IIR half-band filter than the FIR filter [7–9]. Also, in the applications such as the filter bank and wavelet, a condition is required from the regularity condition of the wavelet such that the amplitude characteristic of the half-band filter is flat [3, 6]. Hence, a design is needed for a half-band filter for which the degree of flatness of the amplitude characteristic can be specified arbitrarily.

In this paper, a new design method is proposed for an IIR half-band filter with an arbitrary degree of flatness. First, the relationship of the frequency characteristics in the passband and in the stopband of the IIR half-band filter is studied. It is shown that the filter design problem can be reduced to that of minimization of the maximum amplitude error. Next, from the specified flatness condition, the conditions to be satisfied by the filter coefficients are derived. Hence, in the case of the maximally flat filter, the filter coefficients can be obtained analytically and a closed form solution is given. Also, in the case of an arbitrarily specified degree of flatness, the complex Remez exchange algorithm is applied to the stopband and the maximum amplitude error in the stopband is specified so that the design problem of the filter is formulated in a form of linear equation. Hence, by solving a simple linear equation, the filter coefficients can be derived easily. After several iterations, an equiripple solution satisfying the specified maximum amplitude error can be obtained. In the present design method, the degree of flatness of the filter and the maximum amplitude error in the stopband can be given arbitrarily. Also, stability of the IIR half-band filter is discussed and the minimum group delay for causal stability is clarified. Finally, the present design method is applied to a design of the two-channel perfect reconstruction filter band so that the effectiveness of the method is presented.

2. IIR Half-Band Filter

In the time domain, the impulse response $h(n)$ of a half-band filter must satisfy the following constraint [4, 11]:

$$\begin{cases} h(K) = 0.5 \\ h(K+2k) = 0 \qquad (k = \pm 1, \pm 2, \cdots) \end{cases} \tag{1}$$

where K is the desired group delay of the filter and is an odd number. Also, in the frequency domain, the desired frequency characteristic for a low-pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-jK\omega} & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases} \tag{2}$$

where ω_p and ω_s are the edge frequencies of the passband and the stopband while $\omega_p + \omega_s = \pi$.

The transfer function $H(z)$ of an IIR half-band filter satisfying the time domain condition in (1) is

$$H(z) = \frac{z^{-K}}{2} + \frac{\sum_{n=0}^{N} a_n z^{-2n}}{\sum_{m=0}^{M} b_m z^{-2m}} \tag{3}$$

where N and M are integers while the filter coefficients a_n and b_n are real numbers with $b_0 = 1$. From Eq. (3), a new transfer function $\hat{H}(z)$ can be constructed:

$$\hat{H}(z) = z^K H(z) = \frac{1}{2} + \frac{\sum_{n=0}^{N} a_n z^{K-2n}}{\sum_{m=0}^{M} b_m z^{-2m}} \tag{4}$$

Hence, the frequency characteristic of $\hat{H}(z)$ is

$$\hat{H}(e^{j\omega}) = \frac{1}{2} + \frac{\sum_{n=0}^{N} a_n e^{j(K-2n)\omega}}{\sum_{m=0}^{M} b_m e^{-j2m\omega}} \tag{5}$$

Also, from Eq. (2), the desired frequency characteristic of $\hat{H}(z)$ is

$$\hat{H}_d(e^{j\omega}) = \begin{cases} 1 & (0 \le \omega \le \omega_p) \\ 0 & (\omega_s \le \omega \le \pi) \end{cases} \tag{6}$$

From Eq. (5), the frequency characteristic of $\hat{H}(z)$ satisfies the following relationship:

$$\hat{H}(e^{j\omega_0}) + \hat{H}^*(e^{j(\pi-\omega_0)}) \equiv 1 \tag{7}$$

where x^* denotes the complex conjugate of x. Equation (7) implies that the sum of the frequency characteristics at the frequency points ω_0 and $\pi - \omega_0$ is constant and does not depend on the filter coefficients a_n and b_m. If ω_0 is a frequency point within the passband, then $\pi - \omega_0$ is found to be located in the stopband. Hence, from Eq. (7), it is sufficient to approximate the frequency characteristic of $\hat{H}(z)$ by that either in the passband or in the stopband. In this paper, the maximum amplitude error in the stopband is assumed δ_s. Hence, the maximum amplitude error δ_p and the maximum phase error $\Delta\theta_p$ in the passband are

$$\begin{cases} \delta_p \le \delta_s \\ \Delta\theta_p \le \sin^{-1}\delta_s \end{cases} \tag{8}$$

Therefore, it is found that the amplitude error and the phase error in the passband are governed by the maximum amplitude error δ_s in the stopband. Therefore, the problem of design of a half-band filter is reduced to the minimization problem of the amplitude error in the stopband. In what follows, an approximation of $\hat{H}(z)$ in the stopband is considered.

3. Design of IIR Half-Band Filter

3.1. Maximally flat half-band filter

In the applications of filter banks and wavelets, it is required that the amplitude characteristic of the half-band filter be flat based on the regularity condition of the wavelet [3, 6]. This flatness condition is given by

$$\left.\frac{\partial^r \hat{H}(e^{j\omega})}{\partial\omega^r}\right|_{\omega=\pi} = 0 \quad (r = 0, 1, \cdots, L-1) \tag{9}$$

where L is the degree of flatness of the filter such that $0 \le L \le N + M + 1$. In order to satisfy flatness condition (9), it is necessary to place L multiple zeros at $\omega = \pi$. Here, the frequency characteristic in Eq. (5) is rewritten as

$$\hat{H}(e^{j\omega}) = \frac{N(\omega)}{D(\omega)} \tag{10}$$

where

$$N(\omega)=\sum_{n=0}^{N} a_n e^{j(K-2n)\omega}+\frac{1}{2}\sum_{m=0}^{M} b_m e^{-j2m\omega} \tag{11}$$

Therefore, flatness condition (9) is equivalent to

$$\left.\frac{\partial^r N(\omega)}{\partial\omega^r}\right|_{\omega=\pi} = 0 \quad (r = 0, 1, \cdots, L-1) \tag{12}$$

Hence, when the numerator polynomial $N(\omega)$ of $\hat{H}(e^{j\omega})$ is differentiated r times and is substituted into flatness condition (12), we obtain

$$\begin{cases} 2\sum_{n=0}^{N} a_n - \sum_{m=1}^{M} b_m = 1 \\ 2\sum_{n=0}^{N} a_n (K-2n)^r - \sum_{m=1}^{M} b_m (-2m)^r = 0 \\ \qquad\qquad (r = 1, 2, \cdots, L-1) \end{cases} \tag{13}$$

In the case of $L = N + M + 1$, or the maximally flat filter, there exists a unique solution to the above linear equations. When Eq. (13) is solved, the filter coefficients a_n and b_m can be derived easily. Also, by making use of Cramer's rule and the Vandermonde determinant, linear equations (13) can be solved analytically. The closed form solution is given by

$$\begin{cases} a_n = \dfrac{(-1)^{N-n}}{2} \dfrac{M!}{n!(N-n)!} \dfrac{\prod_{i=0}^{N}\left(\frac{K}{2}-i\right)}{\prod_{i=0}^{M}\left(\frac{K}{2}-n+i\right)} \\ b_m = (-1)^m \dbinom{M}{m} \prod_{i=0}^{N} \dfrac{\frac{K}{2}-i}{\frac{K}{2}+m-i} \end{cases} \tag{14}$$

When $M = 0$, the FIR maximally flat half-band filter is obtained and the solution is identical to that in Ref. 12.

3.2. Filter with an arbitrary degree of flatness

As shown in Refs. 9 and 11, the maximally flat filter has both a passband and stopband that are flat, but the transition region is wider. Hence, a design is needed for a filter with a sharper characteristic (namely, a narrower transition region) while the specified degree of flatness is satisfied. In the following, the design of a half-band filer with an arbitrary degree of flatness, namely, the case with $L \le N + M$, is considered.

3.2.1. Initial values of the independent zeros

Since there are $N + M + 1$ coefficients in the IIR half-band filter, the number of independent zeros is $N + M + 1$. As shown in Fig. 1, the number of independent zeros other than $z = -1$ is $N + M + 1 - L$ in the case of $L \le N + M$. In the case of a real coefficient filter, zeros on the unit circle other than $z = \pm 1$ necessarily form complex conjugate pairs. Therefore, $N + M + 1 - L$ must be an even number. Hence, $N + M + 1 - L = 2I$. In order to minimize the amplitude error in the stopband, it is necessary to place all of these remaining independent zeros on the unit circle in the z plane. Hence, in the stopband, $2I$ independent zeros are set as follows:

$$z_i = e^{\pm j\hat{\omega}_i} \quad \left(\frac{\pi}{2} < \hat{\omega}_1 < \hat{\omega}_2 < \cdots < \hat{\omega}_I < \pi\right) \tag{15}$$

From Eq. (4),

$$\hat{H}(z_i) = \frac{1}{2} + \frac{\sum_{n=0}^{N} a_n e^{\pm j(K-2n)\hat{\omega}_i}}{\sum_{m=0}^{M} b_m e^{\pm j(-2m)\hat{\omega}_i}} = 0 \tag{16}$$

If Eq. (16) is separated into the real part and the imaginary part, we obtain

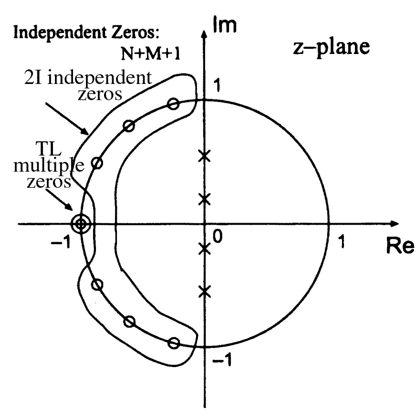

Fig. 1. Pole-zero locations of IIR half-band filters.

$$\begin{cases} 2\sum_{n=0}^{N} a_n \cos\{(K-2n)\hat{\omega}_i\} \\ \qquad + \sum_{m=1}^{M} b_m \cos(2m\hat{\omega}_i) = -1 \\ 2\sum_{n=0}^{N} a_n \sin\{(K-2n)\hat{\omega}_i\} \\ \qquad - \sum_{m=1}^{M} b_m \sin(2m\hat{\omega}_i) = 0 \end{cases} \tag{17}$$

When Eqs. (13) and (17) are combined, a total of $N + M + 1$ linear equations is obtained. Hence, by solving these linear equations, the initial values of the filter coefficients a_n and b_m are obtained.

3.2.2. Formulation by complex Remez algorithm

It is not guaranteed that the amplitude characteristic of the filter with the filter coefficients obtained in Section 3.2.1 provides equal ripples in the stopband. Hence, in order to obtain an equiripple characteristic in the stopband, the filter coefficients obtained in Section 3.2.1 are used as the initial values and then formulated by means of the complex Remez exchange algorithm. From the initial values of the filter coefficients, the frequency characteristic of $\hat{H}(z)$ is derived. In the stopband, I optimum frequency points ω_I are sought as follows and then their phase $\theta(\omega_i)$ is derived:

$$\frac{\pi}{2} < \omega_1 < \omega_2 < \cdots < \omega_I < \pi \tag{18}$$

Next, at these frequency points, the following formulation is used:

$$\hat{H}(e^{j\omega_i}) = \delta e^{j\theta(\omega_i)} \tag{19}$$

where δ is the specified amplitude error and is known. Hence, the design is performed with

$$|\hat{H}(e^{j\omega})| \le \delta \qquad (\omega \in [\omega_s, \pi]) \tag{20}$$

However, the stopband edge frequency ω_s cannot be specified here. When Eq. (5) is substituted into Eq. (19), the following is obtained:

$$\hat{H}(e^{j\omega_i}) = \frac{1}{2} + \frac{\sum_{n=0}^{N} a_n e^{j(K-2n)\omega_i}}{\sum_{m=0}^{M} b_m e^{j(-2m)\omega_i}} = \delta e^{j\theta(\omega_i)} \tag{21}$$

If Eq. (21) is separated into the real part and the imaginary part, we have

$$\begin{cases} 2\sum_{n=0}^{N} a_n \cos\{(K-2n)\omega_i\} + \sum_{m=1}^{M} b_m \{\cos(2m\omega_i) \\ \quad -2\delta\cos\{\theta(\omega_i) - 2m\omega_i\}\} = 2\delta\cos\theta(\omega_i) - 1 \\ 2\sum_{n=0}^{N} a_n \sin\{(K-2n)\omega_i\} - \sum_{m=1}^{M} b_m \{\sin(2m\omega_i) \\ \quad +2\delta\sin\{\theta(\omega_i) - 2m\omega_i\}\} = 2\delta\sin\theta(\omega_i) \end{cases} \tag{22}$$

When Eqs. (13) and (22) are combined, $N + M + 1$ linear equations are obtained. Hence, by solving these linear equations, the filter coefficients a_n and b_m are derived. With the obtained filter coefficients, the frequency characteristic of $\hat{H}(z)$ is derived and the optimum frequencies Ω_i are sought in the stopband and the phase $\theta(\Omega_i)$ is computed. As a result, it is not always the case that the frequency points ω_i and the optimum frequency points Ω_i coincide. Therefore, the obtained optimum frequency points Ω_i are used as the sample frequency points ω_i the next time, and Eqs. (13) and (22) are solved again to derive the filter coefficients. This process is repeated in the iterative calculations. Once the sample frequency points ω_i and the optimum frequency points Ω_i coincide, the equiripple characteristic in the stopband is presumed to have been obtained. In this paper, the filter coefficients obtained in Section 3.2.1 are used as the initial values, and the design algorithm converges after several iterations.

3.2.3. Design algorithm

1. Provide the order of the numerator N, the order of the denominator M, the desired group delay K, the degree of flatness L, and the maximum amplitude error δ in the stopband.

2. The initial zeros $\hat{\omega}_i$ in the stopband are set at an equal interval as in Eq. (15).

3. By solving linear equations (13) and (17), the initial values a_n and b_m of the filter coefficients are derived.

4. With the obtained a_n and b_m, the frequency characteristic of $\hat{H}(z)$ is derived. In the stopband, the optimum frequency points Ω_i are sought and the phase $\theta(\Omega_i)$ is computed.

5. Let $\omega_i = \Omega_i$ $(i = 1, 2, \ldots, I)$.

6. By solving linear equations (13) and (22), the filter coefficients a_n and b_m are derived.

7. With the obtained a_n and b_m, the frequency characteristic of $\hat{H}(z)$ is derived, the optimum frequency points Ω_i are sought in the stopband, and the phase $\theta(\Omega_i)$ is computed.

8. If $|\Omega_i - \omega_i| \le \varepsilon$ $(i = 1, 2, \ldots, I)$ is satisfied, the process is completed. Otherwise the process is returned to

Table 1. Minimum desired group delay K_{min} for stable IIR half-band filters

	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$
$M=0$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$M=1$	1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
$M=2$	1	1	3	5	5	7	7	9	11	11	13	13	15	15	17	19
$M=3$	1	3	3	5	7	7	9	11	11	13	15	15	17	17	19	21
$M=4$	1	3	5	5	7	9	9	11	13	13	15	17	17	19	21	21
$M=5$	1	3	5	5	7	9	11	11	13	15	17	17	19	21	21	23
$M=6$	×	3	5	7	7	9	11	13	15	15	17	19	19	21	23	25
$M=7$	×	3	5	7	9	9	11	13	15	17	17	19	21	23	23	25
$M=8$	×	×	5	7	9	11	11	13	15	17	19	19	21	23	25	25
$M=9$	×	×	5	7	9	11	11	13	15	17	19	21	21	23	25	27
$M=10$	×	×	×	7	9	11	13	13	15	17	19	21	23	23	25	27
$M=11$	×	×	×	7	9	11	13	15	15	17	19	21	23	25	25	27
$M=12$	×	×	×	×	9	11	13	15	17	17	19	21	23	25	27	27
$M=13$	×	×	×	×	×	11	13	15	17	19	19	21	23	25	27	29
$M=14$	×	×	×	×	×	11	13	15	17	19	21	21	25	25	27	29
$M=15$	×	×	×	×	×	×	13	15	17	19	21	23	25	25	27	29

5. Here, ε is the allowable convergence value given (in general $\varepsilon = 1.0 \times 10^{-10}$).

4. Filter Stability

For the filter to be causally stable, all poles must be located within the unit circle. In the design of a digital filter, it is known that the group delay increases if more poles of the filters are located inside the unit circle rather than outside the unit circle. Hence, if all poles are placed in the unit circle, it is necessary to provide a group delay larger than a certain value. In this paper, the stability of the IIR half-band filter designed here is studied by changing the group delay K. Table 1 shows the minimum desired group delay K_{min} when the IIR half-band filter is causally stable. First, for $M = 0$, the filter is always stable because it is a FIR half-band filter. In order to satisfy causality, the minimum group delay is $K_{min} = 1$. Next, in the case of $M = N$, the filter coefficients in the numerator and denominator become symmetric by virtue of Eq. (14), so that a half-band filter using an all-pass filter is realized [7, 8]. Thus, the minimum group delay for a stable operation is $K_{min} = 2N - 1$. Also, in the case of $0 < M < N$, it is found that $N \le K_{min} \le 2N - 1$. On the other hand, in the case of $M > N$, it is found that $2N - 1 \le K_{min} \le 2N + 1$. There is no stable filter for $M >> N$. In Table 1, this is indicated by ×. In this case, no stable filter is obtained even if the group delay is made large.

5. Applications to Filter Bank

Recently, research on filter banks and wavelets has been carried out vigorously and has been applied in various areas of signal processing [3, 6]. Here, as an application example of the IIR half-band filter, a design of a two-channel filter bank is described.

In this paper, the method of design of the IIR half-band filter described in Section 3 is applied to the two-channel perfect reconstruction filter bank proposed in Ref. 7. In the two-channel filter bank, if $H_0(z)$ and $H_1(z)$ are considered to be decomposition filters and $G_0(z)$ and $G_1(z)$ are combining filters, then the perfect reconstruction condition of the filter bank is

$$\begin{cases} H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K_o-1} \\ H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \end{cases} \tag{23}$$

where K_0 is an integer. From Ref. 7,

$$\begin{cases} H_0(z) = z^{-2K_2} + Q(z^2)H_1(z) = G_1(-z) \\ H_1(z) = \dfrac{1}{2}\{z^{-2K_1-1} - P(z^2)\} = -G_0(-z) \end{cases} \tag{24}$$

is used, so that perfect reconstruction condition (23) can be satisfied. Here, K_1 and K_2 are integers such that $K_0 = K_1 + K_2$. Also, $P(z)$ and $Q(z)$ are assumed to be IIR filters.

5.1. Design of a high-pass filter

Next, the design of a high-pass filter $H_1(z)$ is described. From Eq. (24), the transfer function of $H_1(z)$ is

$$\begin{aligned} H_1(z) &= \frac{1}{2}\{z^{-2K_1-1} - P(z^2)\} \\ &= z^{-2K_1-1}\hat{H}_1(-z) \end{aligned} \tag{25}$$

Here,

$$\hat{H}_1(z) = \frac{1}{2}\{1 + z^{2K_1+1}P(z^2)\} \tag{26}$$

From Eq. (26), $\hat{H}_1(z)$ becomes an IIR half-band filter. Hence, by using the design method described in Section 3, $\hat{H}_1(z)$ is designed as a low-pass filter, and a high-pass filter $H_1(z)$ can then be obtained from Eq. (25).

5.2. Design of a low-pass filter

Next, let us present the design of a low-pass filter $H_0(z)$. As shown in Ref. 7, $H_0(z)$ becomes a half-band filter similarly. Therefore, it can be designed by the design method presented in Section 3. However, since $H_0(z)$ is affected by $H_1(z)$, equiripple characteristics are not guaranteed in the stopband [9, 11]. Hence, to make the amplitude of $H_0(z)$ equiripple, the effect of $H_1(z)$ is taken into account and $\hat{H}_0(z)$ is designed as a low-pass filter as follows:

$$\begin{aligned} \hat{H}_0(z) &= \frac{z^{2K_2}}{2} H_0(z) \\ &= \frac{1}{2} + z^{2(K_2-K_1)-1} Q(z^2)\hat{H}_1(-z) \end{aligned} \tag{27}$$

In the stopband of $H_0(z)$, the amplitude of $\hat{H}_1(-z)$ is ideally constant. In practice, however, error exists. Hence, when $H_0(z)$ is designed by using the complex Remez exchange algorithm, the formulation is carried out with $\hat{H}_0(z)$ in Eq. (27) containing $\hat{H}_1(-z)$. Then, an equiripple characteristic can be obtained in the stopband [9, 11].

6. Design Examples

[Design example 1] The order of the numerator N = 8, the order of the denominator M = 2, the desired group delay K = 13, the degree of flatness L = 3, and the maximum

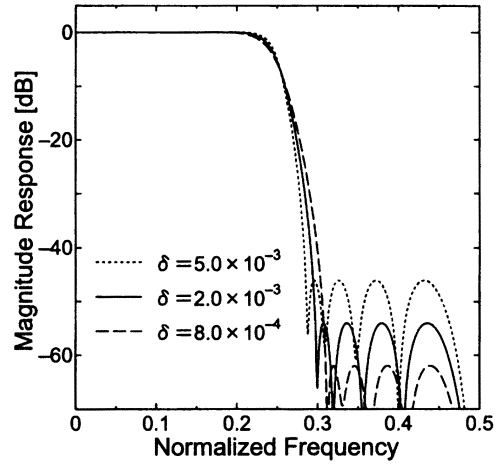

Fig. 3. Magnitude responses of IIR half-band filters in Example 1.

amplitude error $\delta = 2.0 \times 10^{-3}$ are given to design an IIR half-band filter. Its impulse response is shown in Fig. 2. It is found that an IIR half-band filter satisfying the time domain condition and causality stability is obtained. The amplitude characteristics and the group delay characteristics of the obtained filter are shown with solid lines in Figs. 3 and 4. Also, the amplitude and group delay characteristics are presented for $\delta = 5.0 \times 10^{-3}$ and $\delta = 8.0 \times 10^{-4}$. From Figs. 3 and 4, it is found that the specified design conditions are satisfied and the transition region becomes narrower as the specified error becomes larger.

[Design example 2] The order of the numerator N = 5, the order of the denominator M = 3, the desired group delay K = 9, the maximum amplitude error $\delta = 1.0 \times 10^{-2}$, and the degree of flatness L = 5 are provided and an IIR half-band filter is designed. From Table 1, the obtained IIR half-band filter becomes stable if the desired group delay is more than K = 7. The amplitude and group delay charac-

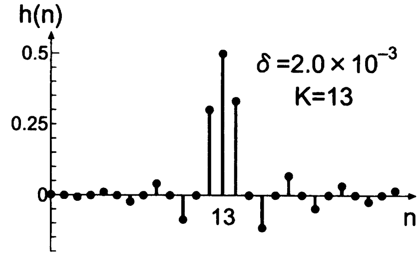

Fig. 2. Impulse response of IIR half-band filter in Example 1.

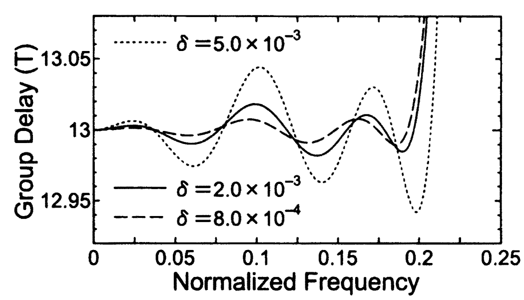

Fig. 4. Group delay of IIR half-band filters in Example 1.

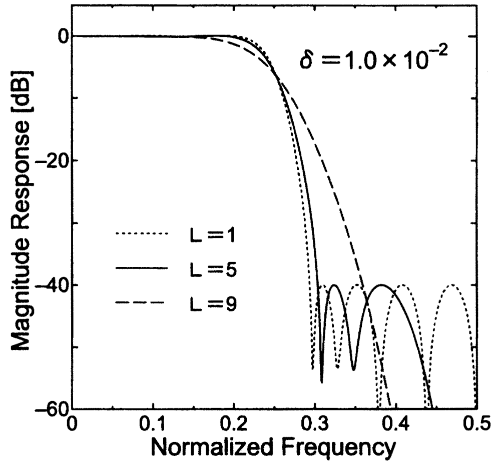

Fig. 5. Magnitude responses of IIR half-band filters in Example 2.

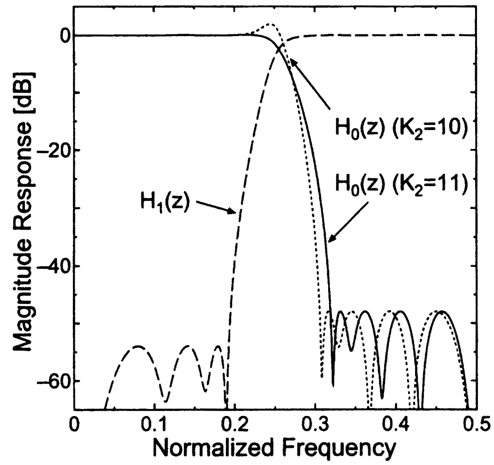

Fig. 8. Magnitude responses of analysis filters in Example 3.

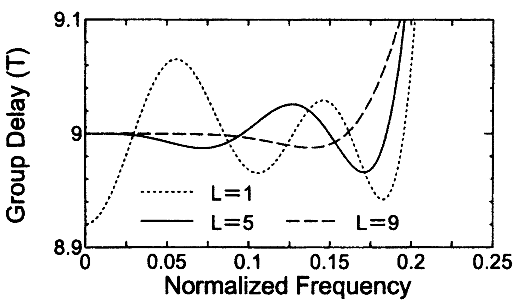

Fig. 6. Group delays of IIR half-band filters in Example 2.

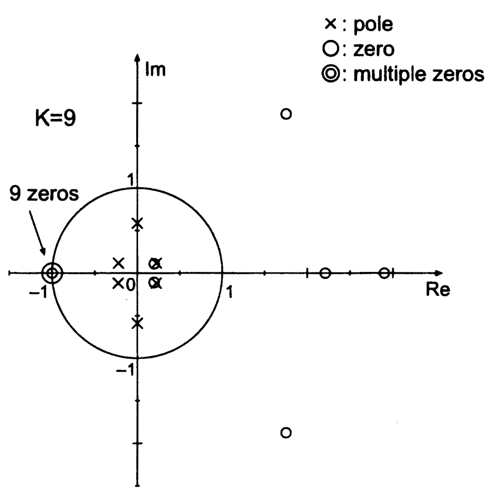

Fig. 7. Pole-zero location of the maximally flat IIR half-band filter in Example 2.

teristics of the obtained filter are shown with solid lines in Figs. 5 and 6. The amplitude and group delay characteristics for $L = 1$ and $L = 9$ are also provided. In the case of $L = 9$, an IIR maximally flat half-band filter is obtained and the transition region is the widest. The pole and zero locations of this maximally flat filter are shown in Fig. 7.

[Design example 3] The order of the numerator $N_1 = 6$ and the order of the denominator $M_1 = 2$ for $P(z)$ and the order of the numerator $N_2 = 7$ and that of the denominator $M_2 = 2$ for $Q(z)$, the degree of flatness $L_1 = 3$ for $H_1(z)$, that of flatness $L_2 = 2$ for $H_0(z)$, their group delays $K_1 = 4$ and $K_2 = 11$, and the maximum amplitude errors of $\delta_1 = 2.0 \times 10^{-3}$ and $\delta_2 = 4.0 \times 10^{-3}$ are given and a two-channel filter bank is designed. Here, from Table 1, the obtained filter bank is stable. The amplitude characteristic of the decomposing filter is shown in Fig. 8 while the group delay characteristic is shown in Fig. 9. When the group delay of

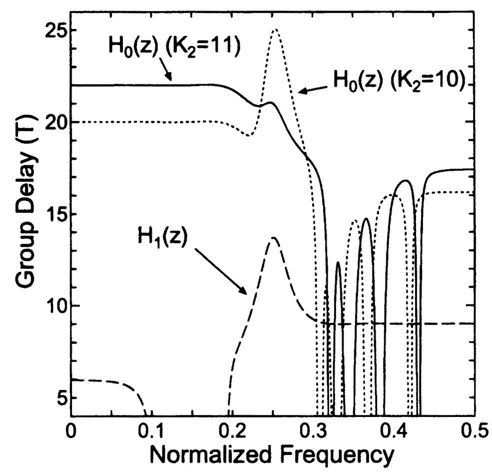

Fig. 9. Group delays of analysis filters in Example 3.

$H_0(z)$ is $K_2 = 10$, the corresponding amplitude and group delay characteristics are shown with dotted lines. It is seen from Fig. 8 that the overshoot of $H_0(z)$ in the transition region can be suppressed by controlling the group delay K_2 of $H_0(z)$.

7. Conclusions

In this paper, a new design method is proposed for an IIR half-band filter that provides an arbitrary given degree of flatness and can be designed with a specified maximum amplitude error in the stopband. First, the relationship of the frequency characteristics of the IIR half-band filter in the passband and the stopband is studied. It is shown that the problem of filter design can be reduced to that of the minimization of the maximum amplitude error in the stopband. Next, it is shown that a maximally flat half-band filter can be obtained analytically by imposing the flatness condition on the stopband of the filter. In the case where the degree of flatness is arbitrary, the complex Remez exchange algorithm is applied to the stopband so that the filter design problem is formulated in the form of linear equations. Hence, by solving simple linear equations, the filter coefficients can be derived easily. After several iterations, an equiripple solution satisfying the specified maximum amplitude error can be obtained. The unique feature of the proposed design method is that the degree of flatness of the filter and the maximum amplitude error in the stopband can be given arbitrarily. Also, the stability of the IIR half-band filter is discussed. The minimum group delay for a causally stable filter is clarified. Finally, the present design method is applied to the design of a two-channel perfect reconstruction filter bank so that the effectiveness of the method is demonstrated.

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