LIFTING IMPLEMENTATION OF IIR ORTHOGONAL WAVELET FILTER BANKS USING ALLPASS FILTERS

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ABSTRACT

The lifting scheme is well-known to be an efficient tool for constructing second generation wavelets, and is often used to design a class of biorthogonal wavelet filter banks. For its efficiency, the lifting implemantation has been also adopted in the international standard JPEG2000. It is known that the orthogonality of wavelets is an important property for many applications. This paper presents how to implement two band IIR orthogonal wavelet filter banks according to the lifting scheme. It is shown that a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. Thus, the proposed filter banks have approximate linear phase responses. Finally, the proposed IIR orthogonal wavelet filter banks are applied to image lossless compression, and the coding performance is investigated.

1. INTRODUCTION

The lifting scheme proposed in [7] and [8] is an efficient tool for constructing second generation wavelets. It is shown in [9] and [10] that every wavelet transform with FIR filters can be decomposed into a finite number of lifting steps, thus this allows the construction of an integer version of the transform. Such integer wavelet transforms are invertible, and then are attractive in lossless coding applications. Due to these properties, the lifting implemantation has been adopted in the international standard JPEG2000 [3],[12]. However, it is not always possible for wavelet transforms with IIR filters to be realized by a finite number of lifting steps. In general, the lifting scheme is used to construct a class of biorthogonal wavelet filter banks. It is known that the orthogonality of wavelets is an important property for many applications of signal and image processing. Therefore, we will consider the realization of orthogonal wavelet filter banks by using the lifting scheme.

This paper presents how to implement two band IIR orthogonal wavelet filter banks by using the lifting scheme. In the proposed method, a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. Thus, the resultant IIR filter banks have approximate linear phase responses. Next, the proposed IIR orthogonal wavelet filter banks are applied to image lossless compression. The coding performance is investigated, and compared with the wavelet transform supported by the baseline codec of JPEG2000. It can be seen from the experimental results that the proposed IIR orthogonal wavelet filter banks can achieve a better coding performance than the conventional wavelet transform.

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2. LIFTING SCHEME

The lifting scheme proposed by W. Sweldens in [7] and [8] is used to construct second generation wavelets, and has many advantages such as faster implementation, fully in-place calculation, reversible integer-to-integer transforms, and so on. In this paper, we consider the lifting scheme with two lifting steps shown in Fig.1. In Fig.1, $P(z)$ is a prediction operator and $Q(z)$ is an update operator. The circles labelled with the symbol R are quantizers that round outputs of $P(z)$ and $Q(z)$. It is clear that a reversible integer-to-integer transform can be easily realized in the lifting scheme, since the inverse wavelet transform is immediately derived. Therefore, the lifting scheme is attractive in lossless coding applications, where the original image can be completely restored after elongating the compressed image.

Let $H_0(z)$ and $H_1(z)$ be a pair of lowpass and highpass filters in the analysis bank. Their transfer functions are given by

$$\begin{cases} H_0(z) = 1 + Q(z^2)H_1(z) \\ H_1(z) = z^{-1} - P(z^2) \end{cases} . \tag{1}$$

Then the design problem of this filter bank is how to determine two transfer functions $P(z)$ and $Q(z)$ to satisfy the given specification. Conventionally, FIR filters are often used in the design of $P(z)$ and $Q(z)$, resulting in a class of biorthogonal wavelet filter banks. In the following, we will describe how to design a class of orthogonal wavelet filter banks by using IIR allpass filters in the lifting scheme.

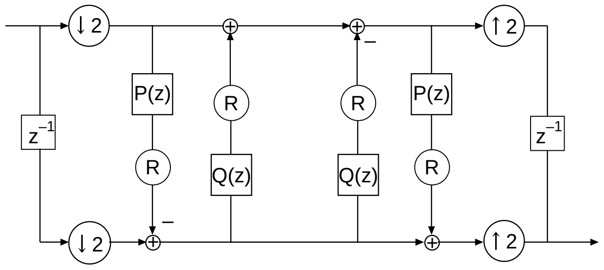

Fig. 1. Lifting scheme

3. IIR ORTHOGONAL WAVELET FILTER BANKS

It is well-known that the orthogonality of wavelets is an important property for many applications of signal and image processing. The orthogonality condition of two band filter banks is given by

$$\begin{cases} H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = c_1 \\ H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) = c_2 \\ H_0(z)H_1(z^{-1}) + H_0(-z)H_1(-z^{-1}) = 0 \end{cases}, \quad (2)$$

where c_1 and c_2 are constants, and $c_1 = c_2 = 2$ for the orthonormal filter banks.

First, by substituting $H_1(z)$ of Eq.(1) into Eq.(2), we have

$$\begin{aligned} &H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) \\ &= 2 + 2P(z^2)P(z^{-2}) \\ &= c_2, \end{aligned} \quad (3)$$

thus,

$$P(z)P(z^{-1}) = \frac{c_2}{2} - 1, \quad (4)$$

that is,

$$|P(e^{j\omega})|^2 = \frac{c_2}{2} - 1, \quad (5)$$

which means that $P(z)$ should possess a constant magnitude response at all frequencies. Therefore, $P(z)$ must be chosen to be an allpass filter. For $H_1(z)$ to be highpass, we set $c_2 = 4$ to ensure $P(z)$ with unit gain, and then, $P(z)$ is obtained by

$$P(z) = z^M A(z) = z^{M-N} \frac{\sum_{n=0}^{N} a_n z^n}{\sum_{n=0}^{N} a_n z^{-n}}, \quad (6)$$

where a_n is real, and $a_0 = 1$. Therefore, we get

$$H_1(z) = z^{-1} - z^{2M} A(z^2). \quad (7)$$

Next, we derive $H_0(z)$ from Eq.(2),

$$\begin{aligned} &H_0(z)H_1(z^{-1}) + H_0(-z)H_1(-z^{-1}) \\ &= 4Q(z^2) - 2P(z^{-2}) \\ &= 0, \end{aligned} \quad (8)$$

thus,

$$Q(z) = \frac{1}{2}P(z^{-1}) = \frac{z^{-M}}{2}A(z^{-1}). \quad (9)$$

Then, $H_0(z)$ becomes

$$H_0(z) = \frac{1}{2}\{1 + z^{-2M-1}A(z^{-2})\}, \quad (10)$$

which satisfies

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 1. \quad (11)$$

Therefore, the orthogonality condition in Eq.(2) is satisfied, and a class of orthogonal wavelet filter banks is obtained by using an allpass filter $A(z)$ in the lifting step.

4. DESIGN OF IIR ORTHOGONAL FILTER BANKS

It can be seen in Eqs.(7) and (10) that the design problem of the proposed IIR filter banks has been reduced to the phase approximation of the allpass filter $A(z)$. Assume that $\theta(\omega)$ is the phase response of $A(z)$. The frequency responses of $H_0(z)$ and $H_1(z)$ are given by

$$\begin{cases} H_0(e^{j\omega}) = \cos[(M + \frac{1}{2})\omega + \frac{\theta(2\omega)}{2}]e^{-j[(M+\frac{1}{2})\omega + \frac{\theta(2\omega)}{2}]} \\ H_1(e^{j\omega}) = \frac{2}{j}\sin[(M + \frac{1}{2})\omega + \frac{\theta(2\omega)}{2}]e^{j[(M-\frac{1}{2})\omega + \frac{\theta(2\omega)}{2}]} \end{cases}. \quad (12)$$

For $H_0(z)$ and $H_1(z)$ to be a pair of lowpass and highpass filters, the phase response $\theta(\omega)$ must satisfy

$$(M + \frac{1}{2})\omega + \frac{\theta(2\omega)}{2} = \begin{cases} 0 & (0 \le \omega \le \omega_p) \\ \pm\frac{\pi}{2} & (\omega_s \le \omega \le \pi) \end{cases}, \quad (13)$$

where ω_p and ω_s are the edge frequencies of $H_0(z)$ in the passband and stopband, respectively, and $\omega_p + \omega_s = \pi$. Therefore, the desired phase response of $A(z)$ is

$$\theta_d(\omega) = -(M + \frac{1}{2})\omega \qquad (0 \le \omega \le 2\omega_p). \quad (14)$$

Once the phase response $\theta(\omega)$ of $A(z)$ is approximated to the desire phase response in Eq.(14), it is clear from Eq.(12) that both $H_0(z)$ and $H_1(z)$ have approximate linear phase responses. Note that $H_0(z)$ and $H_1(z)$ have the passband gain 1 and 2, respectively, which are the same as the wavelet filter banks supported by the baseline codec of JPEG2000.

It is known in [1] that a flat frequency response of the filter is generally required for the regularity of wavelets. In the maxflat design of allpass filters, it has been shown in [11] that the filter coefficients a_n can be analytically determined, and the closed-form solution is given by

$$a_n = \binom{N}{n} \prod_{i=1}^{n} \frac{N - M - i + \frac{1}{2}}{M + i + \frac{1}{2}}. \quad (15)$$

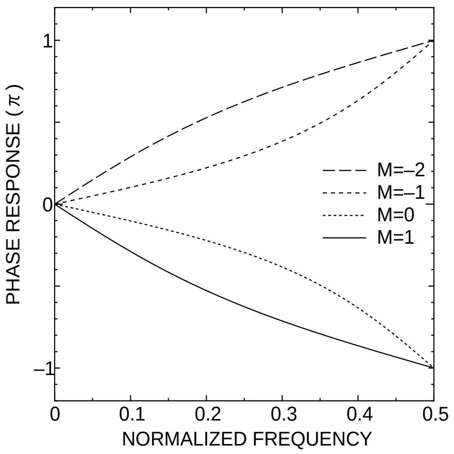

Fig. 2. Phase responses of $A(z)$.

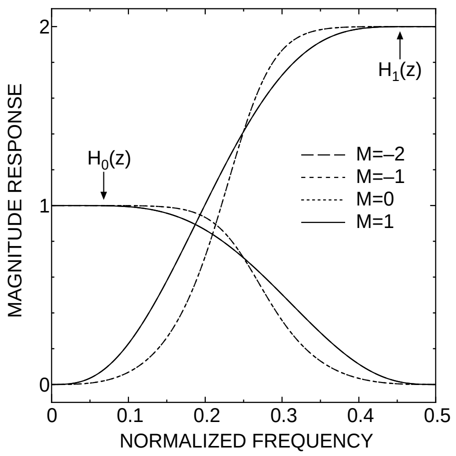

Fig. 3. Magnitude responses of $H_0(z)$ and $H_1(z)$.

Next, we examine the filter property of the proposed IIR orthogonal wavelet filter banks $H_0(z)$ and $H_1(z)$. By way of an example, we consider the design of the filters with $N = 1$, and have designed $A(z)$ with various M. It is found that when $M > N$ or $M < -(N + 1)$, we cannot obtain a pair of reasonable lowpass and highpass filters. We then consider only the filters with $-(N + 1) \leq M \leq N$ below. The obtained phase responses of $A(z)$ are shown in Fig.2, while the magnitude responses of $H_0(z)$ and $H_1(z)$ are shown in Fig.3. It is seen in Fig.2 that the phase responses of $A(z)$ with $M = k$ and $M = -(k + 1)$ for $k = 0, 1, \cdots, N$ are symmetrical, and we have found that their poles each other satisfy the mirror-image relation with respect to the unit circle. Thus, the filter banks $H_0(z), H_1(z)$ with $M = k$ and $M = -(k + 1)$ have the same magnitude responses, as shown in Fig.3. For example, when $M = 1$ and $M = -2$, their magnitude responses are the same in Fig.3. Therefore, we will use only the filter banks with $0 \leq M \leq N$ in image coding application.

Table 1. Lossless coding results: Bit Rate (bpp)

[N=1]	M=0	M=1
Barbara	**4.567**	4.735
Lena	**4.337**	4.414
Mandrill	**6.140**	6.193
Zelda	**3.974**	4.058
Average	**4.754**	4.850

Table 2. Lossless coding results: Bit Rate (bpp)

[N=2]	M=0	M=1	M=2
Barbara	**4.503**	4.512	4.627
Lena	4.333	**4.327**	4.362
Mandrill	**6.128**	**6.128**	6.154
Zelda	3.955	**3.954**	3.998
Average	**4.730**	**4.730**	4.785

5. IMAGE CODING APPLICATION

In this section, we apply the proposed IIR orthogonal wavelet filter banks to image lossless coding, and investigate the lossless coding performance. We have used the reference software of JPEG2000 provided in [14] to evaluate the performance of the proposed filters. Since the proposed IIR filters have only approximate linear phase responses, a simple extension technique where the signals are extended by repetition of their first and last sample values is employed to handle filtering at the signal boundaries, instead of the symmetric extension. The images Barbara, Lena, Mandrill and Zelda of size 512×512, 8 bpp are used as test images. The decomposition level of wavelet transform is set to 6.

5.1. Influence of N and M

First, we investigate the influence of N and M on the lossless coding performance. The lossless coding results of the filters with $N = 1$, $N = 2$ and $N = 3$ are given in Table 1, Table 2 and Table 3, respectively. For each image, the best result has been highlighted. When $N = 1$, it is seen that the filter with $M = 0$ has a lower bit rate than that with $M = 1$. This is because the magnitude response of the filter with $M = 0$ is better, as shown in Fig.3. The similar results can be found also in the cases of $N = 2$ and $N = 3$. Thus, $M = 0$ is chosen in general. It can be seen in Table 4 that the filter with $N = 3$ has the best coding performance. This is because the regularity of wavelets increases with an increasing filter order N of allpass filters. When N is further increased, we can get only a little improvement. However, the computational complexity required in the implementation of the proposed wavelet transforms becomes higher.

Table 3. Lossless coding results: Bit Rate (bpp)

[N=3]	M=0	M=1	M=2	M=3
Barbara	**4.486**	4.496	4.494	4.574
Lena	4.333	4.337	**4.326**	4.345
Mandrill	**6.127**	6.130	**6.127**	6.139
Zelda	**3.946**	3.949	3.949	3.978
Average	**4.723**	4.728	4.724	4.759

Table 4. Comparison of lossless coding results (bpp)

	D-5/3	N1-M0	N2-M0	N3-M0
Barbara	4.697	4.567	4.503	**4.486**
Lena	4.350	4.337	**4.333**	**4.333**
Mandrill	6.151	6.140	6.128	**6.127**
Zelda	4.021	3.974	3.955	**3.946**
Average	4.805	4.754	4.730	**4.723**

5.2. Comparison with the conventional wavelet

Now, we compare the proposed IIR orthogonal wavelet filter banks with the conventional wavelet filter banks. The reversible integer-to-integer wavelet transform D-5/3 supported by the baseline codec of JPEG2000 is chosen as a comparison object. The lossless coding comparison results are shown in Table 4. It is clear from the experimental results in Table 4 that the proposed IIR orthogonal wavelet filter banks with approximate linear phase responses have a better lossless coding performance than the conventional wavelet transform D-5/3.

6. CONCLUSION

In this paper, we have presented how to implement two band IIR orthogonal wavelet filter banks by using the lifting scheme. It has been shown that a class of IIR orthogonal wavelet filter banks can be realized by using allpass filters in the lifting steps. As a result, the proposed IIR orthogonal wavelet filter banks have approximate linear phase responses. Moreover, we have applied the proposed IIR orthogonal wavelet filter banks to image lossless compression, and investigated the lossless coding performance. The coding results have been also compared with the wavelet transform supported by the baseline codec of JPEG2000. It is seen from the experimental results that the proposed IIR orthogonal wavelet filter banks with approximate linear phase responses can achieve a better coding performance than the conventional wavelet transform.

7. REFERENCES

[1] I.Daubechies, "Ten Lectures on Wavelets," SIAM, Philadelphia, PA, 1992.

[2] M.Vetterli and J.Kovacevic, "Wavelets and Subband Coding," Prentice Hall PRT, Upper Saddle River, New Jersey, 1995.

[3] D.S.Taubman and M.W.Marcellin, "JPEG2000: Image Compression Fundamentals, Standards and Practice," Kluwer Academic Publishers, Boston, 2002.

[4] M.J.T.Smith and S.L.Eddins, "Analysis/synthesis techniques for subband image coding," IEEE Trans. Acoust., Speech & Signal Processing, vol.38, no.8, pp.1446–1456, Aug. 1990.

[5] M.Antonini, M.Barlaud, P.Mathieu and I.Daubechies, "Image coding using wavelet transform," IEEE Trans. Image Processing, vol.1, no.2, pp.205–220, April 1992.

[6] C.Herley and M.Vetterli, "Wavelets and recursive filter banks," IEEE Trans. Signal Processing, vol.41, no.8, pp.2536–2556, Aug. 1993.

[7] W.Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," Appl. Comput. Harmon. Anal., vol.3, no.2, pp.186–200, 1996.

[8] W.Sweldens, "The lifting scheme: A construction of second generation wavelets," SIAM J.Math. Anal., vol.29, no.2, pp.511–546, 1997.

[9] I.Daubechies and W.Sweldens, "Factoring wavelet transforms into lifting steps," J.Fourier Anal. Appl., vol.4, pp.247–269, 1998.

[10] A.R.Calderbank, I.Daubechies, W.Sweldens and B.L.Yeo, "Wavelet transforms that map integers to integers," Appl. Comput. Harmon. Anal., vol.5, no.3, pp.332–369, 1998.

[11] X.Zhang, T.Muguruma, and T.Yoshikawa, "Design of orthonormal symmetric wavelet filters using real allpass filters," Signal Processing, Vol.80, No.8, pp.1551–1559, Aug. 2000.

[12] ISO/IEC 155444-1: Information technology – JPEG 2000 image coding system – Part 1: Core coding system, 2000.

[13] ISO/IEC 155444-2: Information technology – JPEG 2000 image coding system – Part 2: Extensions, 2002.

[14] ISO/IEC 155444-5: Information technology – JPEG 2000 image coding system – Part 5: Reference software, 2002.