

Design of Chebyshev-Type IIR Filters with Approximately Linear Phase Characteristics

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SUMMARY

Digital filters with a linear phase characteristic are needed in many applications for signal processing. In this paper, design of a Chebyshev-type IIR filter with approximate linear phase characteristics in the passband is described. First, it is shown that a flat stopband can easily be realized by placing multiple zeros at the specified frequency points in the stopband. Next, the complex Remez exchange algorithm is applied to the passband so that the filter design problem is formulated as an eigenvalue problem. Hence, by solving the eigenvalue problem, the filter coefficients can be derived easily. Further, by means of iterative calculations, an equiripple characteristic of the error function in the passband is obtained. Finally, it is shown that a reverse Chebyshev-type IIR filter with an approximate linear phase characteristic can also be obtained by parallel connection of the proposed Chebyshev-type filter and a delay line. © 2003 Wiley Periodicals, Inc. *Electron Comm Jpn Pt 3*, 87(2): 1–9, 2004; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ecjc.10133

Key words: IIR digital filter; Chebyshev-type filter; approximate linear phase characteristic; eigenvalue problem; complex Remez exchange algorithm.

1. Introduction

Like analog filters, digital filters are classified into four types—maximally flat type, Chebyshev type, reverse

Chebyshev type, and elliptic type—and are widely used in many applications [1, 2]. The maximally flat filter has flat characteristics both in the passband and in the stopband while the elliptic-type filter has equiripple characteristics in both stop- and passbands. On the other hand, the Chebyshev-type and reverse Chebyshev-type filters have an equiripple characteristic in either the pass- or stopband but have a flat characteristic in the other. These filters are needed in many image processing applications in order to suppress ringing and chessboard distortion [2, 3]. In the design of these digital filters, the most general way is to make use of conventional analog filters [1, 2]. However, IIR filters obtained from analog filters by the s - z transform have the same orders in the numerator and the denominator. In addition, the phase characteristics of the filter cannot be specified [1, 2]. Hence, it is necessary to design the digital filters directly in the z domain. Many design methods have been proposed to date [4–14]. Among them, there are design methods taking into consideration the phase characteristics of the passband at the same time [7, 12, 14]. The linear phase characteristic of the filter is needed in many applications for digital signal and image processing. As is well known, a perfect linear phase characteristic can easily be realized in an FIR filter by imposing a symmetry condition on the filter coefficients [8, 11]. In the case of an IIR filter, a perfect linear phase characteristic cannot be realized due to causality. In addition to the amplitude characteristic, the phase characteristic must be approximated [7, 12, 14]. Hence, let us consider the design of Chebyshev-type IIR filters with an approximately linear phase characteristic in the passband.

In this paper, a design method is presented for a Chebyshev-type IIR linear phase filter with a flat charac-

teristic in the stopband and an equiripple characteristic in the passband. In the present design method, it is shown that a flat characteristic in the stopband can easily be realized by placing multiple zeros at specified frequency points in the stopband. Hence, the filter design problem is reduced to a problem of approximation of the frequency characteristic in the passband. Next, the complex Remez exchange algorithm is applied to the passband, formulating the filter design problem in the form of a generalized eigenvalue problem [14]. The eigenvalue minimizing the maximum amplitude of the error function is derived. Its corresponding eigenvector specifies the filter coefficients. Further, by repeating the above calculation process, an equiripple characteristic of the error function in the passband can be obtained. It is also shown that a reverse Chebyshev-type IIR filter with an approximately linear phase characteristic in the passband can be realized at the same time by parallel connection of the proposed Chebyshev-type filter and a delay section. Finally, design examples of the IIR Chebyshev-type low-pass filter and stopband filter are presented, clarifying the usefulness of the present design method.

2. The Chebyshev-Type IIR Filter

The transfer function $H(z)$ of the IIR digital filter is defined as follows:

$$H(z) = \frac{\sum_{n=0}^N a_n z^{-n}}{\sum_{m=0}^M b_m z^{-m}} \quad (1)$$

where N and M are the orders of the numerator and the denominator. The filter coefficients a_n and b_n are real and $b_0 = 1$.

In the Chebyshev-type filter, the amplitude characteristic is required to be flat in the stopband. Hence,

$$\left. \frac{\partial^k |H(e^{j\omega})|}{\partial \omega^k} \right|_{\omega=\omega_s} = 0 \quad (k = 0, 1, \dots, K-1) \quad (2)$$

where ω_s is the frequency point specified in the stopband and K is a parameter expressing the flatness. Also, in the passband, the desired frequency characteristic of the filter is $H_d(e^{j\omega})$, so that

$$H_d(e^{j\omega}) = |H_d(e^{j\omega})| e^{j\theta_d(\omega)} \quad (\omega \in \text{passband}) \quad (3)$$

where $|H_d(e^{j\omega})|$ is the desired amplitude characteristic in the passband and $\theta_d(\omega)$ is the desired phase characteristic. In

the case of a linear phase filter, $\theta_d(\omega)$ has linear phase and the group delay is constant.

The difference between the frequency characteristic of the filter and the desired frequency characteristic is the error function $E(\omega)$ defined as

$$E(\omega) = W(\omega)[H(e^{j\omega}) - H_d(e^{j\omega})] \quad (4)$$

where $W(\omega)$ is a weighting function. Hence, the design target is to minimize the maximum amplitude of the error function $E(\omega)$. Therefore,

$$\min\{\max_{\omega} |E(\omega)|\} \quad (5)$$

3. Design of IIR Low-Pass Filter

Let us next describe the design of an IIR low-pass filter. The flatness condition in the stopband is given by Eq. (2). Here, $\omega_s = \pi$ in the case of the low-pass filter. To satisfy this flatness condition, it is necessary to place K multiple zeros at $\omega = \pi$. Hence, the transfer function $H(z)$ is

$$H(z) = \frac{(1 + z^{-1})^K \sum_{n=0}^{N-K} c_n z^{-n}}{\sum_{m=0}^M b_m z^{-m}} \quad (6)$$

If transfer function (6) is used, it is found that the flatness characteristic in the stopband can be realized. Therefore, the problem of design of a low-pass filter is reduced to the problem of approximation of the frequency characteristic in the passband.

3.1. Setting of the initial value

The desired frequency characteristic of a linear phase filter in the passband is

$$H_d(e^{j\omega}) = e^{-j\tau\omega} \quad (0 \leq \omega \leq \omega_p) \quad (7)$$

where τ is the desired group delay and ω_p is the passband edge frequency. In transfer function (6), the number of unknown filter coefficients is $J = M + N - K + 1$. First, $L = \lceil J/2 \rceil$ initial frequency points $\hat{\omega}_i$ ($\omega_p > \hat{\omega}_1 > \hat{\omega}_2 > \dots > \hat{\omega}_L \geq 0$) are selected in the passband $[0, \omega_p]$. Here, $\lceil * \rceil$ denotes the minimum integer larger than $*$. If J is even, $\hat{\omega}_i$ ($\hat{\omega}_L \neq 0$) is selected at equal intervals as shown in Fig. 1(a). If J is odd, $\hat{\omega}_i$ is selected from $\hat{\omega}_L = 0$ as shown in Fig. 1(b). Next, since the design target is minimization of the maximum amplitude of the error function, the amplitude of the error function at these frequency points $\hat{\omega}_i$ is zero:

$$E(\hat{\omega}_i) = W(\hat{\omega}_i)[H(e^{j\hat{\omega}_i}) - H_d(e^{j\hat{\omega}_i})] = 0 \quad (8)$$

When Eq. (8) is substituted into Eq. (7), we have

$$D(\hat{\omega}_i) \sum_{n=0}^{N-K} c_n e^{-j(n+\frac{K}{2})\hat{\omega}_i} - \sum_{m=0}^M b_m e^{-j(\tau+m)\hat{\omega}_i} = 0 \quad (9)$$

Here, $D(\omega) = (2\cos\frac{\omega}{2})^K$. Since $b_0 = 1$, Eq. (9) is separated into the real part and the imaginary part:

$$D(\hat{\omega}_i) \sum_{n=0}^{N-K} c_n \cos\left(\left(n + \frac{K}{2}\right)\hat{\omega}_i\right) - \sum_{m=1}^M b_m \cos((\tau + m)\hat{\omega}_i) = \cos(\tau\hat{\omega}_i) \quad (10)$$

$$D(\hat{\omega}_i) \sum_{n=0}^{N-K} c_n \sin\left(\left(n + \frac{K}{2}\right)\hat{\omega}_i\right) - \sum_{m=1}^M b_m \sin((\tau + m)\hat{\omega}_i) = \sin(\tau\hat{\omega}_i) \quad (11)$$

If J is even, the numbers of Eqs. (10) and (11) are L in both cases so that the total is $2L = J$. If J is odd, $\hat{\omega}_L = 0$, the number of Eqs. (11) is one fewer than the number of Eqs. (10) and is $L - 1$. Therefore, the total is $2L - 1 = J$. Hence, by solving Eqs. (10) and (11), the filter coefficients c_n and b_m are uniquely determined.

3.2. Formulation by complex Remez exchange algorithm

From the filter coefficients obtained in Section 3.1, the error function $E(\omega)$ is derived and its extremum frequency points ω_i ($\omega_p = \omega_1 > \omega_2 > \dots \geq 0$) are sought. As a result, the obtained error function is not necessarily an equiripple characteristic. Hence, by using the complex Remez exchange algorithm, a formulation is carried out to make the function an equiripple characteristic. As shown in Fig. 1, the number of extremum frequency points ω_i is $L + 1$ if J is even and L if J is odd (including the passband edge frequency ω_p). Let $\theta_e(\omega_i)$ be the phase of the error function at the extremum frequency points ω_i . At these extremum frequency points ω_p , the amplitudes of the error function are made equal in the formulation:

$$E(\omega_i) = W(\omega_i)[H(e^{j\omega_i}) - H_d(e^{j\omega_i})] = \delta e^{j\theta_e(\omega_i)} \quad (12)$$

where δ is the amplitude of the error function. Hence, if Eq. (12) is separated into the real part and the imaginary part, the following are obtained:

$$D(\omega_i) \sum_{n=0}^{N-K} c_n \cos\left(\left(n + \frac{K}{2}\right)\omega_i\right) - \sum_{m=0}^M b_m \cos((\tau + m)\omega_i) = \frac{\delta}{W(\omega_i)} \sum_{m=0}^M b_m \cos(m\omega_i - \theta_e(\omega_i)) \quad (13)$$

$$D(\omega_i) \sum_{n=0}^{N-K} c_n \sin\left(\left(n + \frac{K}{2}\right)\omega_i\right) - \sum_{m=0}^M b_m \sin((\tau + m)\omega_i) = \frac{\delta}{W(\omega_i)} \sum_{m=0}^M b_m \sin(m\omega_i - \theta_e(\omega_i)) \quad (14)$$

It is found from Fig. 1 that the number of Eqs. (14) is fewer by one than the number of Eqs. (13) because $\omega_{L+1} = 0$ if J is even. Hence, when Eqs. (13) and (14) are combined, there are $2L + 1 = J + 1$ equations. When J is odd, the total is $2L = J + 1$, because $\omega_L > 0$. Hence, the number of Eqs. (13) and (14) is $J + 1$ in each case regardless of whether J is even or odd. If Eqs. (13) and (14) are expressed in matrix form, they are reduced to the following generalized eigenvalue problem [14]:

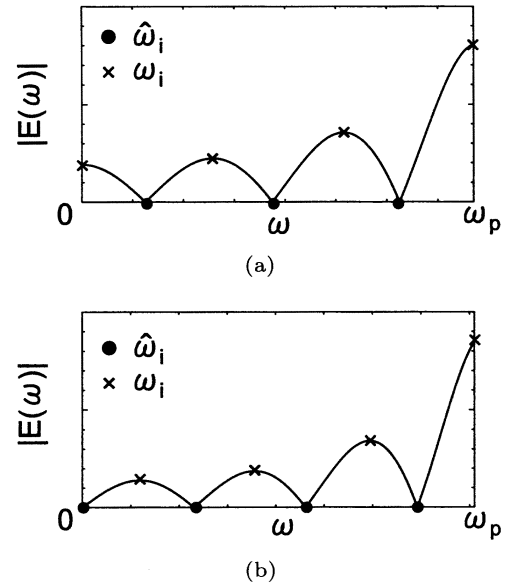


Fig. 1. Selection of initial frequency points for low-pass filters. (a) Even J ; (b) odd J .

$$\mathbf{P}\mathbf{x} = \delta\mathbf{Q}\mathbf{x} \quad (15)$$

where $\mathbf{x} = [b_0, b_1, \dots, b_M, c_0, c_1, \dots, c_{N-K}]^T$. The elements P_{ij} and Q_{ij} of the matrices $\mathbf{P}$ and $\mathbf{Q}$ are

$$\begin{cases} P_{ij} = -\sin((\tau + j)\omega_{i+1}) \\ Q_{ij} = \frac{\sin(j\omega_{i+1} - \theta_e(\omega_{i+1}))}{W(\omega_{i+1})} \end{cases}$$

for $0 \leq i \leq L-1, 0 \leq j \leq M$,

$$\begin{cases} P_{ij} = D(\omega_{i+1}) \sin\left(\left(j - M - 1 + \frac{K}{2}\right)\omega_{i+1}\right) \\ Q_{ij} = 0 \end{cases}$$

for $0 \leq i \leq L-1, M+1 \leq j \leq J$,

$$\begin{cases} P_{ij} = -\cos((\tau + j)\omega_{i+1-L}) \\ Q_{ij} = \frac{\cos(j\omega_{i+1-L} - \theta_e(\omega_{i+1-L}))}{W(\omega_{i+1-L})} \end{cases}$$

for $L \leq i \leq J, 0 \leq j \leq M$, and

$$\begin{cases} P_{ij} = D(\omega_{i+1-L}) \\ \quad \cdot \cos\left(\left(j - M - 1 + \frac{K}{2}\right)\omega_{i+1-L}\right) \\ Q_{ij} = 0 \end{cases}$$

for $L \leq i \leq J, M+1 \leq j \leq J$.

Therefore, solving eigenvalue problem (15) and deriving the eigenvalue minimizing the amplitude δ of the error function yields the corresponding eigenvector that provides the filter coefficients b_m and c_n . From the obtained filter coefficients, the error function is calculated and a new extremum frequency point ω_i is sought. Then, the phase $\theta_e(\omega_i)$ of the error function at such a frequency point is derived. These extremum frequency points are replaced as the sample frequency points of the next iteration. Then, an iterative calculation is carried out until the error function becomes an equiripple characteristic. As shown in Ref. 14, the design algorithm described above uses the complex Remez exchange algorithm so that convergence is not guaranteed. Depending on the design specification, convergence may not be reached. Convergence of the algorithm strongly depends on the initial values described in Section 3.1. Good convergence has been confirmed in many design examples. If there is a case of nonconvergence, the initial frequency points $\hat{\omega}_i$ may be set nearer to the edge of the passband instead of equally spaced, so that convergence can be improved. A specific design algorithm is presented below.

3.3. Design algorithm

(1) Provide the orders N and M of the numerator and denominator, the flatness K , the desired group delay τ , and the passband edge frequency ω_p .

(2) Set L initial frequency points $\hat{\omega}_i$ in the passband as shown in Fig. 1.

(3) By solving linear Eqs. (10) and (11), the filter coefficients c_n and b_m are derived. Then, the extremum frequency points ω_i of $E(\omega)$ are sought and the phases $\theta_e(\omega_i)$ are computed.

(4) The matrices $\mathbf{P}$ and $\mathbf{Q}$ are computed and eigenvalue problem (15) is solved, deriving the filter coefficients c_n and b_m .

(5) Using the obtained c_n and b_m , the extremum frequency points Ω_i are sought and the phases $\theta_e(\omega_i)$ are derived.

(6) The process is terminated if $|\Omega_i - \omega_i| < \varepsilon$. Otherwise, proceed to the next step. Here, ε is the specified convergence tolerance value and is usually $\varepsilon = 10^{-10}$.

(7) With $\omega_i = \Omega_i$, the process returns to step (4).

3.4. Stability of the IIR filter

In the design algorithm presented in Section 3.3, the stability condition of the IIR filter is not considered as a design condition. Hence, there is a possibility that the designed IIR filter may be unstable. However, as is proved in Refs. 4 and 5, the stability of the IIR filter depends on the design specifications. Hence, if a group delay of more than a certain value is given, stability can be guaranteed. Hence, when design specifications are given, a stable IIR filter can be designed provided that a sufficiently large group delay is given. For a specific design example, Ref. 14 should be consulted.

4. Design of IIR Stopband Filter

In Section 3, the design of a low-pass filter is described. A high-pass filter can be derived from the designed low-pass filter by frequency transformation. Hence, a high-pass filter can be readily obtained by changing z to $-z$ in the transfer function. In the following, the design of an IIR stopband filter is considered. The flatness condition in the stopband can be defined by Eq. (2), where $0 < \omega_s < \pi$. In order to satisfy the flatness condition of the stopband, it is necessary to place K zeros at $\omega = \pm\omega_s$:

$$H(z) = \frac{(1 - 2\cos\omega_s z^{-1} + z^{-2})^K \sum_{n=0}^{N-2K} c_n z^{-n}}{\sum_{m=0}^M b_m z^{-m}} \quad (16)$$

By using transfer function (16), a flat stopband can be realized. The desired frequency characteristic $H_d(e^{j\omega})$ in the passband is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\tau\omega} & (0 \leq \omega \leq \omega_{p1}) \\ e^{-j(\tau\omega - \theta_0)} & (\omega_{p2} \leq \omega \leq \pi) \end{cases} \quad (17)$$

In the case of a real coefficient filter, the phase of the filter at $\omega = \pi$ is $\theta(\pi) = n\pi$ (n is an integer). Therefore, it is necessary that $\theta_0 = (n + \tau)\pi$. Therefore, the complex Remez exchange algorithm can be applied to the passband for design as in the case of the low-pass filter. Here, in the case of the stopband filter, there are two passbands. The setting of the initial frequency points is shown in Fig. 2. The number of unknown filter coefficients in Eq. (16) is $J = M + N - 2K + 1$. If J is even, then L of $\hat{\omega}_i$ are selected such that $0 < \hat{\omega}_i < \pi$. The number of generated extremum frequency points ω_i is $L + 2$ (including $\omega_i = 0$ and π). Since the number needed for the formulation by the complex Remez exchange algorithm is $L + 1$, either $\omega_i = 0$ or $\omega_i = \pi$ with a smaller amplitude of the error function is eliminated. The remaining $L + 1$ extremum frequency points are used as the sample frequency points. When J is odd, either $\hat{\omega}_i = 0$ or $\hat{\omega}_i = \pi$ is selected and then L sample frequency points are selected such that $\omega_i \neq 0, \pi$ from the generated extremum frequency points. Also, with regard to the bandpass filter, the design method described above cannot be applied directly; this situation will be studied in the future.

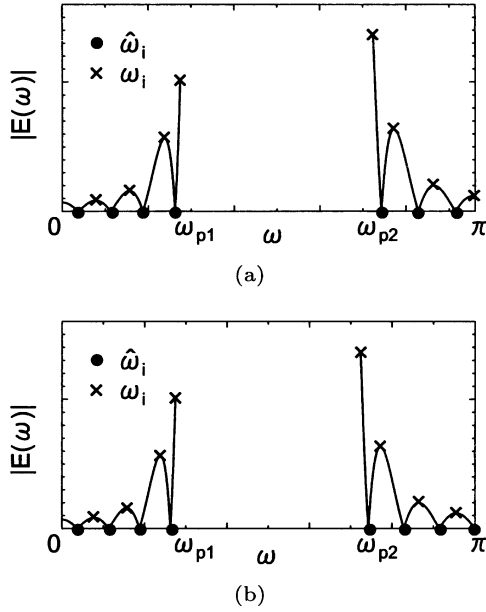


Fig. 2. Selection of initial frequency points for bandstop filters. (a) Even J ; (b) odd J .

5. The IIR Reverse Chebyshev-Type Filter

A Chebyshev-type filter $H(z)$ designed by the above design method and a delay line z^{-1} are connected in parallel as shown in Fig. 3 to construct a new filter $G(z)$. The transfer function of $G(z)$ is

$$G(z) = z^{-I} - H(z) \quad (18)$$

where I is an integer. Now $H(z)$ is designed by letting the desired group delay be $\tau = I$. In the case of the stopband filter, let $\theta_0 = 0$. Hence, it is found from Eq. (18) that the passband of $H(z)$ becomes the stopband of $G(z)$ with an equiripple characteristic. Also, the amplitude of $H(z)$ is 0 in the stopband. Therefore, this band becomes the passband of $G(z)$. Since $H(z)$ has a flat characteristic in the stopband, the amplitude characteristic $|G(e^{j\omega})|$ and the group delay $\tau(\omega)$ of $G(z)$ satisfy

$$\begin{cases} |G(e^{j\omega_s})| = 1 \\ \left. \frac{\partial^k |G(e^{j\omega})|}{\partial \omega^k} \right|_{\omega=\omega_s} = 0 \quad (k = 1, 2, \dots, K-1) \end{cases} \quad (19)$$

$$\begin{cases} \tau(\omega_s) = I \\ \left. \frac{\partial^k \tau(\omega)}{\partial \omega^k} \right|_{\omega=\omega_s} = 0 \quad (k = 1, 2, \dots, K-2) \end{cases} \quad (20)$$

and are flat. Hence, an IIR reverse Chebyshev-type filter with a flat group delay can be realized at the same time. However, since the desired group delay τ is limited by the integer I , a reverse Chebyshev-type filter with a noninteger delay cannot be designed with this configuration.

6. Design Examples

[Design Example 1] As the design specifications, let $N = 15$, $M = 6$, $K = 10$, and $\omega_p = 0.3\pi$. Then, a Chebyshev-type low-pass filter with $\tau = 12$ is designed. The amplitude characteristic of the error function of the obtained filter is plotted in Fig. 4 as the solid line. It is found that an

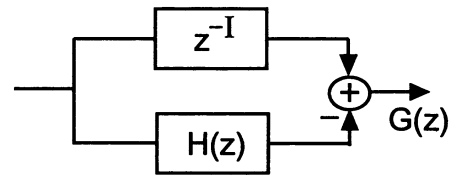


Fig. 3. Parallel connection of $H(z)$ and a delay section.

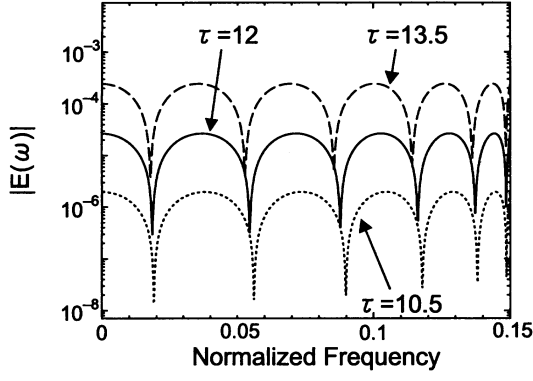


Fig. 4. Magnitude responses of error functions in Example 1.

equiripple characteristic is realized. The amplitude characteristics of the error functions of the filters designed with $\tau = 10.5$ and $\tau = 13.5$ are also presented. The amplitude and group delay characteristics of these filters are shown in Figs. 5 and 6. All of the obtained filters are stable. The pole-zero locations of the filter with $\tau = 12$ are shown in Fig. 7. Also, from the Chebyshev-type low-pass filter $H(z)$ with $\tau = 12$, a reverse Chebyshev-type high-pass filter $G(z)$ is derived. The amplitude and group delay characteristics of $G(z)$ are shown in Figs. 8 and 9.

[Design Example 2] As the design specifications, let $N+M=19$, $K=10$, $\tau=11$, and $\omega_p=0.48\pi$. Design examples of Chebyshev-type low-pass filters with N and M varied are shown. In the case of $M=0$, this yields an FIR filter. Since the group delay is not half the order, the filter is not a

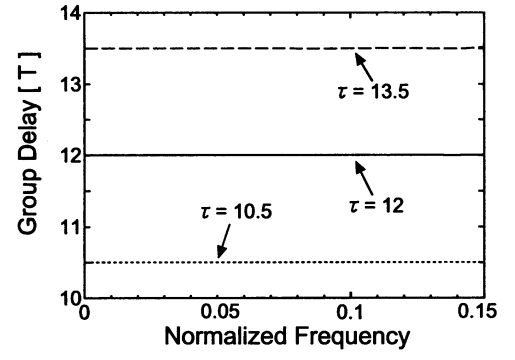


Fig. 6. Group delays of low-pass filters in Example 1.

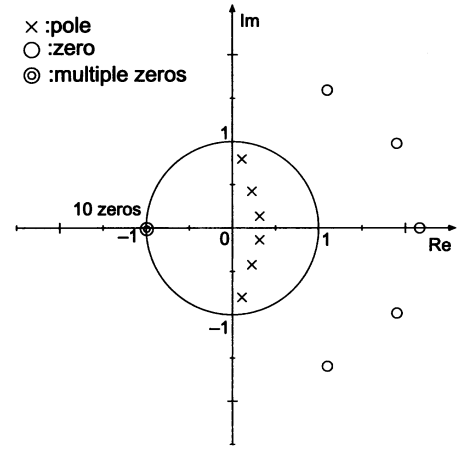


Fig. 7. Pole-zero location of low-pass filter in Example 1.

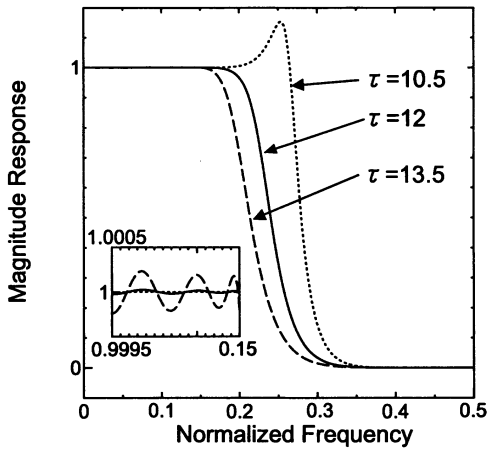


Fig. 5. Magnitude responses of low-pass filters in Example 1.

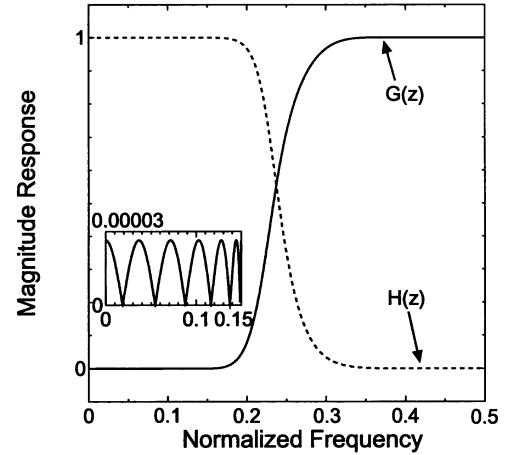


Fig. 8. Magnitude response of high-pass filter in Example 1.

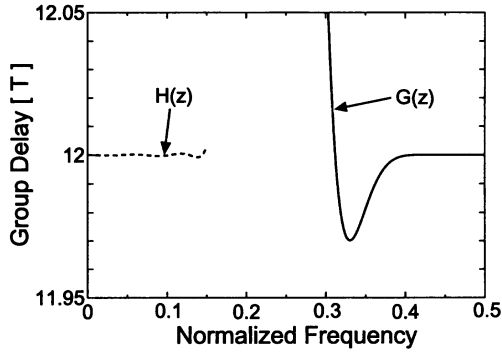


Fig. 9. Group delay of high-pass filter in Example 1.

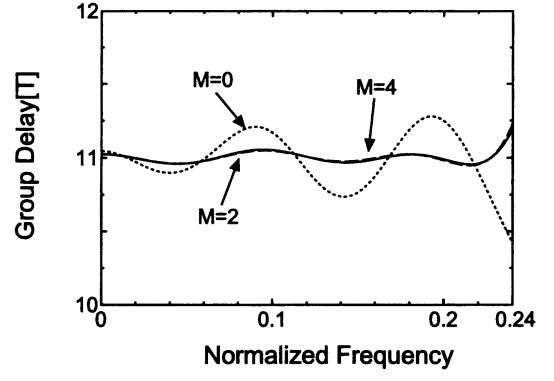


Fig. 12. Group delays of low-pass filters in Example 2.

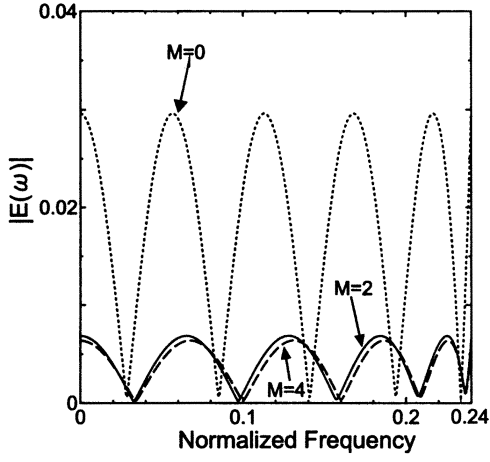


Fig. 10. Magnitude responses of error functions in Example 2.

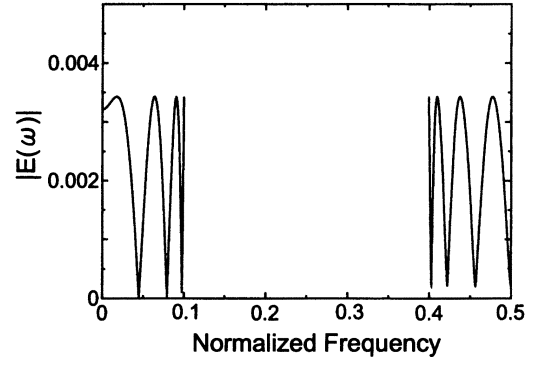


Fig. 13. Magnitude response of error function in Example 3.

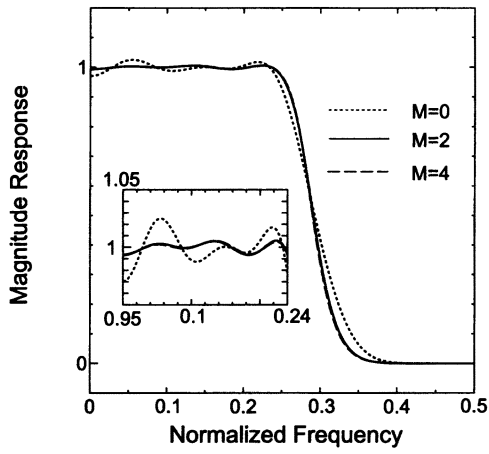


Fig. 11. Magnitude responses of low-pass filters in Example 2.

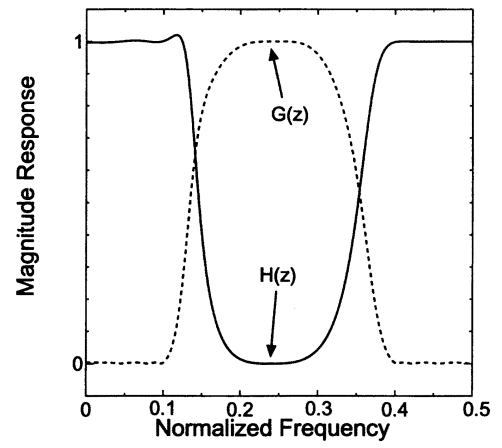


Fig. 14. Magnitude responses of bandstop and bandpass filters in Example 3.

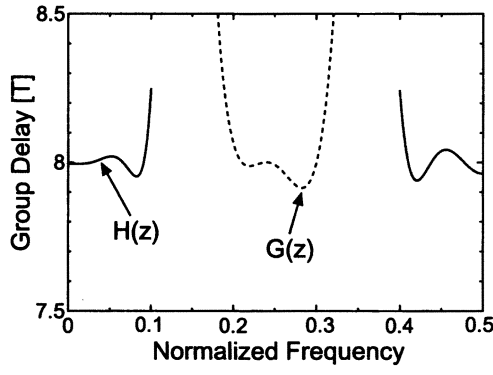


Fig. 15. Group delays of bandstop and bandpass filters in Example 3.

perfectly linear phase FIR filter. The amplitude characteristic of the error function of the designed filter is shown in Fig. 10, where it is seen that an equiripple characteristic can be obtained. In comparison with the FIR filter with $M = 0$, the filter with $M = 4$ has the smallest error. The amplitude and group delay characteristics of the obtained low-pass filter are shown in Figs. 11 and 12. The IIR filters with $M = 2$ and $M = 4$ are found to have smaller amplitude and group delay errors than the FIR filter with $M = 0$.

[Design Example 3] As the design specifications, let $N = M = 10$, $K = 3$, $\tau = 8$, $\theta_0 = 0$, $\omega_{p1} = 0.2\pi$, $\omega_{p2} = 0.8\pi$, and $\omega_s = 0.48\pi$. Then, a Chebyshev-type stopband filter is designed. In this design example, four initial frequency points $\hat{\omega}_i$ are placed in the passband $[0, \omega_{p1}]$ and another four in the passband $[\omega_{p2}, \pi]$. The amplitude characteristic of the error function of the designed filter is shown in Fig. 13 and is found to be of equiripple type. The amplitude and group delay characteristics of the filter are plotted in Figs. 14 and 15 as solid lines. The amplitude and group delay characteristics of the reverse Chebyshev-type bandpass filter $G(z)$ obtained from Eq. (18) are plotted in Figs. 14 and 15 as dotted lines.

7. Conclusions

In this paper, a design method has been proposed for a Chebyshev-type IIR filter with an approximately linear phase characteristic in the passband. In this design method, it is first shown that a flat characteristic in the stopband can be easily realized by placing multiple zeros at the specified frequency points in the stopband. Next, the complex Remez exchange algorithm is applied to the passband and the filter design problem is formulated in the form of a generalized eigenvalue problem. Hence, by solving the eigenvalue problem, the filter coefficients can be readily obtained.

Further, by iterative calculations, an equiripple characteristic of the error function in the passband can be obtained. In addition, by parallel connection of the proposed Chebyshev-type filter and a delay section, a reverse Chebyshev-type IIR filter with an approximately linear phase characteristic in the passband can be realized at the same time. Finally, design examples are presented for IIR Chebyshev-type low-pass filters and a stopband filter, demonstrating the usefulness of the present design method. Future topics of study include the design of a reverse Chebyshev-type linear phase filter with noninteger delay.

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